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THE NASTRAN DEMONSTRATION PROBLEM MANUAL

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INTRODUCTION

The Demonstration Problem Manual is one of four manuals that constitute the documentation for NASTRAN. The other three are the Theoretical Manual, the User's Manual, and the Programmer's Manual.

The Theoretical Manual contains discussions of the mathematical operations and the underlying theory relative to the engineering equations used. There is some discussion relative to data processing techniques, software organization, and accuracy achieved.

The User's Manual is an instructional and encyclopedic reference that describes the finite element modeling features available, defines the input data formats, and shows how to prepare data to obtain solutions in several engineering disciplines.

The Programmer's Manual contains descriptions of the Functional Modules, subroutines, and operating systems from a software point of view. It also contains detailed derivations of the mathematical equations employed by the program.

The Demonstration Problem Manual provides the NASTRAN user with simple solutions to specific problems illustrating applications of all rigid formats. The problems are presented so that the translation of the engineering problem data into a NASTRAN formulation is explained. Theoretical solutions to the problems are included where possible to serve as validation of the NASTRAN results.

This manual is organized into the following main sections:

1. NASTRAN Demonstration Problems on UMF Tape - a list of the problems demonstrated
2. Demonstrated Features of NASTRAN - tables of the features and characteristics of the problems
3. Demonstration Problems by Rigid Format - detailed discussions of each problem

The problems included in the Demonstration Problem Manual illustrate nearly all the available elements, coordinate system types, constraints, loadings, and analytical capabilities available with NASTRAN. A complete listing of NASTRAN features and tabular displays of features versus demonstration problem number are provided for ease of user reference.

The Demonstration Problems by Rigid Format discussions are organized into five subsections. The Description section provides a summary of the engineering aspects of the problem and relates the problem to the appropriate NASTRAN rigid format for solution. The Input section presents the key parameter values for the problem including structural dimensions, material properties, boundary conditions, and loading conditions. The Theory section summarizes theoretical and/or experimental solutions for the problem if available. The Results section provides a summary of answers computed by NASTRAN and results of theoretical calculations or experiments. Under the Driver Decks and Sample Bulk Data section, are the Executive Control, Substructure Control, and Case Control decks for each problem followed by a sample of the bulk data cards used to formulate the model. The bulk data card lists are not complete since this data is not input in card form but is obtained from the UMF tape.

The input data for the demonstration problems described herein are provided to the user on a NASTRAN User Master File (UMF) which is a part of the NASTRAN system delivery. This file is provided so that users may check the system installation by executing characteristic problems with NASTRAN. (The method of accessing these data is described in the UMF Tape section of this manual and in Section 2.5 of the User's Manual.) When executing a problem from the UMF, the user must allocate required operating system resources for necessary tape and disk files.

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NASTRAN DEMONSTRATION PROBLEMS ON UMF TAPE

The Bulk Data Decks required to execute the demonstration problems are provided on tape as a NASTRAN User's Master File (UMF). Other files provided as part of the NASTRAN system tape delivery include the driver decks which contain the Executive Control, Substructure Control, and Case Control Decks and a print file of the execution results. The delivery tapes are made compatible with each of the three computer systems used. See the Programmer's Manual Section 5 for descriptions of the delivered tapes for each computer system.

The UMB provides a convenient means for storing large volumes of NASTRAN bulk data in a standardized format. Since there is a data edit capability, the complete bulk data card listings may be obtained by printing the contents of the UMF. A complete description of how to use the UMF is found in Section 2.5 of the User's Manual. Executing any of the demonstration problems described in this manual requires the corresponding driver decks from the delivered tape and then running a standard NASTRAN job using the UMF file as described in the User's Manual. (An alternative to executing a NASTRAN UMF job is to output the delivered print file for the problem via operating system utilities.)

Each demonstration problem is assigned a problem number to key it to the Rigid Format. The UMF problem identification (pid) is an adaptation of that problem number. The UMF tape identification (tid) is the year in which the set of demonstration problems was generated. This would tend to change from one release of NASTRAN to the next as new capabilities are introduced into the system. Furthermore, it may not always be possible to execute a given UMF on a previous level due to changes in data handling techniques.

The UMF problem identification number (pid) is made up of four elements: The Rigid Format number, the problem number, the version number, and a trailing dummy zero. Thus, the general UMF number is xxyyz0. The Rigid Format number is the first one or two digits (easily seen by ignoring the last four digits); the problem number is always two digits; the version number is always one digit; and the 0 always trails to allow insertion of additional problem versions in the future. A UMF pid of 10210 means the problem runs on Rigid Format 1 (xx = 1~~0210~~), it is the second demonstration problem on that Rigid Format (yy = 02), and it is version 1 (z = 1) of that problem. Another example, 110110, is a problem for Rigid Format 11, problem 1, version 1. A table of pid numbers for each demonstration problem follows:

Restart problem driver decks do not contain a UMF card because the data is already stored on a checkpoint tape which must have been created by the user. The restart problems thus do not have a pid and are indicated by RESTART in the table. For the restart problems, all changes to the sorted UMF bulk data cards are shown with the driver decks.

UMF pid	NASTRAN DEMONSTRATION PROBLEMS ON UMF TAPE
10110	Delta Wing with Biconvex Cross Section, Load on Trailing Edge
RESTART	Delta Wing with Biconvex Cross Section, Load on Leading Edge
RESTART	Delta Wing with Biconvex Cross Section, Switch to Rigid Format 3
10120	Delta Wing with Biconvex Cross Section Using QDMEM1 and QDMEM2 Elements
10130	Delta Wing with Biconvex Cross Section Using QDMEM1 Elements
10140	Delta Wing with Biconvex Cross Section Using QDMEM2 Elements
10210	Spherical Shell with Pressure Loading, No Moments on Boundary
RESTART	Spherical Shell with Pressure Loading, Clamped Boundary
10310	Free Rectangular QDMEM Plate with Thermal Loading
10320	Free Rectangular QDMEM1 Plate with Thermal Loading
10330	Free Rectangular QDMEM2 Plate with Thermal Loading
10410	Long, Narrow, 5x50 Orthotropic Plate
RESTART	Long, Narrow, 5x90 Orthotropic Plate, Modified Model
10420	Long, Narrow, 5x60 Orthotropic Plate
10430	Long, Narrow, 5x50 Orthotropic Plate (via INPUT Module)
10440	Long, Narrow, 5x60 Orthotropic Plate (via INPUT Module)
10510	Nonsymmetric Bending of a Cylinder of Revolution
10610	Solid Disc with Radially Varying Thermal Load
10710	Shallow Spherical Shell Subjected to External Pressure Loading
10810	Bending of a Beam Fabricated with HEXA1 Solid Elements
10910	Thermal and Applied Loads on HEXA2 Solid Elements
10920	Thermal and Applied Loads on TRIM6 Higher Order Membrane Elements
11010	Thermal Bending of a Bar
11110	Simply-Supported Rectangular Plate with a Thermal Gradient
11120	Simply-Supported Rectangular Plate with a Thermal Gradient (via INPUT Module)
11210	Linear Steady State Heat Conduction Through a Washer Using Solid Elements
11220	Linear Steady State Heat Conduction Through a Washer Using Ring Elements
11310	Thermal and Pressure Loads on a Long Pipe Using Linear Isoparametric Elements
11320	Thermal and Pressure Loads on a Long Pipe Using Quadratic Isoparametric Elements
11330	Thermal and Pressure Loads on a Long Pipe Using Cubic Isoparametric Elements
11410	Static Analysis on a Beam Using General Elements

UMF pid	NASTRAN DEMONSTRATION PROBLEMS ON UMF TAPE
11510	Asymmetric Pressure Loading of an Axisymmetric Cylindrical Shell
11610	Fully Stressed Design of a Plate with a Reinforced Hole
11710	Rectangular Plate with Variable Moduli of Elasticity
20110	Inertia Relief Analysis of A Circular Ring Under Concentrated and Centrifugal Loads
20210	Windmill Panel Sections for Multi-stage Substructuring (Run 1, Phase 1)
20220	Windmill Panel Sections for Multi-stage Substructuring (Run 2, Phase 1)
20230	Windmill Panel Sections for Multi-stage Substructuring (Run 3, Phase 1)
20240	Windmill Panel Sections for Multi-stage Substructuring (Run 4, Phase 2)
20250	Windmill Panel Sections for Multi-stage Substructuring (Run 5, Phase 3)
20260	Windmill Panel Sections for Multi-stage Substructuring (Run 6, Phase 3)
20270	Windmill Panel Sections for Multi-stage Substructuring (Run 7, Phase 2)
30110	Vibration of a 10x20 Plate
30120	Vibration of a 20x40 Plate
30130	Vibration of a 10x20 Plate (via INPUT Module)
30140	Vibration of a 20x40 Plate (via INPUT Module)
30210	Vibration of a Compressible Gas in a Rigid Spherical Tank
30310	Vibration of a Liquid in a Half Filled Rigid Sphere
30410	Acoustic Cavity Analysis
30510	Nonlinear Heat Transfer in an Infinite Slab
30610	Nonlinear Radiation and Conduction of a Cylinder
30710	Vibrations of a Linearly Tapered Cantilever Plate
30810	Helicopter Main Rotor Pylon on a Rigid Body Fuselage
40110	Differential Stiffness Analysis of a Hanging Cable
50110	Symmetric Buckling of a Cylinder
50210	Buckling of a Tapered Column Fixed at the Base
60110	Piecewise Linear Analysis of a Cracked Plate
70110	Complex Eigenvalue Analysis of a 500-Cell String
70120	Complex Eigenvalue Analysis of a 500-Cell String (via INPUT Module)
70210	Third Harmonic Complex Eigenvalue Analysis of a Gas-Filled Thin Elastic Cylinder
70220	Fifth Harmonic Complex Eigenvalue Analysis of a Gas-Filled Thin Elastic Cylinder
80110	Frequency Response of a 10x10 Plate
80120	Frequency Response of a 20x20 Plate

UMF pid	NASTRAN DEMONSTRATION PROBLEMS ON UMF TAPE
80130	Frequency Response of a 10x10 Plate (via INPUT Module)
80140	Frequency Response of a 20x20 Plate (via INPUT Module)
90110	Transient Analysis with Direct Matrix Input
90210	Transient Analysis of a 1000-Cell String, Traveling Wave Problem
90220	Transient Analysis of a 1000-Cell String, Traveling Wave Problem (via INPUT Module)
90310	Transient Analysis of a Fluid-Filled Elastic Cylinder
90410	Linear Transient Heat Transfer in a Plate
100110	Complex Eigenvalue Analysis of a Rocket Control System
100210	Aeroelastic Flutter Analysis of a 15° Swept Wing
110110	Frequency Response and Random Analysis of a Ten Cell Beam
RESTART	Frequency Response and Random Analysis of a Ten Cell Beam, Enforced Deformation and Gravity Load
110210	Frequency Response of a 500-Cell String
110310	Jet Transport Wing Dynamic Analysis, Frequency Response
110320	Jet Transport Wing Dynamic Analysis, Transient Response
120110	Transient Analysis of a Free One Hundred Cell Beam
130110	Normal Modes Analysis of a One Hundred Cell Beam with Differential Stiffness
140110	Static Analysis of a Circular Plate Using Dihedral Cyclic Symmetry
150110	Normal Modes Analysis of a Circular Plate Using Rotational Cyclic Symmetry

DEMONSTRATED FEATURES OF NASTRAN

This section provides a summary of the modeling, execution, and output control features which are demonstrated. Tables are provided to give the user a convenient reference for locating the Demonstration Problem Number for a problem which illustrates a particular feature. The features are categorized according to:

A. Physical Problems

B. Solution Methods

C. Element Types

D. Constraints

E. Geometry and Property Definitions

F. Special Matrix Options

G. Loading Options

H. Execution Options

I. Output Options

Each of these categories is expanded to include available options which may be correlated to the demonstration problem(s) illustrating the feature. The demonstration problem numbers are indicated by two- or three-digit reference numbers. A two-digit number is used to indicate there is more than one demonstration problem version exhibiting the same feature. A three-digit number means the demonstration problem is the only one used to illustrate a particular feature. For example, there are three versions of NASTRAN Demonstration Problem No. 1-13 (1-13-1, 1-13-2, and 1-13-3). Since all three versions use solid elements, the entry for this modeling feature is indicated by the series designation, 1-13. However, since only two of these versions use element congruency, that feature is indicated by referencing the problem versions 1-13-1 and 1-13-2.

DEMONSTRATED FEATURES OF NASTRAN

A. PHYSICAL PROBLEMS

Structures

1. Line
2. Plate or Shell
3. Solids
4. Rotational Symmetry

Fluid Dynamics

5. Flexible Boundary
6. Rigid Boundary
7. Sloshing
8. Acoustic
9. Aeroelastic

Heat Transfer

10. Conduction
11. Convection
12. Radiation

B. SOLUTION METHODS

Steady State

1. Linear Statics
2. Inertia Relief
3. Nonlinear Geometry
4. Material Plasticity
5. Fully Stressed Design
6. Linear Heat Transfer
7. Nonlinear Heat Transfer

Eigenvalue Analysis

8. Real Modes
9. Complex Modes
10. Inverse Power
11. FEER
12. Determinant
13. Givens
14. Upper Hessenberg

Dynamic Response

15. Direct Formulation
16. Modal Formulation
17. Transient Response
18. Frequency Response
19. Random Analysis
20. Flutter Analysis

C. ELEMENT TYPES

1. Bar, Rod, Tube or Conrod
2. Shear or Twist Panel
3. Plate or Membrane
4. Scalar Springs, Mass and Dampers
5. Concentrated Mass
6. Viscous Dampers
7. Plot (PLØTEL)
8. General (GENEL)
9. Conical Shell
10. Toroidal Shell
11. Axisymmetric Solids
12. Linear Solids
13. Isoparametric Solids

14. Solid Heat Conductors
15. Heat Transfer Boundary Elements
16. Fluid Elements
17. Acoustic Elements
18. Aerodynamic Elements
19. Rigid Elements

D. CONSTRAINTS

1. Single-Point Constraints
2. Multipoint Constraints
3. Omitted Coordinates
4. Free-Body Supports
5. Fluid Free Surface
6. Symmetry Used on Boundary
7. "Grounded" Stiffness Terms

E. GEOMETRY AND PROPERTY DEFINITIONS

1. Property ID Default
2. Local Coordinate System
3. Resequenced Grid Points
4. Thermal Dependent Materials
5. Nonlinear Materials
6. Anisotropic Materials
7. Offset BAR Connections
8. Structural Mass
9. Nonstructural Mass
10. Structural Element Damping
11. Compressibility of Fluid
12. Fluid Gravity Effects
13. Multiple Fluid Harmonics

F. SPECIAL MATRIX OPTIONS

1. General Element (GENEL)
2. Direct Input Matrices
3. Transfer Functions
4. Extra Points
5. Direct Damping Matrix Input
6. Modal Damping
7. Substructuring
8. Cyclic Symmetry
9. Uniform Structure Damping
10. Element Congruency

G. LOADING OPTIONS

Static

1. Concentrated Loads
2. Pressure Loads
3. Gravity Loads
4. Thermal Loads
5. Harmonic Loads
6. Centrifugal Field Loads
7. Enforced Element Deformation
8. Enforced Displacement

Dynamic Excitation

9. Tabular Loads vs. Frequency or Time
10. Direct Time Function Loads
11. Loading Phase Angles
12. Loading Time Lags
13. Load Combinations (DLØAD)
14. Transient Initial Conditons
15. Random Analysis Power Spectral Density Functions

16. Aerodynamic Gust

Heat Transfer

17. Volume Heating
18. Area Heating
19. Radiation Heating
20. Enforced Boundary Temperature

H. EXECUTION OPTIONS

Multiple Solution Techniques

1. Loads
2. Boundary Constraints
3. Cyclic Symmetry
4. Direct Input Matrices
5. Aerodynamic Coefficients

Operational Techniques

6. Checkpoint
7. Restart with Modified Case Control
8. Restart with Rigid Format Change
9. Restart with Modified Bulk Data
10. Altered Rigid Format Using DMAP Statements
11. Multi-stage Substructuring

I. OUTPUT OPTIONS

Print and/or Punch

1. Point Output Selections
2. Element Output Selections
3. Subcase Level Request Changes
4. Sorted by Frequency or Time (SØRT2)

5. Magnitude and Phase of Complex Numbers
6. Mode Acceleration Data Recovery
7. Solution Set Output Requests
8. Frequency Set Selections
9. Punched Output Selections
10. Weight and Balance
11. Grid Point Force Balance
12. Element Strain Energy
13. Element Stress Precision

<u>Plot</u>

14. Structures Plot of Undeformed Structure
15. Structures Plot of Deformed Structure
16. Curve Plotting vs. Frequency
17. Curve Plotting vs. Time
18. Curve Plotting vs. Subcase

FEATURES VERSUS PROBLEMS

Item	Feature	1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9-1	1-9-2	1-10	1-11	1-12-1	1-12-2	1-13	1-14	1-15	1-16	1-17	2-1	2-2	3-1	3-2	3-3	3-4	3-5
A	PHYSICAL PROBLEMS																										
	Structures																										
1	line	x							x	x		x					x				x	x					x
2	plate or shell	x	x	x	x	x		x			x		x						x	x			x				
3	solids						x		x	x				x	x	x		x									
4	rotational symmetry		x			x	x	x						x	x	x		x			x						
	Fluid Dynamics																										
5	flexible boundary																										
6	rigid boundary																							x	x		
7	sloshing																								x		
8	acoustic																							x		x	
9	aeroelastic																										
	Heat Transfer																										
10	conduction													x	x												x
11	convection														x												
12	radiation																										

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	3-6	3-7	3-8	4-1	5-1	5-2	6-1	7-1	7-2	8-1	9-2	9-3	9-4	10-1	10-2	10-3	11-1	11-2	11-3	12-1	13-1	14-1	15-1
A	PHYSICAL PROBLEMS																							
	Structures																							
1	line	X		X	X				X			X		X	X	X		X	X	X	X	X		
2	plate or shell		X			X	X	X		X	X		X										X	X
3	solids																							
4	rotational symmetry					X				X			X										X	X
	Fluid Dynamics																							
5	flexible boundary									X			X											
6	rigid boundary																							
7	sloshing																							
8	acoustic									X														
9	aeroelastic															X				X				
	Heat Transfer																							
10	conduction	X												X										
11	convection																							
12	radiation	X																						

Note: Two-digit problem numbers refer to all problems in the series: three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	1-11	1-12	1-13	1-14	1-15	1-16	1-17	2-1	3-5	3-6	4-1	5-1	6-1	9-4	11-1-1A	13-1	14-1
B	SOLUTION METHODS																											
	<u>Steady State</u>																											
1	linear statics	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x								x		x
2	inertia relief																		x									
3	nonlinear geometry																					x	x				x	
4	material plasticity																							x				
5	fully stressed																x											
6	linear heat transfer												x												x			
7	non-linear heat transfer																			x	x							

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

Item	Feature	Problem Numbers: 1-1-1B	3-1-1	3-1-2	3-1-3	3-1-4	3-2	3-3	3-4	3-7	3-8	5-1	5-2	7-1	7-2	10-1	10-2	10-3	11-1	11-2	11-3	12-1	13-1	15-1
8	SOLUTION METHODS																							
	<u>Eigenvalue Analysis</u>																							
8	real modes	X	X	X	X	X	X	X	X	X	X	X	X			X	X		X	X	X	X	X	X
9	complex modes													X	X	X	X							
10	inverse power	X	X		X		X	X	X	X			X			X			X			X	X	X
11	FEER			X		X														X				
12	determinant											X		X	X	X								
13	Givens										X						X				X			
14	upper Hessenberg																X							

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

Item	Feature	Poblem Numbers														
		7-1	7-2	8-1	9-1	9-2	9-3	9-4	10-2	10-2	10-3	11-1	11-2	11-3-1	11-3-2	12-1
B	SOLUTION METHODS															
	<u>Dynamic Response</u>															
15	direct formulation	X	X	X	X	X	X	X								
16	modal formulation								X	X		X	X	X	X	X
17	transient response				X	X	X	X							X	X
18	frequency response			X								X	X	X		
19	random analysis											X		X		
20	flutter analysis									X						

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

Item	Feature	Problem Numbers																				
		1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9-1	1-9-2	1-10	1-11	1-12-1	1-12-2	1-13	1-14	1-15	1-16	1-17	2-1	2-2
C	ELEMENT TYPES																					
1	bar, rod, tube or conrod	X										X					X		X		X	
2	shear or twist panels	X																				
3	plate or membrane	X	X	X	X						X		X						X	X		X
4	scalar																					
5	concentrated mass																					
6	viscous damping																					
7	plot (PLØTEL)		X																			
8	general (GENEL)																X					
9	conical shell					X																
10	toroidal shell							X														
11	axisymmetric solids						X								X			X				
12	linear solids								X	X				X								
13	isoparametric solids															X						
14	solid heat conductors													X	X							
15	heat transfer boundary elements													X	X							
16	fluid elements																					
17	acoustic elements																					
18	aerodynamic elements																					
19	rigid elements																					

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	Problem Numbers																											
		3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	4-1	5-1	5-2	6-1	7-1	7-2	8-1	9-2	9-3	9-4	10-1	10-2	10-3	11-1	11-2	11-3	12-1	13-1	14-1	15-1
C	ELEMENT TYPES																												
1	bar, rod, tube or conrod					X			X	X									X	X	X		X			X	X	X	X
2	shear or twist panels																												
3	plate or membrane	X						X			X	X	X		X	X		X										X	X
4	scalar								X					X			X		X	X	X			X	X				
5	concentrated mass								X					X			X			X	X		X	X		X			
6	viscous damping													X															
7	plot (PLØTEL)	X																											
8	general (GENEL)																						X		X				
9	conical shell																												
10	toroidal shell																												
11	axisymmetric solids																												
12	linear solids																												
13	isoparametric solids																												
14	solid heat conductors																		X										
15	heat transfer boundary elements					X	X												X										
16	fluid elements		X	X											X			X											
17	acoustic elements				X																								
18	aerodynamic elements																				X				X				
19	rigid elements								X																				

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

Item	Feature	Problem Numbers																	
		1-1	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	1-11	1-12	1-13	1-14	1-15	1-16	1-17	2-1
D	CONSTRAINTS																		
1	single-point	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
2	multi-point																		
3	omitted																		
4	free body supports																		X
5	fluid free surface																		
6	boundary symmetry	X	X	X	X				X	X		X	X	X			X		
7	grounded stiffness														X				

Item	Feature	Problem Numbers																											
		3-1	3-3	3-4	3-5	3-6	3-7	3-8	4-1	5-1	5-2	6-1	7-1	7-2	8-1	9-1	9-2	9-3	9-4	10-1	10-2	10-3	11-1	11-2	11-3	12-1	13-1	14-1	15-1
D	CONSTRAINTS																												
1	single-point	X			X	X	X	X	X	X	X	X		X	X			X	X	X	X		X		X	X	X	X	X
2	multi-point																			X					X				
3	omitted							X	X																	X			
4	free body supports			X				X												X					X	X			
5	fluid free surface		X											X				X											
6	boundary symmetry	X								X		X			X														
7	grounded stiffness												X			X	X		X					X					

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Problem Numbers	E GEOMETRY & PROPERTY	1 property id. default	2 local coord. system	3 resequenced grid point	4 thermal dep. material	5 nonlinear materials	6 anisotropic materials	7 offset bar connection	8 structural mass	9 nonstructural mass	10 structural elem. damping	11 fluid compressibility	12 fluid gravity	13 multiple fluid harmonic
15-1			x	x					x					
14-1			x	x										
13-1									x					
12-1									x					
11-3			x							x				
11-1									x					
10-3														
10-2			x	x					x					
10-1														
9-4														
9-3			x	x					x					
8-1										x				
7-2			x	x					x			x		
7-1											x			
6-1						x								
5-1			x	x										
4-1			x						x					
3-8										x				
3-7									x					
3-6			x											
3-5					x	x								
3-4												x		
3-3			x										x	
3-2			x									x		x
3-1									x	x				
2-2			x											
2-1			x					x	x					
1-16		x			x									
1-13			x											
1-12-2				x										
1-12-1			x	x										
1-4				x			x							
1-3					x									
1-2			x											
1-1-1B									x					

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	1-3	1-8	1-9	1-11-1	1-13-1	1-13-2	1-14	2-2	3-1-1	3-1-2	5-1	7-1	8-1-1	8-1-2	9-1	10-1	11-1	11-3	12-1	14-1	15-1
		Problem Numbers																				
F	MATRIX OPTIONS																					
1	general element							X										X	X			
2	direct input matrices															X						
3	transfer functions																X					
4	extra points															X	X					
5	direct damping matrix input															X	X					
6	modal damping																	X	X	X		
7	substructuring								X													
8	cyclic symmetry																				X	X
9	uniform damping												X									
10	element congruency	X	X	X	X	X	X			X	X	X		X	X						X	X

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	Problem Numbers																										
		1-1	1-1-1A	1-2	1-3	1-4	1-5	1-6	1-7	1-8	1-9	1-10	1-11	1-13-1	1-13-2	1-13-3	1-14	1-15	1-16	1-17	2-1	4-1	5-1	5-2	6-1	11-1-1A	13-1	14-1
G	LOADING																											
	<u>Static</u>																											
1	concentrated	X	X			X	X		X	X	X						X		X	X	X		X	X	X		X	
2	pressure			X										X	X	X		X										X
3	gravity																					X				X		
4	thermal				X			X			X	X	X	X	X	X				X								
5	harmonic						X											X										
6	centrifugal field																				X							
7	enf. elem. deform.																									X		
8	enf. displacement																											

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	Problem Numbers									
		8-1	9-1	9-2	9-3	9-4	11-1-1	11-2	11-3-1	11-3-2	12-1
G	LOADING										
	Dynamic										
9	tabular	X	X				X	X	X	X	
10	direct time func.				X	X	X				X
11	phase angles						X				
12	time lags		X				X				
13	combinations						X				
14	initial conditions		X	X		X					
15	random						X		X		
16	aerodynamic gust								X	X	

Item	Feature	Problem Numbers			
		1-12	3-5	3-6	9-4
G	LOADING				
	Heat Transfer				
17	volume heating		X		
18	area heating	X			X
19	radiation heating			X	
20	enf. bdy. temp.	X	X	X	X

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

Item	Feature	Problem Numbers												
		1-1-2	1-9	1-10	1-13	1-14	1-16	9-1	10-2	10-3	11-1	11-3	14-1	15-1
H	EXECUTION OPTIONS													
	Multiple Solutions													
1	loads	X	X		X	X	X				X			
2	boundary constraints			X										
3	cyclic symmetry												X	X
4	direct input matrices							X						
5	aerodynamic coefficients								X			X		

Item	Feature	Problem Numbers														
		1-1-1	1-1-1A	1-1-1B	1-2-1	1-2-1A	1-4-1	1-4-1A	1-17	2-2	3-8	10-1-1	10-2-1	10-3	11-1-1	11-1-1A
H	EXECUTION OPTIONS															
	Operational															
6	checkpoint	X			X		X								X	
7	restart-C.C. change		X	X		X		X								X
8	restart-R.F. change			X												X
9	restart-B.D. change			X				X								X
10	DMAP alters								X		X	X	X		X	
11	substructuring									X						

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	1-1-1	1-1-2	1-1-3	1-1-4	1-2	1-2-1A	1-3	1-4	1-4-1A	1-5	1-6	1-7	1-8	1-9	1-10	1-11	1-12	1-13	1-14	1-15	1-16	1-17	2-1
I	**OUTPUT**																							
	Print/Punch																							
1	point	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
2	element	X	X	X	X	X	X	X		X	X	X	X	X	X	X	X	X	X	X		X	X	X
3	subcase level change														X							X		
4	SØRT2		X																					
5	magnitude/phase																							
6	mode acceleration																							
7	solution set																							
8	frequency set																							
9	punched output	X				X			X									X				X		
10	weight and balance																					X		X
11	grid point force balance			X																				
12	element strain energy				X																			
13	element stress precision																						X	

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

FEATURES VERSUS PROBLEMS

Item	Feature	3-1	3-2	3-3	3-4	3-5	3-6	3-7	3-8	4-1	5-1	5-2	6-1	7-1	7-2	8-1	9-1	9-2	9-3	9-4	10-1	10-3	11-1	11-1-1A	11-2	11-3	12-1	13-1	14-1	15-1
I	OUTPUT																													
	<u>Print/Punch</u>																													
1	point	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X		X	X	X	X	X	X	X	X
2	element				X	X	X			X	X	X	X										X	X			X	X		
3	subcase level change									X	X																	X		
4	SØRT2																X	X	X	X			X		X		X			
5	magnitude/phase															X					X		X		X					
6	mode acceleration																								X					
7	solution set																X				X				X	X	X			
8	frequency set															X														
9	punched output					X																	X							
10	weight and balance	X							X	X											X		X				X			
11	grid point force balance																													
12	element strain energy																													
13	element stress precision																													

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

Item	Feature	Problem Numbers														
		1-2-1	1-15	1-16	3-1	3-4	5-1	9-1	9-3	10-2	10-3	11-1	11-2	11-3-1	11-3-2	12-1
I	OUTPUT															
	Plot															
14	structure-undeformed	X	X	X	X	X	X									
15	structure-deformed	X		X	X	X	X									X
16	curve vs. frequency												X	X		
17	curve vs. time							X	X						X	X
18	curve vs. subcase															
19	curve-aerodynamic									X						

Note: Two-digit problem numbers refer to all problems in the series; three-digit numbers refer to a specific version.

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RIGID FORMAT No. 1, Static Analysis

Delta Wing with Biconvex Cross Section (1-1-1)

Delta Wing with Biconvex Cross Section Using QDMEM1 and QDMEM2 Elements (1-1-2)

Delta Wing with Biconvex Cross Section Using QDMEM1 Elements (1-1-3)

Delta Wing with Biconvex Cross Section Using QDMEM2 Elements (1-1-4)

A. Description

This series illustrates the use of various NASTRAN elements in the solution of an actual structural problem. Figure 1 shows the delta wing to be modeled and Figures 2 and 3 shows the finite element model. The delta wing model is composed of membrane, shear panel and rod elements. Due to the existence of symmetry or antisymmetry in the structure and loading conditions, only one-quarter of the wing needs to be modeled. The midplane of the wing (the plane dividing the wing into upper and lower halves) is a plane of symmetry as is the center plane (the plane that divides the wing into left and right halves). The loading conditions are antisymmetrical with respect to the midplane of the wing and symmetric with respect to the center plane.

B. Input

The surface skin of the wing is modeled with membrane elements while the ribs and spars are modeled with a combination of shear panels and rods. The shear load carrying capability of ribs and spars is represented by shear panels. The bending stiffness of the ribs and spars is modeled with rod elements placed in the plane of the skin surface.

Since a quarter model is used, the loading conditions require that an antisymmetric boundary be provided on the midplane and a symmetric boundary must be provided on the center plane. These boundary conditions are provided by constraining all grid points on the midplane in the x and y directions and all grid points on the center plane in the x direction. Supports for the structure are provided by constraining grid points 13, 33, 53, 73 and 93 in the z direction. Since no rotational rigidity is provided by the elements used in the model, all rotational degrees of freedom have been removed by the use of the GRDSET card.

Figure 4 shows the two loading conditions analyzed. The problem is first modeled (Problem 1-1-1) with a load on the trailing edge and a checkpoint is requested. The modified restart (Problem 1-1-1A) capability is used to perform the analysis associated with the leading edge loading condition. The ability of NASTRAN to change rigid formats on a restart is demonstrated by the third case (Problem 1-1-1B). The natural modes of the structure are extracted using the Inverse

Power method. Since the symmetric boundary conditions are used, only the modes with symmetric motion about the center line will be extracted. If the unsymmetric modes were required, a separate run with the appropriate boundary conditions could be submitted.

A second variation (Problem 1-1-2) of the basic problem is obtained by replacing the quadrilateral membrane elements (QDMEM) with the QDMEM1 and QDMEM2 elements. This modification demonstrates the ability to reproduce previously derived theoretical results. The SØRT2 format of the printed output allows the results obtained with a leading and trailing load to be compared. A third case (Problem 1-1-3) is modeled with all QDMEM elements replaced by QDMEM1 (Reference 26) elements. A grid point force balance is requested to verify the static equilibrium of forces at a grid point (due to the load, constraints, and element forces) is zero. A fourth modeling of the wing (Problem 1-1-4) uses QDMEM2 elements in place of the QDMEM elements. In this case, element strain energy is requested to exhibit the energy transmitted by each of the elements due to the load and resultant deflections.

1. Parameters

$E = 10.4 \times 10^6$ lb/in^2	(modulus of elasticity)
$G = 4.0 \times 10^6$ lb/in^2	(shear modulus)
$\rho = 2.523 \times 10^{-4}$ lb sec^2/in^4	(density)

2. Constraints

$\theta_x = \theta_y = \theta_z = 0.0$	All grid points
$U_z = 0.0$	Grids 13, 33, 53, 73 and 93
$U_x = 0.0$	Grids 11, 31, 51, 71 and 91
$U_x = U_y = 0.0$	Grids 1, 2, 3, 4, 5, 6, 21, 22, 23, 24, 25, 26, 41, 42, 43, 44, 45, 61, 62, 63, 64, 81, 82 and 83

3. Loads

Problems 1-1-1, 1-1-2, 1-1-3, 1-1-4

Grid 16 $F_z = -500.0$ (trailing edge)

Problem 1-1-2

Grid 36 $F_z = -500.0$ (leading edge)

4. Eigenvalue extraction data

Method: Inverse Power

Region of interest: $30.0 \leq f \leq 160.0$

Number of desired roots: 3

Number of estimated roots: 1

C. Results

No closed-form or theoretical solution exists for this problem. However, a passive analog computer simulation (Reference 1) and a laboratory test (Reference 2) have been performed for this structural model. The displacements calculated by NASTRAN and the experimentally measured and simulated displacements are shown in Tables 1 and 2. The natural frequencies and modal displacements are shown in Tables 3 and 4. Table 5 presents the displacements for the static loading conditions when elements 1 through 9 are CQDMEM1 elements and the other quadrilaterals are CQDMEM2 elements.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=(UMF,NPTP)
  1   ID         DEM1011,NASTRAN
  2   UMF        1977 10110
  3   CHKPNT     YES
  4   APP        DISPLACEMENT
  5   SØL        1,1
  6   TIME       5
  7   CEND
  8   TITLE = DELTA WING WITH BICØNVEX CRØSS SECTION
  9   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ.  1-1-1
 10   LABEL = LØAD ØN TRAILING EDGE
 11      SPC = 1
 12       LØAD = 1
 13   ØUTPUT
 14   $    SET 1  HAS GRIDS ON THE UPPER SURFACE  * * * * * * * * * * * * * * *
 15   $     SET 2  HAS TOP SURFACE ELEMENTS. SHEAR(TRAILING AND LEADING EDGE).
 16   $      SHEAR(CENTERLINE - BØTH DIRECTIØNS). SHEAR(TIP) * * * * * * * * * *
 17   $
 18           SET 1 = 11 THRU 16,31 THRU 36,51 THRU 55,71 THRU 74,91 THRU 93
 19           SET 2 = 1 THRU 22,28 THRU 31, 35, 36, 41, THRU 44, 50
 20   $
 21           DISPLACEMENTS = 1
 22           SPCFØRCE = ALL
 23       ELSTRESS = 2
 24   BEGIN BULK
 25   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØNRØD	100	11	12	1	.035				
CQDMEM	1	1	11	12	32	31			
CRØD	60	5	1	11	61	6	2	12	
CSHEAR	18	2	1	2	12	11			
CTRMEM	10	3	35	36	55				
FØRCE	1	16	0	-1.	.0	.0	500.		
GRDSET							456		
GRID	1		.0	.0	.0				
MAT1	1	10.4+6	4.+6						
PARAM	IRES	1							
PQDMEM	1	2	.16	.0					
PRØD	5	1	2.1						
PSHEAR	2	2	.14	.0					
PTRMEM	3	2	.16	.0					
SPC1	1	1	11	31	51	71	91		

```
Card
 No.

  0   NASTRAN FILES=ØPTP
  1   ID         DEM1011A,RESTART
  2   APP        DISPLACEMENT
  3   SØL        1,1
  4   DIAG  14
  5   TIME       5
  6   CEND
  7   TITLE = DELTA WING         RESTART
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-1-1A
  9   LABEL = LØAD ØN LEADING EDGE
 10           ECHØ = SØRT
 11           LØAD = 2
 12           SPC = 1
 13   ØUTPUT
 14   $       SET 1 HAS GRIDS ØN THE UPPER SURFACE  * * * * * * * * * * * * * * *
 15   $        SET 2 HAS TOP SURFACE ELEMENTS. SHEAR(TRAILING AND LEADING EDGE).
 16   $         SHEAR(CENTERLINE - BØTH DIRECTIØNS). SHEAR(TIP)  * * * * * * * *
 17   $
 18             SET 1 = 11 THRU 16,31 THRU 36,51 THRU 55,71 THRU 93
 19             SET 2 = 1 THRU 22,28 THRU 31, 35, 36, 41 THRU 44, 50
 20             DISPLACEMENTS = 1
 21             SPCFØRCE = ALL
 22             ELSTRESS = 2
 23   BEGIN BULK
 24   ENDDATA
```

Note: The Restart Dictionary from Problem 1-1-1 is required in the Executive Control Deck.

Card
No.

```
 0  NASTRAN FILES=ØPTP
 1  ID      DEM1011B,RESTART
 2  TIME    5
 3  SØL     3,1
 4  DIAG 14
 5  APP     DISPLACEMENT
 6  CEND

 7  TITLE = DELTA WING        RESTART, REAL  EIGENVALUE ANALYSIS
 8  SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-1-1B
 9     LABEL = RIGID FØRMAT SWITCH FRØM 1 TO 3
10     ECHØ = BØTH
11     SPC = 1
12     METHØD = 12
13  ØUTPUT
14     SET 1 = 11 THRU 16,31 THRU 36,51 THRU 55,71 THRU 74,91 THRU 93
15     SET 2 = 1 THRU 22,28 THRU 31, 35, 36, 41 THRU 44, 50
16  $
17     DISPLACEMENTS = 1
18     SPCFØRCE = ALL
19     ELSTRESS = 2
20  BEGIN BULK
```

	1	2	3	4	5	6	7	8	9	10
21	EIGR	12	INV	30.0	160.0	1	3	0	1.-4	+EIGR12
22	+EIGR12	MAX								
23	ENDDATA									

Note: The Restart Dictionary from Problem 1-1-1 is required in the Executive Control Deck.

Card
No.

```
 0  NASTRAN FILES=UMF
 1  ID       DEM1012,NASTRAN
 2  UMF      1977      10120
 3  APP DISPLACEMENT
 4  SØL      1,0
 5  TIME       5
 6  CEND
 7  TITLE = STATIC ANALYSIS ØF A DELTA WING WITH BICØNVEX CRØSS SECTIØN
 8  SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-1-2
 9  LABEL = QDMEM1 AND QDMEM2 ELEMENTS
10  SET 1 = 11 THRU 16,31 THRU 36,51 THRU 55,71 THRU 74,91 THRU 93
11  SET 2 = 1 THRU 22,28 THRU 31,35,36,41 THRU 44,50
12  DISPLACEMENTS (SØRT2) = 1
13  SPCF (SØRT2) = ALL
14  ELSTRESS (SØRT2) = 2
15  SPC = 1
16  SUBCASE 1
17  LØAD = 1
18  SUBCASE 2
19  LØAD = 2
20  BEGIN BULK
21  ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØNRØD	100	11	12	1	.035				
CQDMEM1	1	1	11	12	32	31			
CRØD	60	5	1	11	61	6	2	12	
CSHEAR	26	2	24	25	35	34			
CTRMEM	10	3	35	36	55				
FØRCE	1	16	0	-1.0	.0	.0	500.		
GRDSET							456		
GRID	1		.0	.0	.0				
MAT1	1	10.4+6	4.+6						
PARAM	IRES	1							
PQDMEM1	1	2	.16	.0					
PQDMEM2	1	2	.16	.0					
PRØD	5	1	2.1						
PSHEAR	2	2	.14	.0					
PTRMEM	3	2	.16	.0					
SPC1	1	1	11	31	51	71	91		

```
Card
 No.

 0   NASTRAN FILES=UMF
 1   ID       DEM1013,NASTRAN
 2   UMF      1977     10130
 3   APP      DISPLACEMENT
 4   SØL      1,1
 5   TIME     5
 6   CEND

 7   TITLE = DELTA WING WITH BICØNVEX CRØSS SECTIØN USING QDMEM1 ELEMENTS
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-1-3
 9   LABEL = LØAD ØN TRAILING EDGE
10      SPC = 1
11      LØAD = 1
12   ØUTPUT
13   $  SET 1 HAS GRIDS ØN THE UPPER SURFACE  * * * * * * * * * * * * * * * *
14   $  SET 2 HAS TØP SURFACE ELEMENTS, SHEAR(TRAILING AND LEADING EDGE),
15   $  SHEAR(CENTERLINE - BØTH DIRECTIONS), SHEAR(TIP) * * * * * * * *
16   $
17      SET 1 = 11 THRU 16,31 THRU 36,51 THRU 55,71 THRU 74,91 THRU 93
18      SET 2 = 1 THRU 22,28 THRU 31, 35, 36, 41 THRU 44, 50
19   $
20      DISPLACEMENTS = 1
21      SPCFØRCE = ALL
22      GPFØRCE = ALL
23      FØRCE = ALL
24      ELSTRESS = 2
25   BEGIN BULK
26   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØNRØD	100	11	12	1	.035				
CQDMEM1	1	1	11	12	32	31			
CRØD	60	5	1	11	61	6	2	12	
CSHEAR	18	2	1	2	12	11			
CTRMEM	10	3	35	36	55				
FØRCE	1	16	0	-1.	.0	.0	500.		
GRDSET							456		
GRID	1		.0	.0	.0				
MAT1	1	10.4+6	4.+6						
PARAM	IRES	1							
PQDMEM1	1	2	.16	.0					
PRØD	5	1	2.1						
PSHEAR	2	2	.14	.0					
PTRMEM	3	2	.16	.0					
SPC1	1	1	11	31	51	71	91		

Card No.

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 0  NASTRAN FILES=UMF
 1  ID       DEM1014,NASTRAN
 2  UMF      1977    10140
 3  APP      DISPLACEMENT
 4  SØL      1,1
 5  TIME     5
 6  CEND
 7  TITLE = DELTA WING WITH BICØNVEX CRØSS SECTIØN USING QDMEM2 ELEMENTS
 8  SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-1-4
 9  LABEL = LØAD ØN TRAILING EDGE
10     SPC = 1
11     LØAD = 1
12  ØUTPUT
13  $  SET 1 HAS GRIDS ØN THE UPPER SURFACE * * * * * * * * * * * * * * * *
14  $  SET 2 HAS TØP SURFACE ELEMENTS, SHEAR(TRAILING AND LEADING EDGE),
15  $  SHEAR(CENTERLINE - BØTH DIRECTIØNS), SHEAR(TIP) * * * * * * * *
16  $
17     SET 1 = 11 THRU 16,31 THRU 36,51 THRU 55,71 THRU 74,91 THRU 93
18     SET 2 = 1 THRU 22,28 THRU 31, 35, 36, 41 THRU 44, 50
19  $
20     DISPLACEMENTS = 1
21     SPCFØRCE = ALL
22     ESE = ALL
23  ELSTRESS = 2
24  BEGIN BULK
25  ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØNRØD	100	11	12	1	.035				
CQDMEM2	1	1	11	12	32	31			
CRØD	60	5	1	11	61	6	2	12	
CSHEAR	18	2	1	2	12	11			
CTRMEM	10	3	35	36	55				
FØRCE	1	16	0	-1.	.0	.0	500.		
GRDSET							456		
GRID	1		.0	.0	.0				
MAT1	1	10.4+6	4.+6						
PARAM	IRES	1							
PQDMEM2	1	2	.16	.0					
PRØD	5	1	2.1						
PSHEAR	2	2	.14	.0					
PTRMEM	3	2	.16	.0					
SPC1	1	1	11	31	51	71	91		

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Table 1. NASTRAN and Experimental Deflections - Concentrated Load on Outboard Trailing Edge.

GRID NUMBER	Z DISPLACEMENT		
	NASTRAN	EXPERIMENTAL	ANALOG
14	-.082	-.08	-.080
15	-.221	-.22	-.210
16	-.424	-.39	-.400
34	-.063	-.07	-.061
35	-.162	-.16	-.157
36	-.293	-.28	-.286
54	-.043	-.05	-.044
55	-.104	-.12	-.144
74	-.025	-.03	-.030

Table 2. NASTRAN and Experimental Deflections - Concentrated Load on Outboard Leading Edge.

GRID NUMBER	Z DISPLACEMENT		
	NASTRAN	EXPERIMENTAL	ANALOG
14	-.063	-.06	-.060
15	-.163	-.15	-.157
16	-.293	-.28	-.286
34	-.057	-.06	-.057
35	-.148	-.15	-.150
36	-.280	-.30	-.290
54	-.046	-.05	-.048
55	-.118	-.13	-.127
74	-.030	-.04	-.035

TABLE 3. NASTRAN and analog computer analysis eigenvalues.

Mode No.	NASTRAN (cps.)	ANALOG (cps.)
1	40.9	41.3
2	115.3	131.0
3	156.2	167.0

TABLE 4. Mode displacements for first mode.

GRID NUMBER	Z DISPLACEMENT	
	NASTRAN	ANALOG
14	.250	.273
15	.601	.630
16	1.000	1.000
34	.210	.239
35	.504	.558
36	.854	.902
54	.162	.192
55	.391	.462
74	.112	.148

TABLE 5. Comparison of Z Displacements

Grid Point	Trailing Edge Load		Leading Edge Load	
	CQDMEM Elements	CQDMEM1 and CQDMEM2 Elements	CQDMEM Elements	CQDMEM1 and CQDMEM2 Elements
14	-.082	-.082	-.063	-.064
15	-.221	-.224	-.163	-.167
16	-.424	-.433	-.293	-.300
34	-.063	-.064	-.057	-.059
35	-.162	-.166	-.148	-.154
36	-.293	-.300	-.280	-.294
54	-.043	-.044	-.046	-.047
55	-.104	-.108	-.118	-.123
74	-.025	-.026	-.030	-.031

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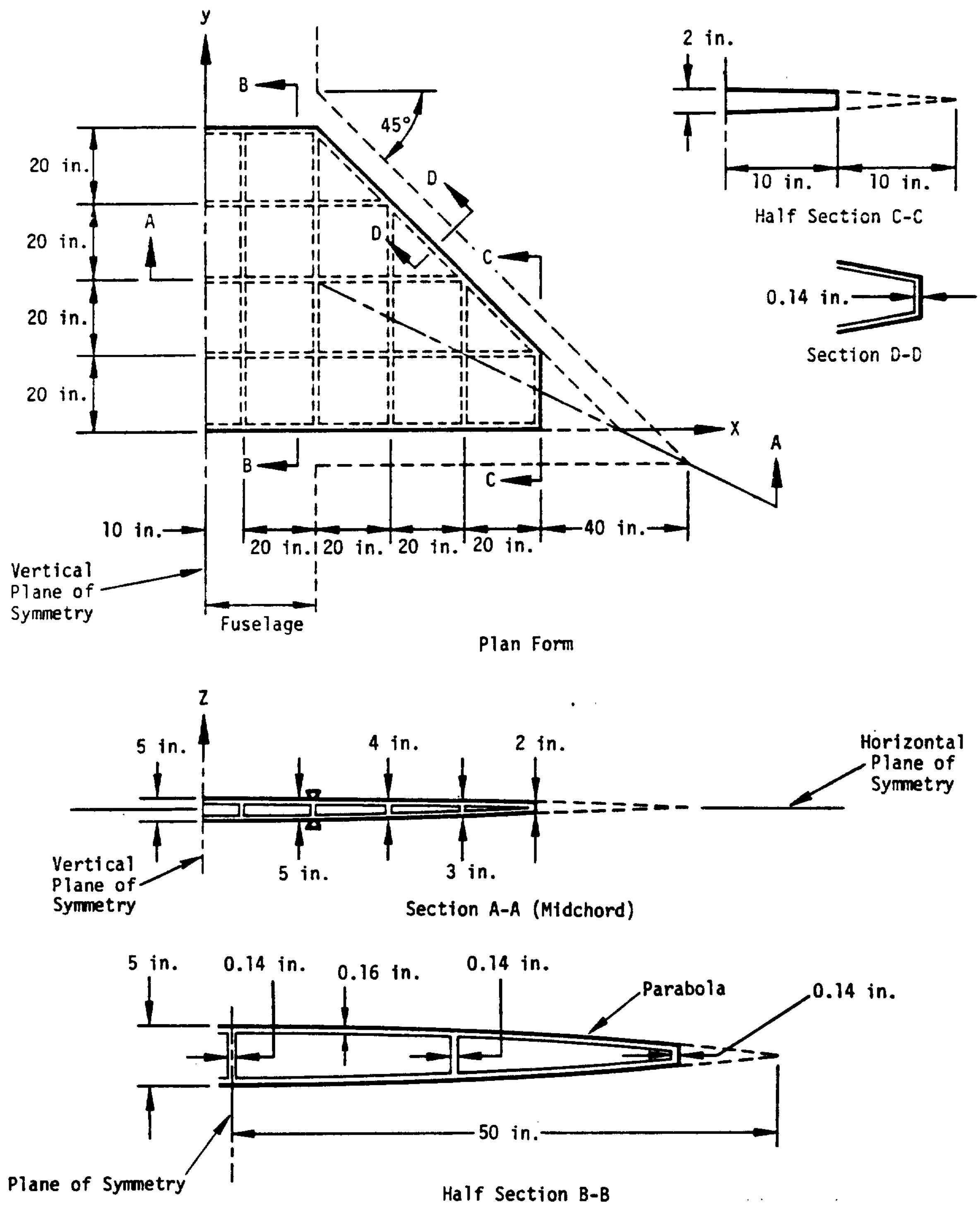


Figure 1. Delta wing with biconvex section.

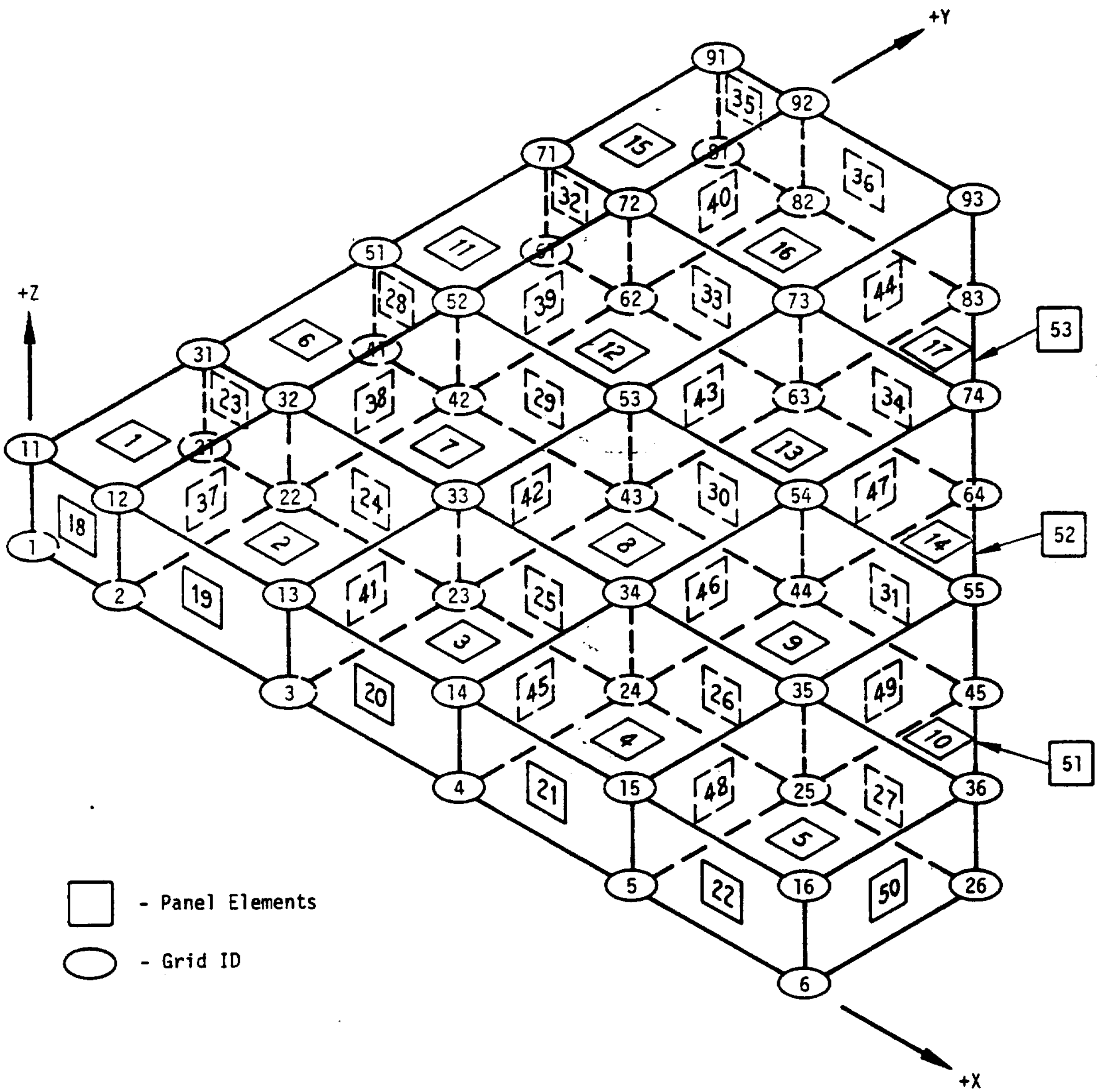


Figure 2. Delta wing with biconvex section model.

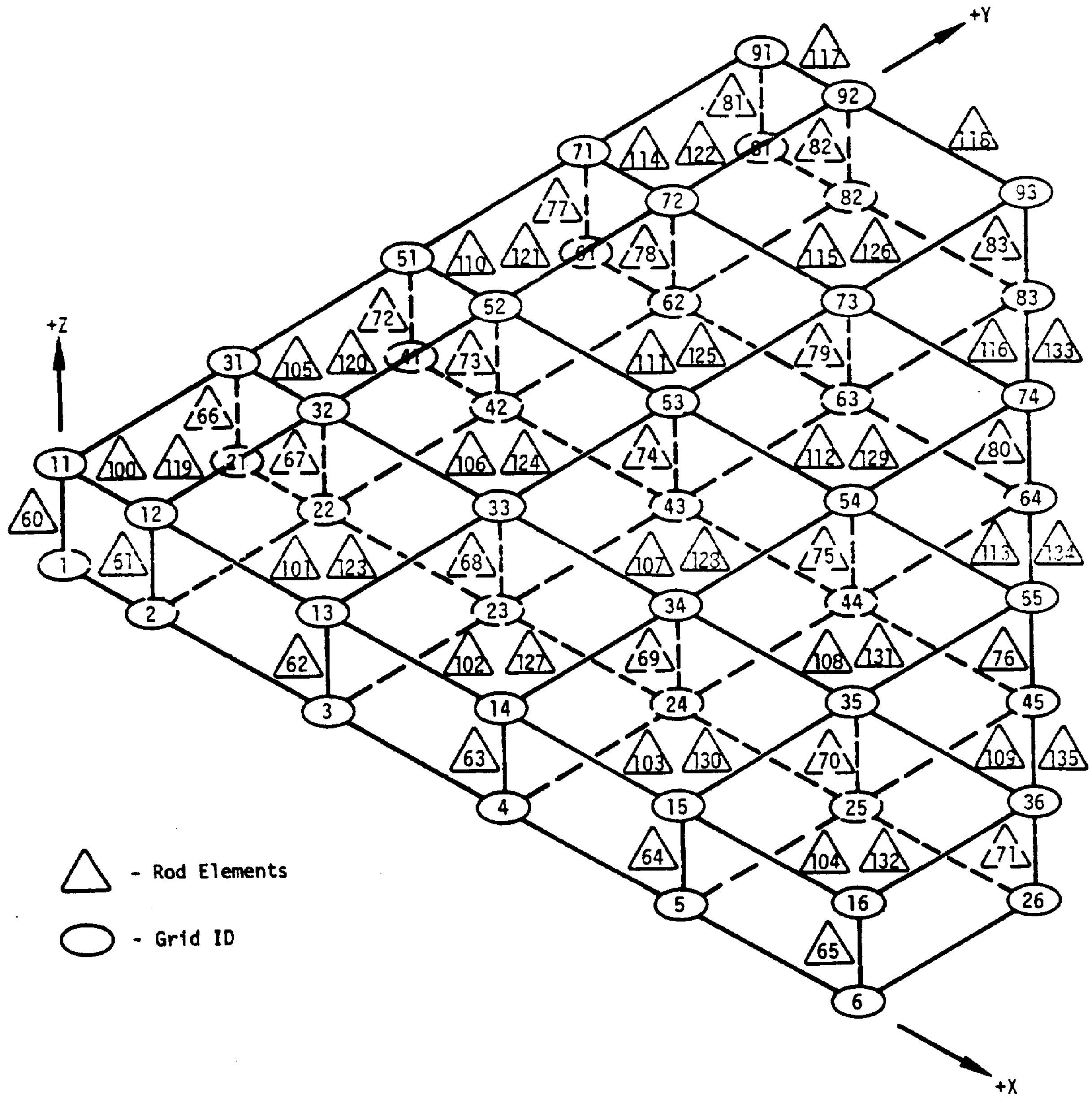
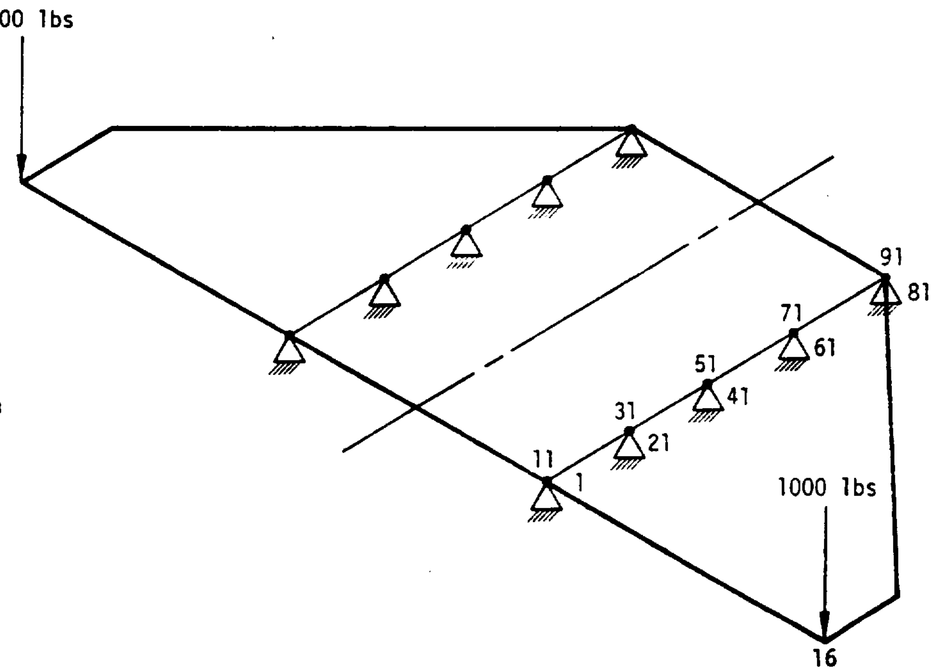
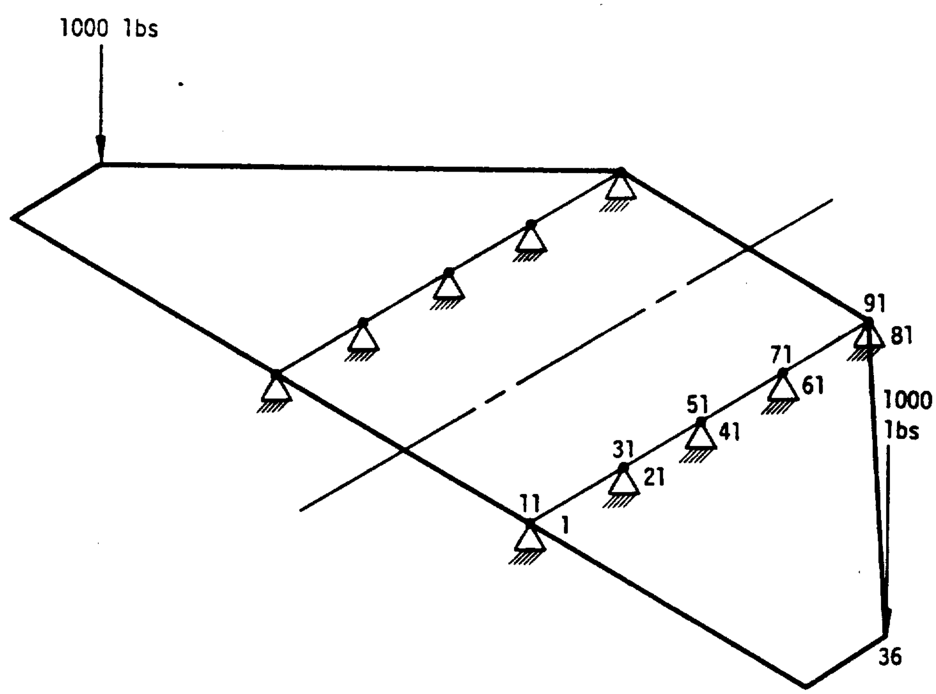


Figure 3. Delta wing with biconvex section model.



Loading Condition on
Trailing Edge



Loading Condition on
Leading Edge

Figure 4. Loading conditions for Delta wing.

RIGID FORMAT No. 1, Static Analysis

Spherical Shell with Pressure Loading (1-2-1)

A. Description

This problem demonstrates the finite element approach to the modeling of a uniform spherical shell (Problem 1-2-1). A spherical coordinate system is chosen to describe the location and displacement degrees of freedom at each grid point. Triangular plate elements are chosen to provide a nearly uniform pattern. Two symmetric boundaries are used to analyze the structure with a symmetric pressure load. Figure 1 describes the quarter model.

Two different sets of boundary conditions are used on the outside edge to demonstrate the ability of NASTRAN to restart (Problem 1-2-1A) with different constraint sets by simply changing the case control request. The effective boundary constraints are shown in Figure 2. The membrane support, under a uniform inward pressure load, results in uniform in-plane compression in two directions. The clamped support produces bending moments in addition to in-plane stresses.

The grid point numbering sequence used minimizes the computer time required to perform the triangular decomposition of the constrained stiffness matrix. This numbering sequence results in a partially banded matrix with all terms outside the band located in a single column. The grid points are arranged to form five rings; the center point is sequenced last.

Orthographic and perspective plots of the deformed and undeformed structure are requested. For the orthographic projections the plots are fully labeled to aid in checking the model. The perspective projection uses the symmetric plotting capability to plot all four quadrants of the shell. A region request is used to find an origin location that will allow all quadrants to be plotted. The deformed plot uses plot elements to simplify the presentation. Underlays of the undeformed structure are also shown for both projections.

B. Input

1. Parameters

$r = 90.0$ in.	(radius)
$t = 3.0$ in.	(thickness)
$E = 3.0 \times 10^6$ lb/in^2	(modulus of elasticity)
$\nu = .1666$	(Poisson's ratio)

2. Constraints

Problem 1-2-1

a) Grids at $\phi = 0^\circ$ and $\phi = 90^\circ$ are constrained $u_\phi = \Theta_r = 0.0$

b) Grids at $\Theta = 35^\circ$ are constrained $u_\theta = 0.0$ only

Problem 1-2-1A

a) Grids at $\Phi = 0^\circ$ and $\Phi = 90^\circ$ are constrained $u_\theta = \Theta_r = 0.0$

b) Grids at $\Theta = 35^\circ$ are constrained $u_r = u_\phi = u_\theta = \Theta_r = \Theta_\phi = \Theta_\theta = 0.0$

3. Loads

A uniform pressure load of 1 lb/in^2 is applied in the -R direction (acting inward).

C. Theory

Theoretical solutions for the continuum shell were obtained from Reference 4 using the first 20 terms of the series shown in Equation (j) of Section 94.

D. Results

Results obtained using NASTRAN and the theoretical solution for the membrane boundary condition are shown in Figures 3 and 4. Also included on these figures are the NASTRAN answers obtained using a 10-ring model. Figures 5 thru 7 present the NASTRAN answers and the theoretical solution for the shell with a clamped boundary.

The slight differences between theoretical and computed answers are due to the combined effects of the finite element theory and the structural behavior in the region of the clamped boundary. In the region of the clamped boundary, in-plane stresses and bending moments are predicted to have large variations. However, the elements used in the model assume a constant in-plane stress and linearly varying bending moment and do not accurately represent the structural response. In addition, the irregularities of the finite element model cause extra coupling between bending and membrane action. Since the elements are planar, the curvature is modeled, in effect, by the dihedral angles between elements. Since the elements are different sizes and shapes, these dihedral angles vary, which results in slight differences in curvature that cause small errors.

E. Driver Decks and Sample Bulk Data

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UMF       1977    10210
CHKPNT    YES
TIME      5
APP       DISPLACEMENT
SØL       1,1
CEND

TITLE = SPHERICAL SHELL WITH PRESSURE LØADING, NØ MØMENTS ØN BØUNDARY
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-2-1
          LØAD = 1
          SPC = 2
ØUTPUT
DISP = ALL
SPCF = ALL
STRESS = ALL
PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-2-1
ØUTPUT(PLØT)
PLØTTER SC
          MAXIMUM DEFØRMATIØN 6.0
$
$    ALL ELEMENTS
     SET 1 = ELEMENTS TRIA2
$
$    PLØTEL - EDGES AND CENTERLINE
     SET 2 = PLØTEL
$
     VIEW 20.0, 30.0, 0.0
     FIND SCALE ØRIGIN 1 SET 1
PTITLE = UNDEFØRMED SECTIØN TRIA2 ELEMENTS
PLØT LABEL(BOTH), SYMBØLS 6
PTITLE = SECTIØN TRIA2 ELEMENTS WITH UNDERLAY
     PLØT STATIC DEFØRMATIØN 0,1, SET 1, ØRIGIN 1,SHAPE, LABELS
$
$
          PERSPECTIVE PRØJECTIØN
$
     FIND SCALE, SET 2, ØRIGIN 1000
     FIND SCALE,ØRIGIN 1000, SET 1,VANT PØINT,REGIØN 0.35,0.1, 0.9, 0.8
PTITLE = SECTIØN PLØTEL ELEMENTS (PERSPECTIVE PRØJECTION)
PLØT SET 2, ØRIGIN 1000, LABELS
PTITLE = FULL MØDEL (VIA SYMMETRY) TRIA2 ELEMENTS - PERSPECTIVE
PLØT      SET 1, ØRIGIN 1000, SYMBØLS 9, SHAPE,
          SET 1, ØRIGIN 1000 SYMBØLS 9 SHAPE     SYMMETRY X,
          SET 1, ØRIGIN 1000 SYMBØLS 9 SHAPE     SYMMETRY Y,
          SET 1, ØRIGIN 1000 SYMBØLS 9 SHAPE     SYMMETRY XY
PTITLE = FULL MØDEL (VIA SYMMETRY) PLØTEL ELEMENTS - PERSPECTIVE
PLØT STATIC DEFØRMATIØN 0,1,
          SET 2, ØRIGIN 1000, SHAPE,
          SET 2, ØRIGIN 1000, SHAPE,      SYMMETRY X,
          SET 2, ØRIGIN 1000, SHAPE,      SYMMETRY Y,
          SET 2, ØRIGIN 1000, SHAPE,      SYMMETRY XY
BEGIN BULK
ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØRD2S	2		.0	.0	.0	.0	.0	1.	+CØR1
+CØR1	1.000	.000	.000						
CTRIA2	1	31	1	6	26	.0			
GRDSET		2				2			
GRID	1		90.0	7.	.0				
MAT1	1	3.+6		.1666					
PLØAD2	1	-1.0	1	2	3	4	5	6	
PLØTEL	50	26	1		51	1	2		
PTRIA2	31	1	3.						
SPC	1	26	12456	.0					
SPC1	1	345	1	2	3	4	11	16	+SPC1-2
+SPC1-2	20	23							

```
Card
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  2   TIME      5
  3   APP       DISPLACEMENT
  4   SØL       1,1
  5   DIAG 14
  6   CEND
  7   TITLE = SPHERICAL SHELL     RESTART WITH CLAMPED BØUNDARY
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-2-1A
  9       ECHØ = BØTH
 10       LØAD = 1
 11       SPC = 1
 12   ØUTPUT
 13       DISPLACEMENT = ALL
 14       SPCFØRCE = ALL
 15       ELFØRCE = ALL
 16       STRESSES = ALL
 17   BEGIN BULK
 18   ENDDATA
```

Note: The Restart Dictionary from Problem 1-2-1 is required in the Executive Control Deck.

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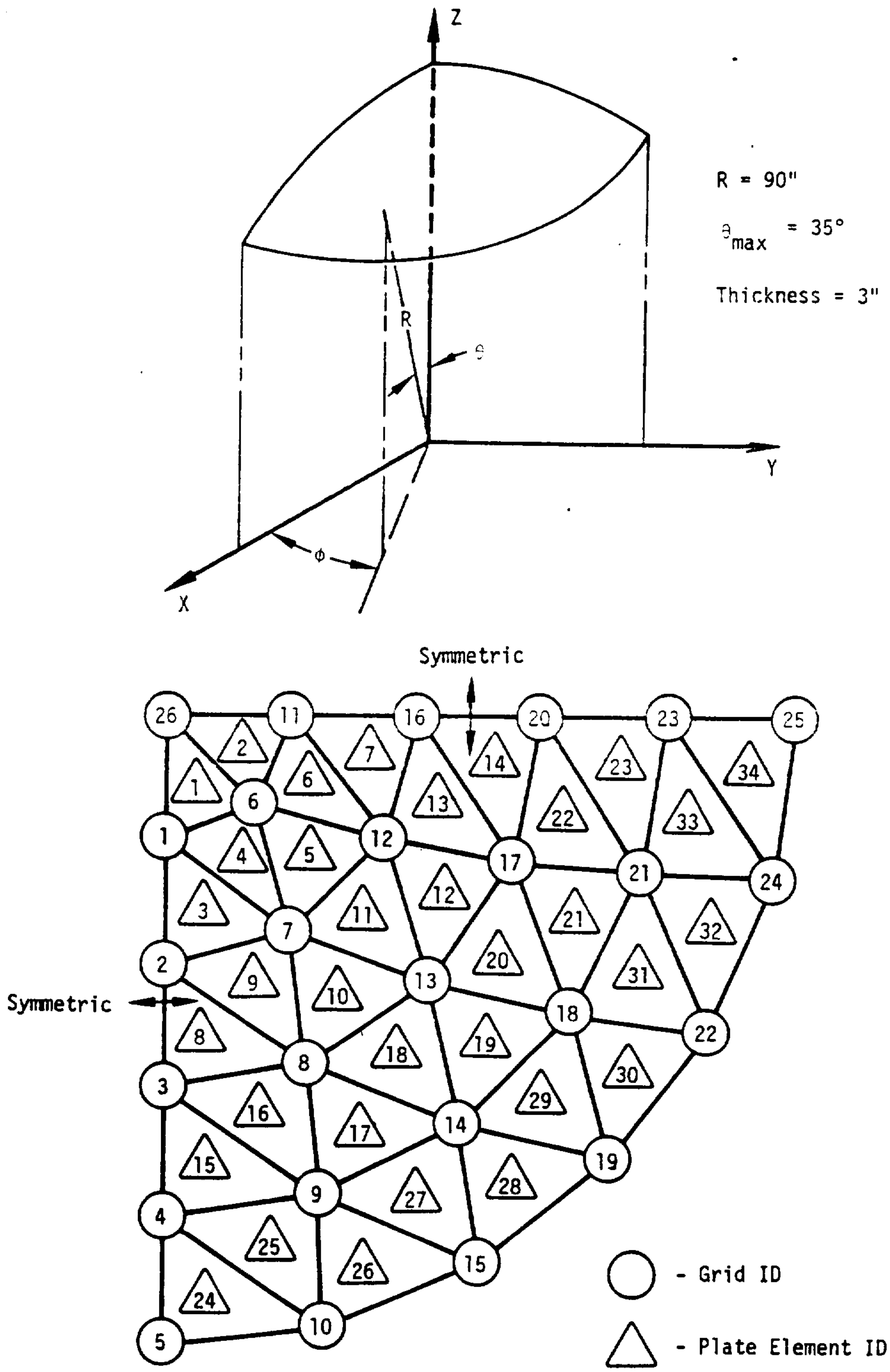

Figure 1. 5 ring spherical shell model.

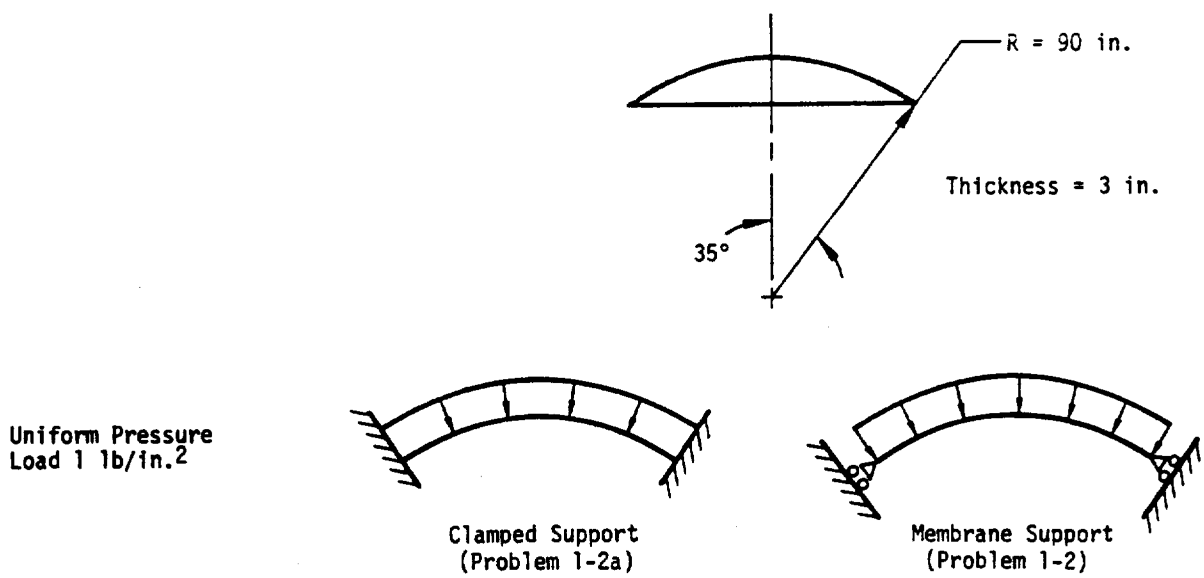

Figure 2. Spherical shell loading and edge support conditions.

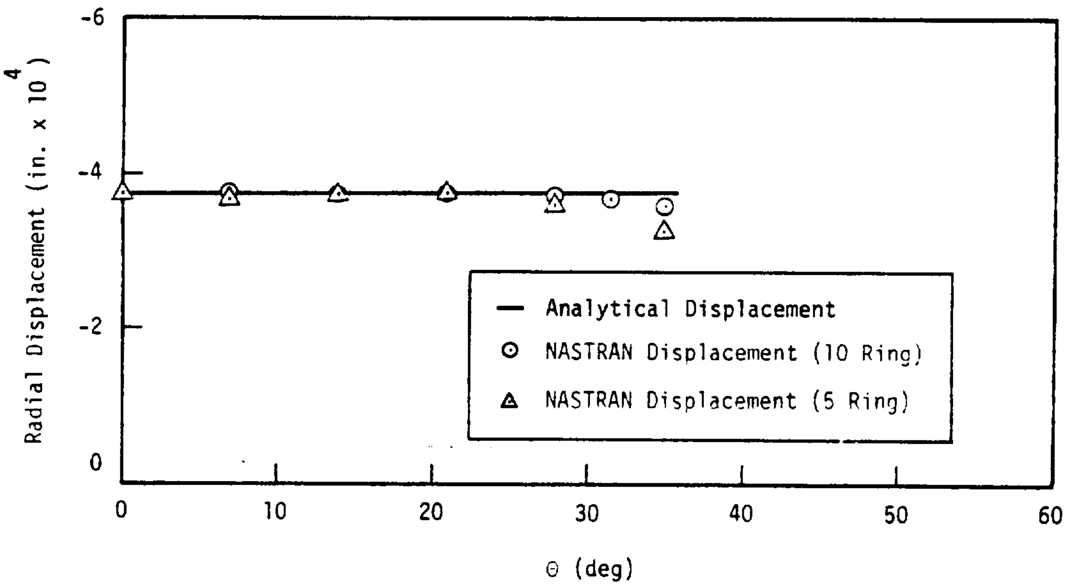

Figure 3. Comparison of NASTRAN and analytical displacements for spherical shell - membrane boundary

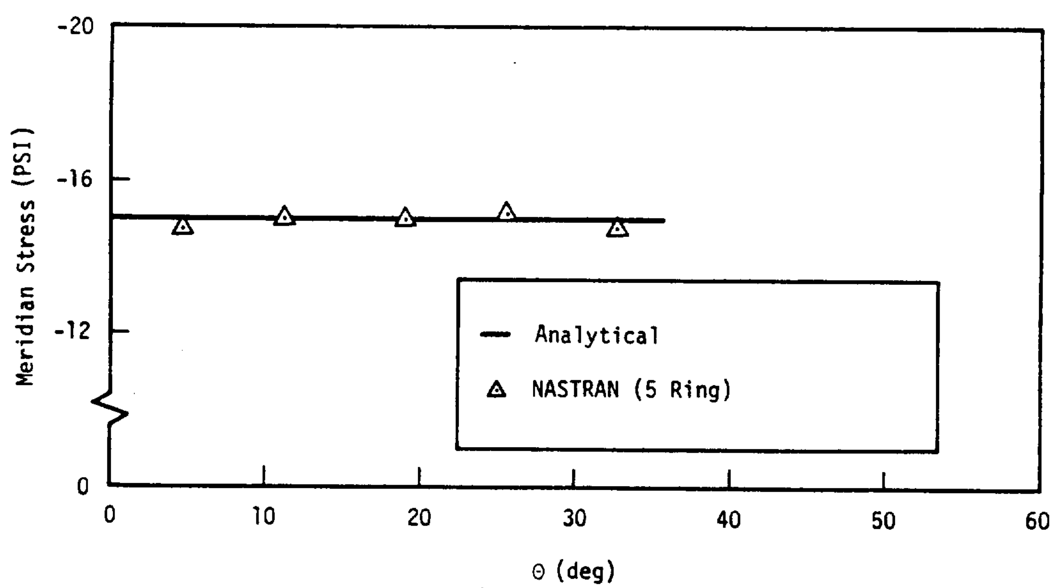

Figure 4. Comparison of NASTRAN and analytical stresses for spherical shell - membrane boundary

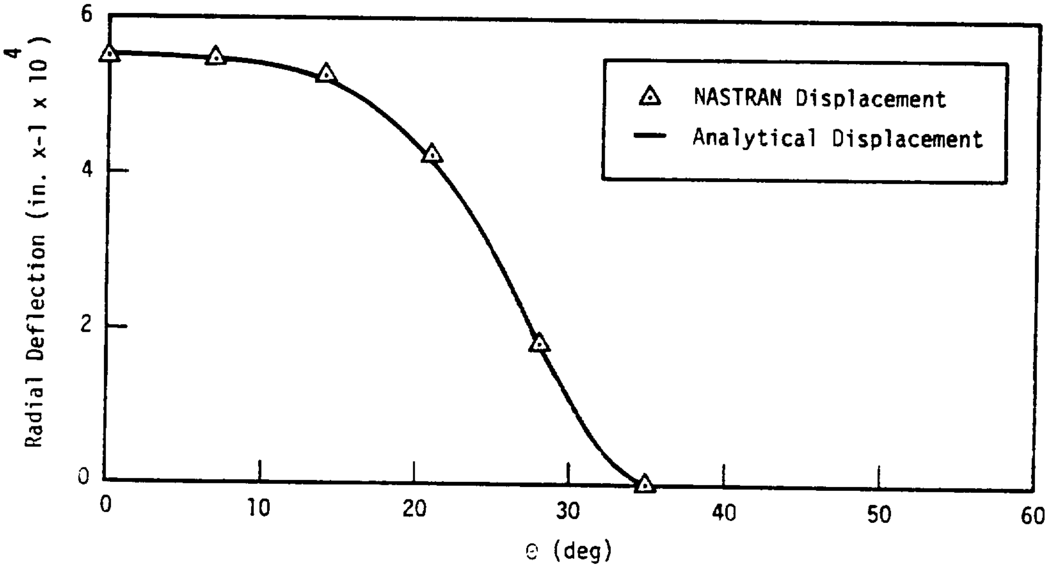

Figure 5. Comparison of NASTRAN and analytical displacements for 5 ring spherical shell - clamped boundary.

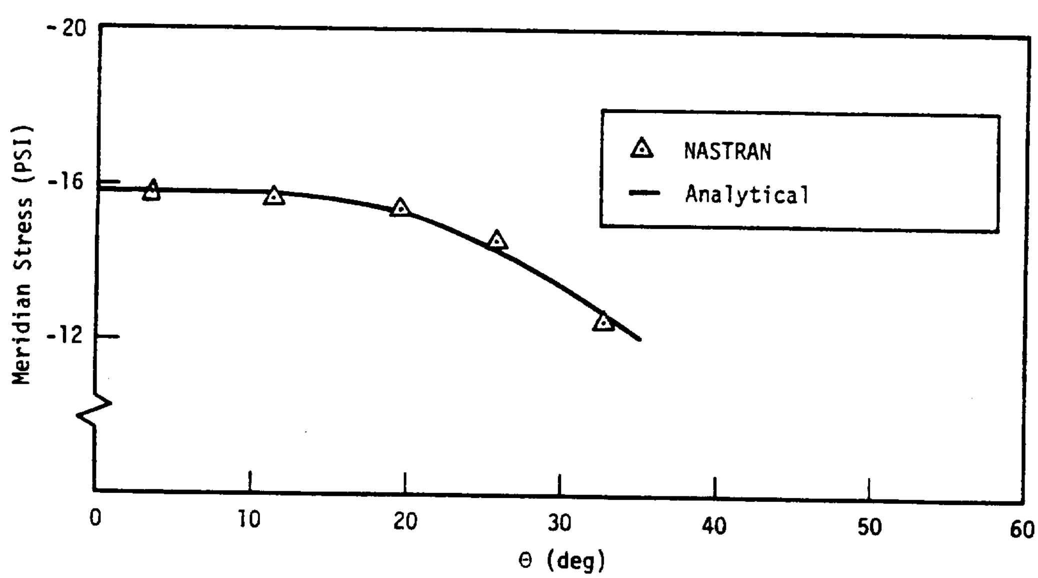

Figure 6. Comparison of NASTRAN and analytical meridian stress for 5 ring spherical shell - clamped boundary.

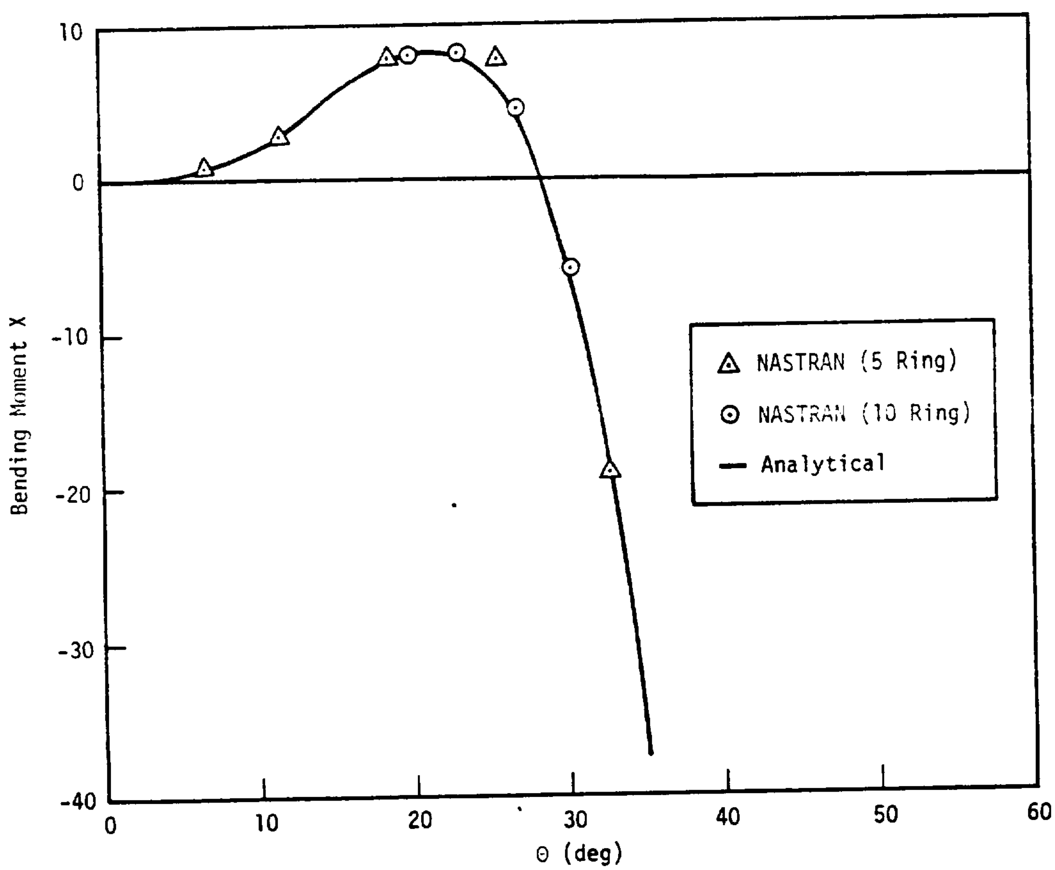

Figure 7. Comparison of NASTRAN and analytical bending moment for 5 ring spherical shell - clamped boundary.

Free Rectangular (QDMEM) Plate with Thermal Loading (1-3-1)
Free Rectangular (QDMEM1) Plate with Thermal Loading (1-3-2)
Free Rectangular (QDMEM2) Plate with Thermal Loading (1-3-3)

A. Description

Problem 1-3-1 demonstrates the use of thermal loading conditions and temperature-dependent materials. The model, a rectangular plate shown in Figure 1, is given a temperature gradient which causes internal loads and elastic deflections. Since there are two planes of symmetry, only one-quarter of the structure needs to be modeled (the shaded portion shown in Figure 1). The analysis has been performed using three different NASTRAN membrane plate elements. The two variations of this problem are obtained by replacing the quadrilateral membrane elements, QDMEM, with QDMEM1 and QDMEM2 membrane elements to illustrate their application to this type of problem (Problems 1-3-2 and 1-3-3, respectively).

B. Input

The finite element model for the quarter section is shown in Figure 2. Figure 3 shows the thermal loading condition. The temperature load is constant in the y direction and symmetric about the y-axis. Since membrane elements are used to model the structure, it is necessary to remove all rotational degrees of freedom and translational degrees of freedom normal to the membrane. The symmetric boundary conditions were modeled by constraining the displacements normal to the planes of symmetry. The material used has temperature-dependent elasticity (as defined in Reference 5) therefore, the INPUT module cannot be used for this application. The CNGRNT bulk data card can be used if the congruency is defined in one direction.

1. Parameters

$L = 36.0$ in	(length)
$W = 24.0$ in	(width)
$t = 0.25$ in	(thickness)
$E = 10.4 \times 10^6$ lb/in^2	(modulus of elasticity at T_o)
$\nu = 0.3$	(Poisson's ratio)
$\alpha = 12.7 \times 10^{-6}$ in/in/°F	(thermal expansion coefficient)
$T_o = 75.0$ °F	(thermal expansion reference temperature)

2. Constraints

$u_x = 0.0$ at $x = 0.0$

$u_y = 0.0$ at $y = 0.0$

$u_z = \Theta_x = \Theta_y = \Theta_z = 0.0$ at all Grids

3. Loads

The thermal loading is specified with TEMP Bulk Data cards. Young's modulus is specified as a function of temperature with MATT1 and TABLEM1 cards.

C. Results

There is no theoretical solution to this problem. However, this problem represents a model of a laboratory experiment described in Reference 5. Figures 4 and 5 present the NASTRAN stresses and the experimentally measured stresses reported in the reference.

D. Driver Decks and Sample Bulk Data

```
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 No.

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 2   UMF       1977    10310
 3   APP       DISPLACEMENT
 4   SØL       1,1
 5   TIME      6
 6   CEND
 7   TITLE = FREE RECTANGULAR PLATE WITH THERMAL LØADING
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-3-1
 9       LABEL = LINEARLY VARYING THERMAL LØAD - TEMPERATURE DEPENDENT MATERIAL
10       SPC = 1
11       TEMPERATURE = 1
12   ØUTPUT
13   SET 1 = 1 THRU 13, 79 THRU 91, 157 THRU 169, 235 THRU 247
14   SET 2 = 1 THRU 26
15       DISPLACEMENTS = 1
16       ØLØAD = 2
17   $   STRESSES FØR PØINTS ØN PUBLISHED CURVES
18       SET 3 = 1 THRU 12, 15,20, 28,33, 41,46, 54,59, 67,72, 80,85, 93,98,
19       106,111, 118 THRU 129, 132,137, 145,150, 158,163, 171,176,
20       184,189, 197,202, 210,215, 223,228
21       STRESSES = 3
22   BEGIN BULK
23   ENDDATA
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CNGRNT	1	14	27	40	53	66	79	92	+CNG11
+CNG11	105	118	131	144	157	170	183	196	+CNG12
+CNG12	209	222							
CQDMEM	1	21	1	2	15	14	.00		
GRDSET							3456		
GRID	1		.0	.0	.0				
MAT1	75	10.400+6		.3		12.700-6	75.		
MATT1	75	100							
PARAM	IRES	1							
PQDMEM	21	75	.25						
SPC1	1	1	1	14	27	40	53	66	CSPC-A
+SPC-A	79	92	105	118	131	144	157	170	CSPC-B
+SPC-B	183	196	209	222	235				
TABLEM1	100								+TM1
+TM1	80.	10.4+6	150.	10.15+6	200.	9.84+6	250.	9.51+6	+TM2
+TM2	300.	9.15+6	ENDT						
TEMP	1	1	245.000	2	232.500	3	220.000		

Card No.

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 2   UMF      1977     10320
 3   APP      DISPLACEMENT
 4   SØL      1,1
 5   TIME     6
 6   CEND
 7   TITLE = FREE RECTANGULAR PLATE WITH THERMAL LØADING (QDMEM1 ELEMENTS)
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-3-2
 9       LABEL = LINEARLY VARYING THERMAL LØAD - TEMPERATURE DEPENDENT MATERIAL
10           SPC = 1
11           TEMPERATURE = 1
12       ØUTPUT
13       SET 1 = 1 THRU 13, 79 THRU 91, 157 THRU 169, 235 THRU 247
14       SET 2 = 1 THRU 26
15           DISPLACEMENTS = 1
16           ØLØAD = 2
17   $       STRESSES FØR PØINTS ØN PUBLISHED CURVES
18       SET 3 = 1 THRU 12, 15,20, 28,33, 41,46, 54,59, 67,72, 80,85, 93,98,
19           106,111, 118 THRU 129, 132,137, 145,150, 158,163, 171,176,
20           184,189, 197,202, 210,215, 223,228
21           STRESSES = 3
22   BEGIN BULK
23   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	14	27	40	53	66	79	92	+CNG11
+CNG11	105	118	131	144	157	170	183	196	+CNG12
+CNG12	209	222							
CQDMEM1	1	21	1	2	15	14	.00		
GRDSET							3456		
GRID	1		.0	.0	.0				
MAT1	75	10.400+6		.3		12.700-6	75		
MATT1	75	100							
PARAM	IRES	1							
PQDMEM1	21	75	.25						
SPC1	1	1	1	14	27	40	53	66	CSPC-A
+SPC-A	79	92	105	118	131	144	157	170	CSPC-B
+SPC-B	183	196	209	222	235				
TABLEM1	100								+TM1
+TM1	80.	10.4+6	150.	10.15+6	200.	9.84+6	250.	9.51+6	+TM2
+TM2	300.	9.15+6	ENDT						
TEMP	1	1	245.000	2	232.500	3	220.000		

```
Card
 No.

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 2   UMF       1977    10330
 3   APP       DISPLACEMENT
 4   SØL       1,1
 5   TIME      6
 6   CEND

 7   TITLE = FREE RECTANGULAR PLATE WITH THERMAL LØADING (QDMEM2 ELEMENTS)
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-3-3
 9       LABEL = LINEARLY VARYING THERMAL LØAD - TEMPERATURE DEPENDENT MATERIAL
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11            TEMPERATURE = 1
12       ØUTPUT
13       SET 1 = 1 THRU 13, 79 THRU 91, 157 THRU 169, 235 THRU 247
14       SET 2 = 1 THRU 26
15            DISPLACEMENTS = 1
16            ØLØAD = 2
17   $   STRESSES FØR PØINTS ØN PUBLISHED CURVES
18       SET 3 = 1 THRU 12, 15,20, 28,33, 41,46, 54,59, 67,72, 80,85, 93,98,
19            106,111, 118 THRU 129, 132,137, 145,150, 158,163, 171,176,
20            184,189, 197,202, 210, 215, 223,228
21            STRESSES = 3
22   BEGIN BULK
23   ENDDATA
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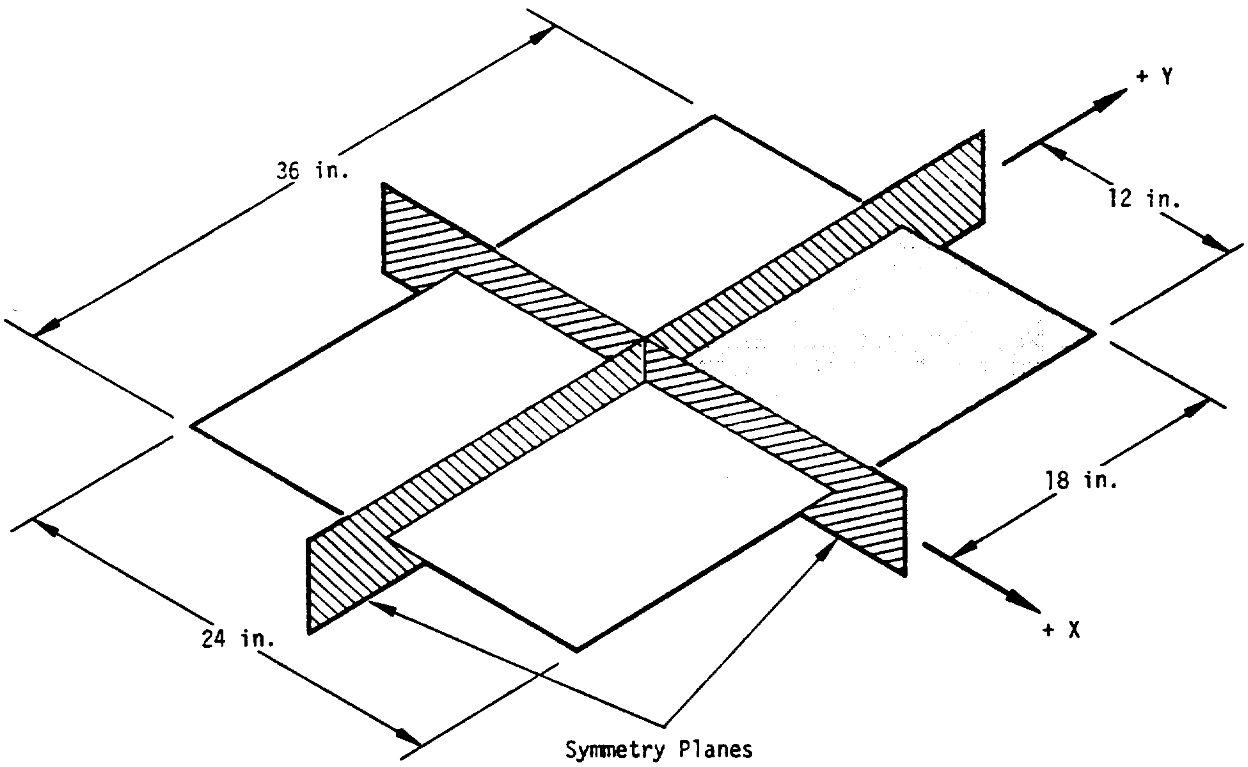
1	2	3	4	5	6	7	8	9	10
CNGRNT	1	14	27	40	53	66	79	92	+CNG11
+CNG11	105	118	131	144	157	170	183	196	+CNG12
+CNG12	209	222							
CQDMEM2	1	21	1	2	15	14	.00		
GRDSET							3456		
GRID	1		.0	.0	.0				
MAT1	75	10.400+6		.3		12.700-6	75.		
MATT1	75	100							
PARAM	IRES	1							
PQDMEM2	21	75	.25						
SPC1	1	1	1	14	27	40	53	66	CSPC-A
+SPC-A	79	92	105	118	131	144	157	170	CSPC-B
+SPC-B	183	196	209	222	235				
TABLEM1	100								+TM1
+TM1	80.	10.4+6	150.	10.15+6	200.	9.84+6	250.	9.51+6	+TM2
+TM2	300.	9.15+6	ENDT						
TEMP	1	1	245.000	2	232.500	3	220.000		



Note: Shaded area is quarter of plate modeled.

Figure 1. Free plate structure.

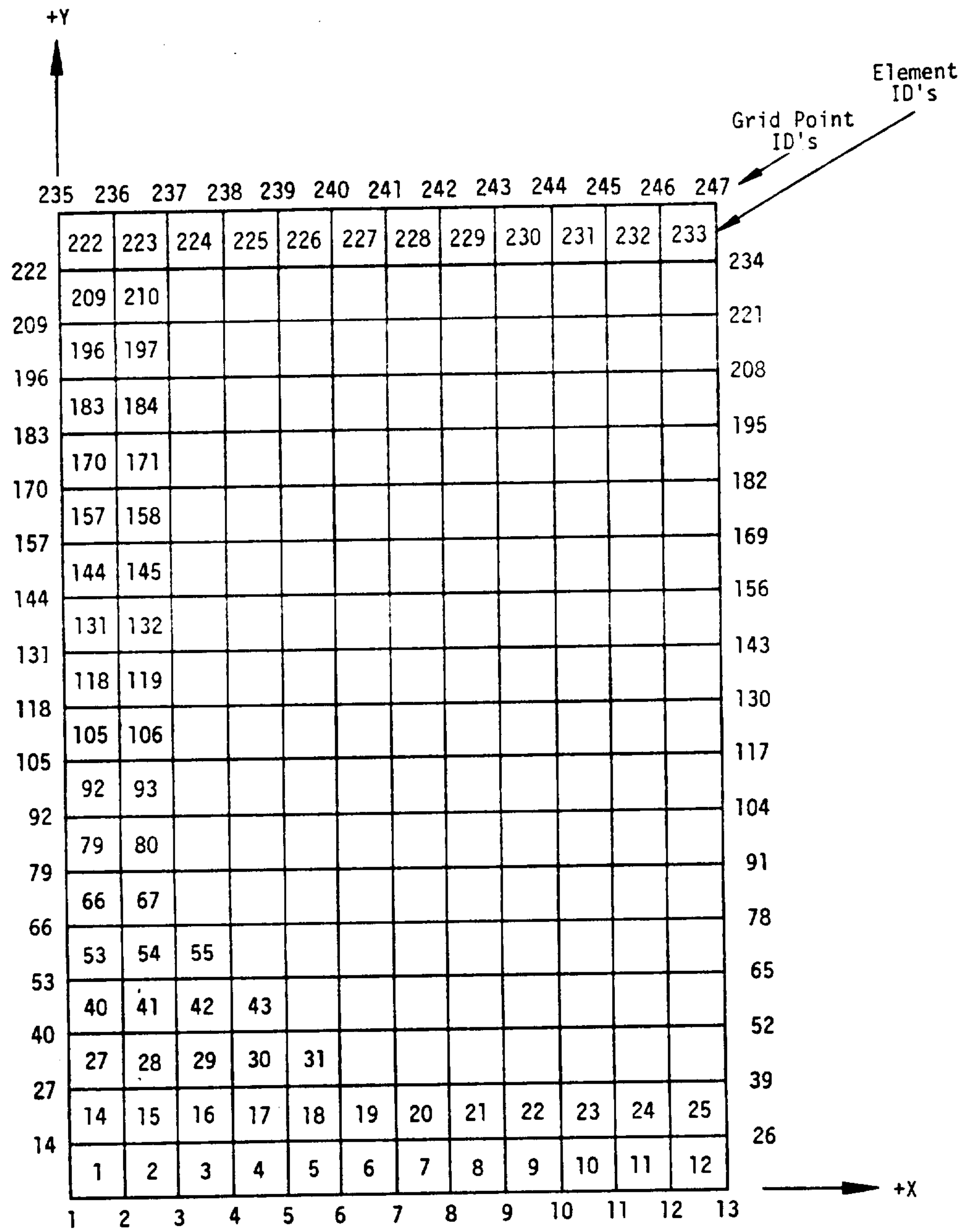


Figure 2. Free rectangular plate model.

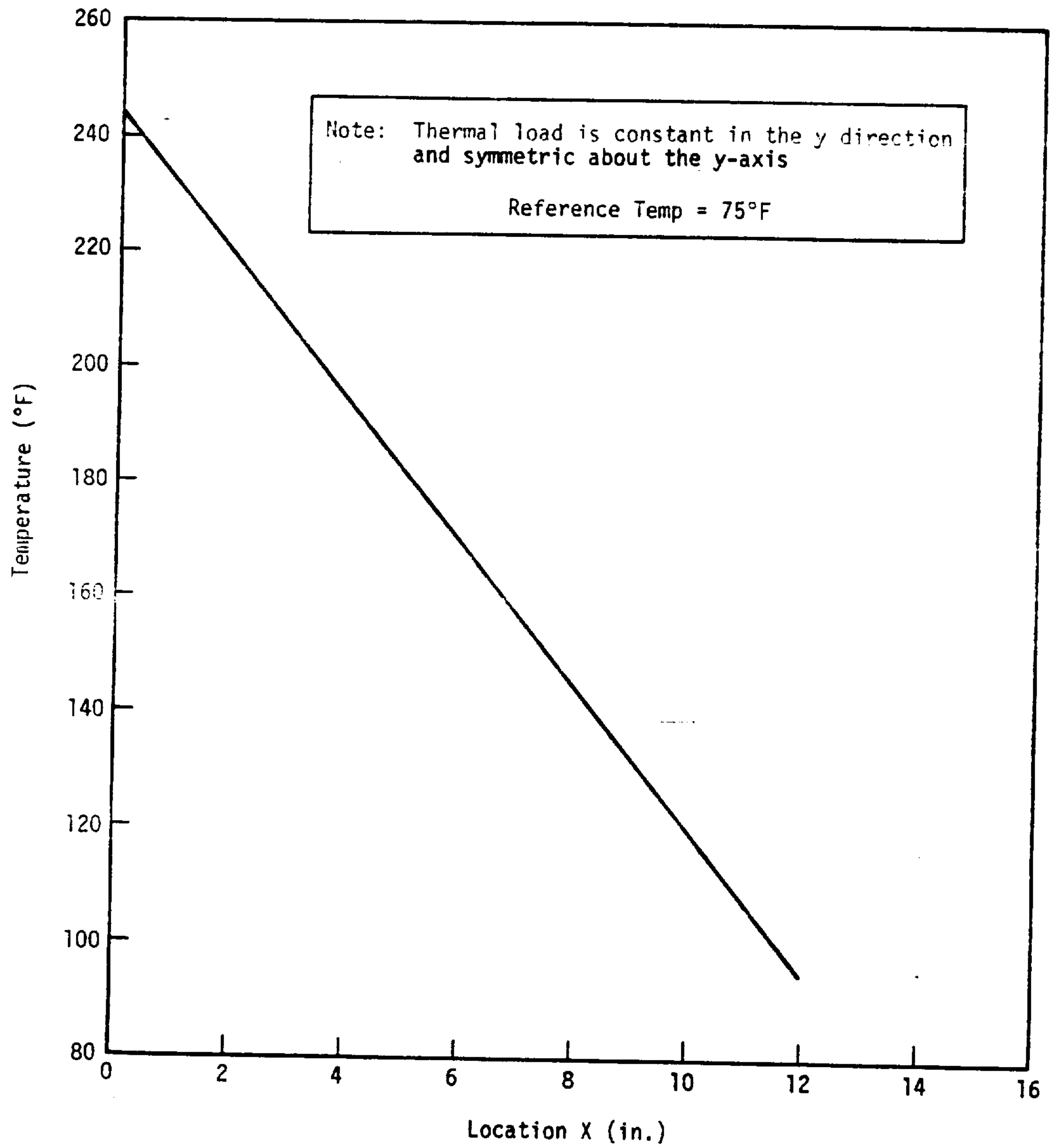


Figure 3. Thermal load applied to free rectangular plate.

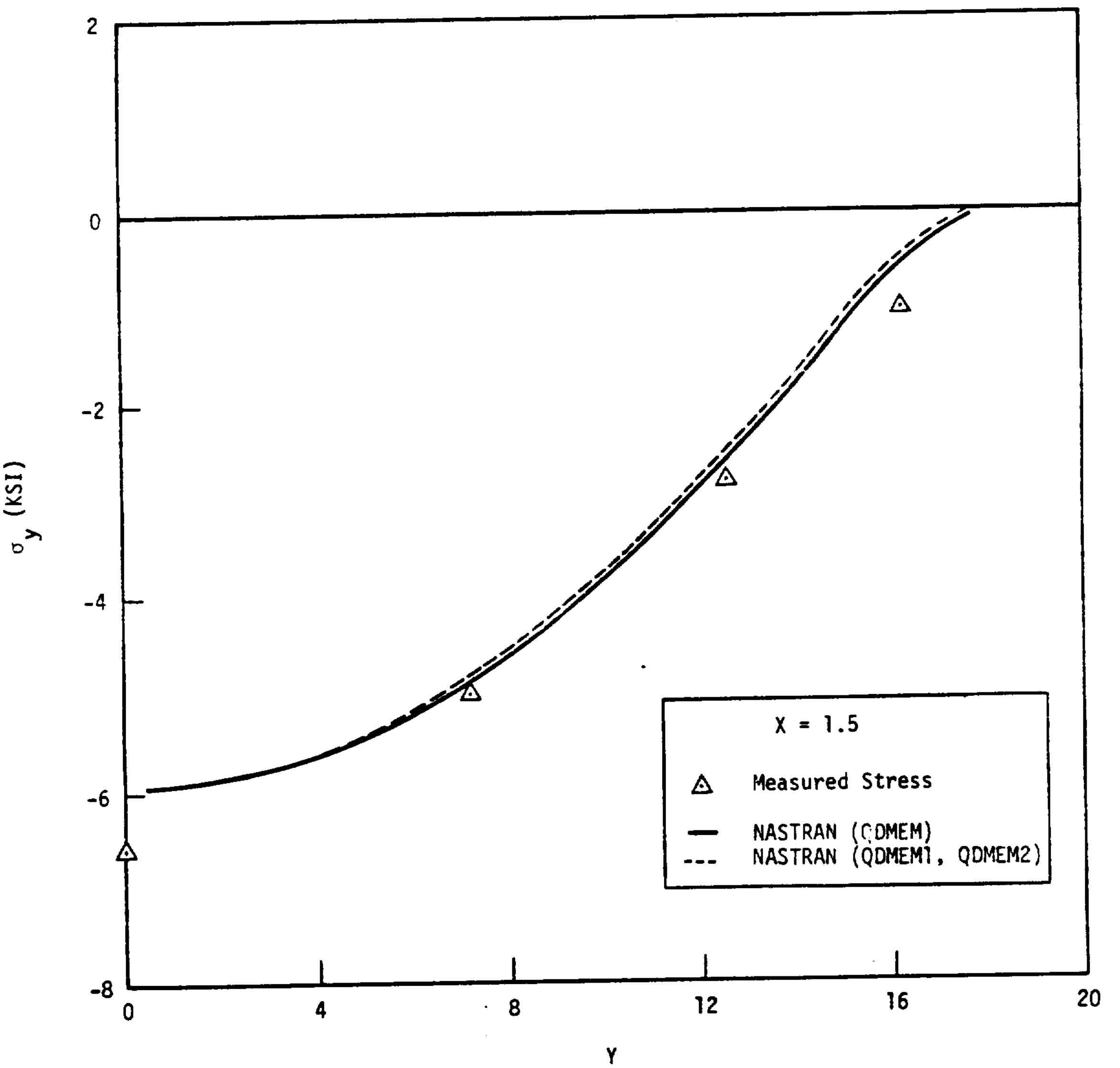


Figure 4. Comparison of NASTRAN and experimental stresses for free rectangular plate with thermal loading - temperature dependent properties.

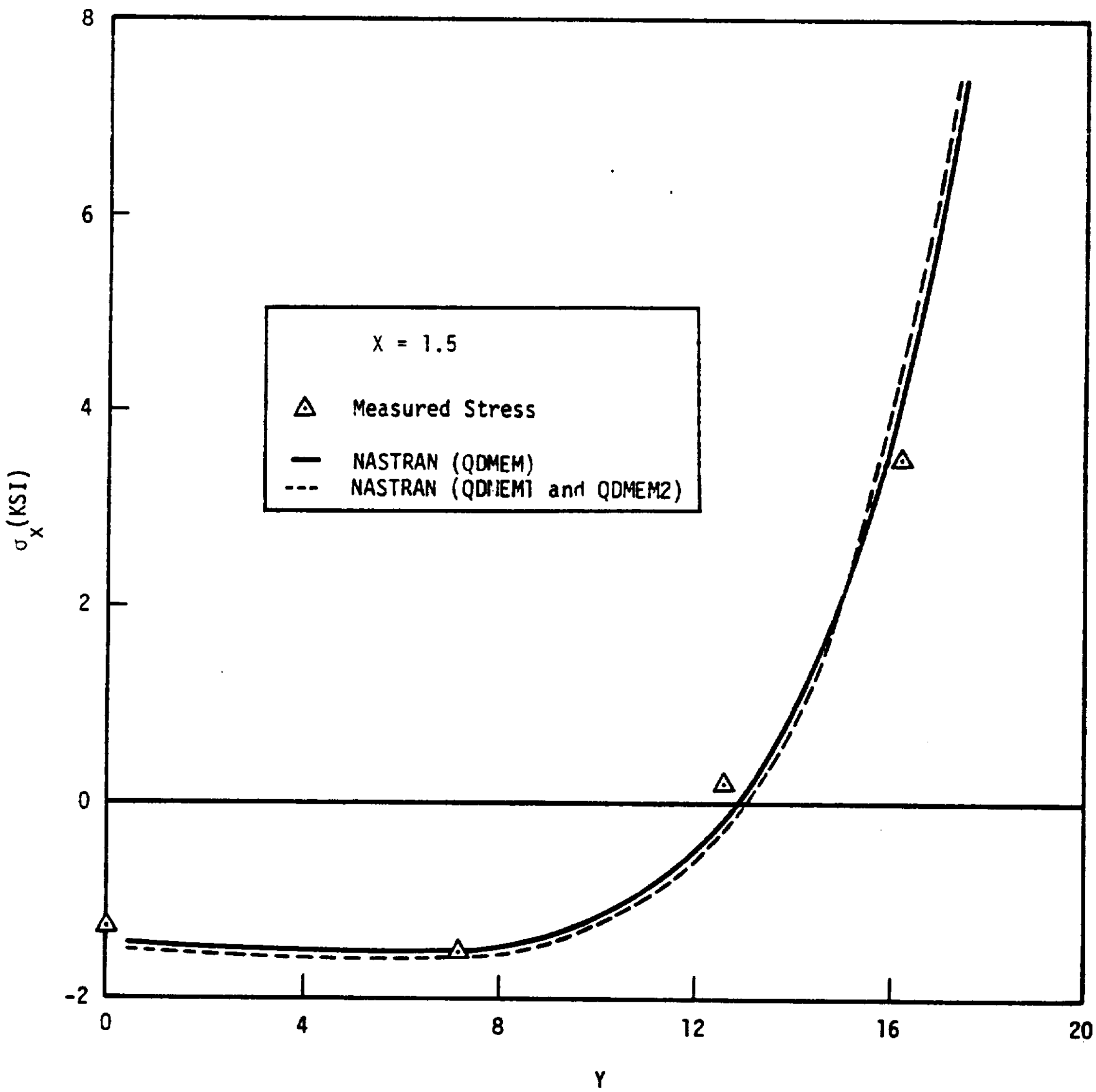


Figure 5. Comparison of NASTRAN and experimental stresses for free rectangular plate with thermal loading - temperature dependent properties.

RIGID FORMAT No. 1, Static Analysis

Long, Narrow, 5 x 50 Orthotropic Plate (1-4-1)
Long, Narrow, 5 x 60 Orthotropic Plate (1-4-2)
Long, Narrow, 5 x 50 Orthotropic Plate (INPUT, 1-4-3)
Long, Narrow, 5 x 60 Orthotropic Plate (INPUT, 1-4-4)

A. Description

A long, narrow, orthotropic plate is modeled and analyzed to illustrate NASTRAN operations with spill logic for problems too large for available core. Other features of this problem include grid point resequencing, use of orthotropic materials, application of quarter symmetry, use of the INPUT module, and modified restart to obtain additional output.

The plate to be modeled and its loading are shown in Figure 1. The 5 x 50 finite element quarter model is presented in Figure 2, showing minimum bandwidth. The same model is resequenced using SEQGP cards as shown in Figure 3 to create the poorest bandwidth.

The 5 x 50 model is presented as Problem 1-4-1. A restart driver deck (Problem 1-4-1A) is provided for this problem to include additional data that expands the model to nearly double its original length. A subsequent poor bandwidth resequence forces the exercise of spill logic. For Problem 1-4-2, the model is extended to a 5 x 60 grid pattern and it too is resequenced for poor bandwidth. Problems 1-4-3 and -4 are duplications of the two problems described above using the INPUT module to generate the grid point and element data cards.

These models could be run if desired with their optimal bandwidths by simply deleting the SEQGP cards from the bulk data.

B. Input

1. Parameters

Material Elastic Properties

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} 4.0+6 & 2.0+6 & 0. \\ 2.0+6 & 6.0+6 & 0. \\ 0. & 0. & 3.0+6 \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

I = .0833333 (area moment of inertia per unit width)

C. Results

The displacement and stress results from NASTRAN are presented along with theoretical results in Tables 1 and 2. The theoretical results are from an infinitely long continuous plate analyzed in Section 37 of Reference 4.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=(UMF,NPTP)
  1   ID        DEM1041,NASTRAN
  2   UMF       1977     10410
  3   APP       DISPLACEMENT
  4   TIME      30
  5   SØL       1,1
  6   CHKPNT    YES
  7   CEND

  8   TITLE = 5X50 LØNG, NARRØW, ØRTHØTRØPIC PLATE
  9   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-4-1
 10       LABEL = SEQUENCED FØR WIDE BAND
 11       SPC = 37
 12       LØAD = 17
 13   ØUTPUT
 14       ØLØAD = ALL
 15       SPCFØRCE = ALL
 16   BEGIN BULK
 17   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CQUAD1	1	23	1	2	8	7	.00		
FØRCE	17	1		1.0			.9958928		
GRDSET							126		
GRID	1		.0	.0	.0				
MAT2	1234	4.0+6	2.0E6		6.E+06		3.0+06	1.0	+MAT2
+MAT2	.5	1.0	.05	10.0	.004	1.+12	2.+12	3.+12	
PARAM	IRES	1							
PQUAD1	23			1234	.0833333				+PQD
+PQD	.005+2	-500.-3							
SEQGP	1	1	2	52	3	103	4	154	
SPC1	337	34	6	12	18	24	30	36	+1
+1	42	48	54	60	66	72	78	84	+2
SPCADD	37	337	347	354					

```
NASTRAN FILES=ØPTP
ID          DEM1041A,RESTART
APP         DISPLACEMENT
TIME        5
SØL         1,1
DIAG 14
CEND

TITLE = 5 X 90 LØNG NARRØW ØRTHØTRØPIC PLATE - RESTART, MØDIFIED MØDEL
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-4-1A
LABEL = SEQUENCED FØR WIDE BAND TØ SPILL
    ECHØ = BØTH
    SPC = 37
    LØAD = 17
ØUTPUT
    SET 1 = 1 THRU 37,42,43,49,55,61,67,73,79,85,91,97,103,
                        109,115,121,
         127,133,139,145,150 THRU 157,163,169,175,181,187,193,199,205,
         211,217,223,229,235,241,247,253,259,265,271,277,283,289,295,
         301 THRU 306
    SET 2 = 1 THRU 37,151 THRU 161,295 THRU 300
    SET 3 = 1 THRU 36, 151 THRU 162, 295 THRU 306
         DISPLACEMENTXSET = 1
         STRESSXSET = 2
         ELFØRCEXSET = 2
         SPCFØRCEXSET = 3
BEGIN BULK
ENDDATA
```

Note: The Restart Dictionary from Problem 1-4-1 is required in the Executive Control Deck.

Data Cards to reconfigure the model are required in the Bulk Data Deck.

```
Card
 No.

 0   NASTRAN FILES=UMF
 1   ID        DEM1042,NASTRAN
 2   UMF       1977    10420
 3   APP       DISPLACEMENT
 4   TIME      30
 5   SØL       1,1
 6   CEND

 7   TITLE = 5X60 LØNG, NARRØW, ØRTHØTRØPIC PLATE
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-4-2
 9             LABEL = SEQUENCED FØR WIDE BAND
10             SPC = 37
11             LØAD = 17
12   ØUTPUT
13             ØLØAD = ALL
14             SPCFØRCE = ALL
15   BEGIN BULK
16   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CQUAD1	1	23	1	2	8	7	.00		
FØRCE	17	1		1.0			.9958928		
GRDSET							126		
GRID	1		.0	.0	.0				
MAT2	1234	4.0+6	2.0E6		6.E+06		3.0+06	1.0	+MAT2
+MAT2	.5	1.0	.05	10.0	.004	1.+12	2.+12	3.+12	
PARAM	IRES	1							
PQUAD1	23			1234	.0833333				+PQD
+PQD	.005+2	-500.-3							
SEQGP	1	1	2	62	3	123	4	184	
SPC1	337	4	1	2	3	4	5	6	
SPCADD	37	337	347	354					

Card
No.

```
 0  NASTRAN FILES=UMF
 1  ID        DEM1043,NASTRAN
 2  UMF       1977     10430
 3  APP       DISPLACEMENT
 4  TIME      30
 5  SØL       1,1
 6  DIAG 14
 7  ALTER  1
 8  PARAM    //C,N,NØP/V,N,TRUE=-1 $
 9  INPUT,  GEØM1,,,,/G1,G2,,G4,/C,N,3/C,N,1 $
10  EQUIV   G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE $
11  ENDALTER
12  CEND

13  TITLE = 5X50 LØNG, NARRØW, ØRTHØTRØPIC PLATE
14  SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-4-3
15      LABEL = SEQUENCED FØR WIDE BAND
16      SPC = 5050
17      LØAD = 17
18  ØUTPUT
19      ØLØAD = ALL
20      SPCFØRCE = ALL
21  BEGIN BULK
22  ENDDATA

23          5       50      2.0      2.0     126      0.0      0.0
24          4        5        0       34       0        0
```

1	2	3	4	5	6	7	8	9	10
FØRCE	17	1		1.0			.9958928		
MAT2	1234	4.0+6	2.0E6		6.E+06		3.0+06	1.0	+MAT2
+MAT2	.5	1.0	.05	10.0	.004	1.+12	2.+12	3.+12	
PARAM	IRES	1							
PQUAD1	101			1234	.0833333				+PQD
+PQD	.005+2	-500.-3							
SEQGP	1	1	2	52	3	103	4	154	

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1044,NASTRAN
  2   UMF       1977    10440
  3   APP       DISPLACEMENT
  4   TIME      30
  5   SØL       1,1
  6   DIAG 14
  7   ALTER 1
  8   PARAM //C,N,NØP/V,N,TRUE=-1 $
  9   INPUT     GEØM1,,,,/G1,G2,,G4,/C,N,3/C,N,1 $
 10   EQUIV     G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE $
 11   ENDALTER
 12   CEND

 13   TITLE = 5X60 LØNG, NARRØW, ØRTHØTRØPIC PLATE
 14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-4-4
 15       LABEL = SEQUENCED FØR WIDE BAND
 16       SPC = 5060
 17       LØAD = 17
 18   ØUTPUT
 19       ØLØAD = ALL
 20       SPCFØRCE = ALL
 21   BEGIN BULK
 22   ENDDATA

 23          5        4       60      2.0       2.0      126      0.0      0.0
 24          4        5        0       34
```

1	2	3	4	5	6	7	8	9	10
FØRCE	17	1		1.0			.9958928		
MAT2	1234	4.0+6	2.0E6		6.E+06		3.0+06	1.0	+MAT2
+MAT2	.5	1.0	.05	10.0	.004	1.+12	2.+12	3.+12	
PARAM	IRES	1							
PQUAD1	101			1234	.0833333				+PQD
+PQD	.005+2	-500.-3							
SEQGP	1	1	2	62	3	123	4	184	

Table 1. NASTRAN and Theoretical Displacements for Long, Narrow, Orthotropic Plate.

GRID	Z DISPLACEMENT x 10^4	
	THEORY	NASTRAN
1	3.048	3.037
2	2.899	2.889
3	2.466	2.457
4	1.792	1.785
5	0.942	0.939
7	2.949	2.940
13	2.723	2.714
19	2.446	2.435
25	2.157	2.145
31	1.880	1.866
37	1.625	1.611
43	1.397	1.383

Table 2. NASTRAN and Theoretical Stresses for Long, Narrow, Orthotropic Plate.

EL. ID.	STRESS X		STRESS Y		SHEAR STRESS	
	THEORY	NASTRAN	THEORY	NASTRAN	THEORY	NASTRAN
1	19.05	18.90	20.35	20.40	-0.39	-0.39
2	17.19	17.05	18.36	18.40	-1.12	-1.13
3	13.64	13.53	14.57	14.60	-1.74	-1.76
4	8.76	8.69	9.35	9.38	-2.19	-2.22
5	3.02	2.99	3.22	3.23	-2.43	-2.46
7	15.86	15.76	12.91	12.90	-0.84	-0.88
13	13.27	13.20	8.28	8.23	-1.03	-1.06
19	11.14	11.08	5.38	5.33	-1.07	-1.09
25	9.37	9.33	3.55	3.51	-1.02	-1.04
31	7.90	7.86	2.38	2.36	-0.94	-0.95
37	6.67	6.63	1.64	1.63	-0.84	-0.85

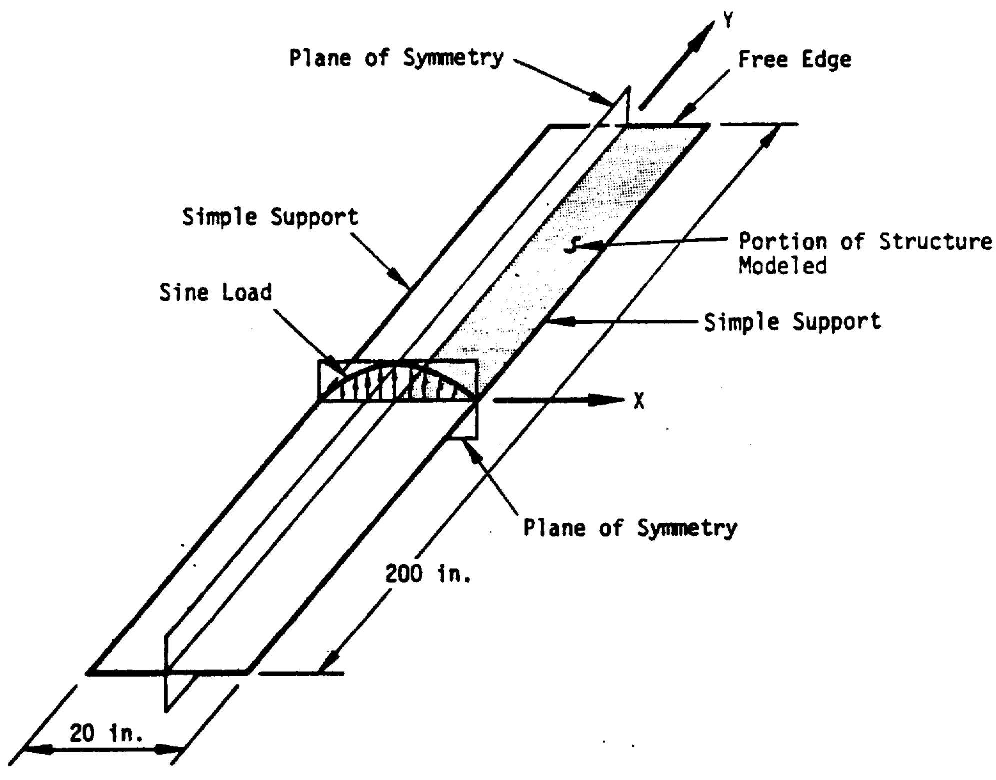


Figure 1. Simply-supported long narrow orthotropic plate

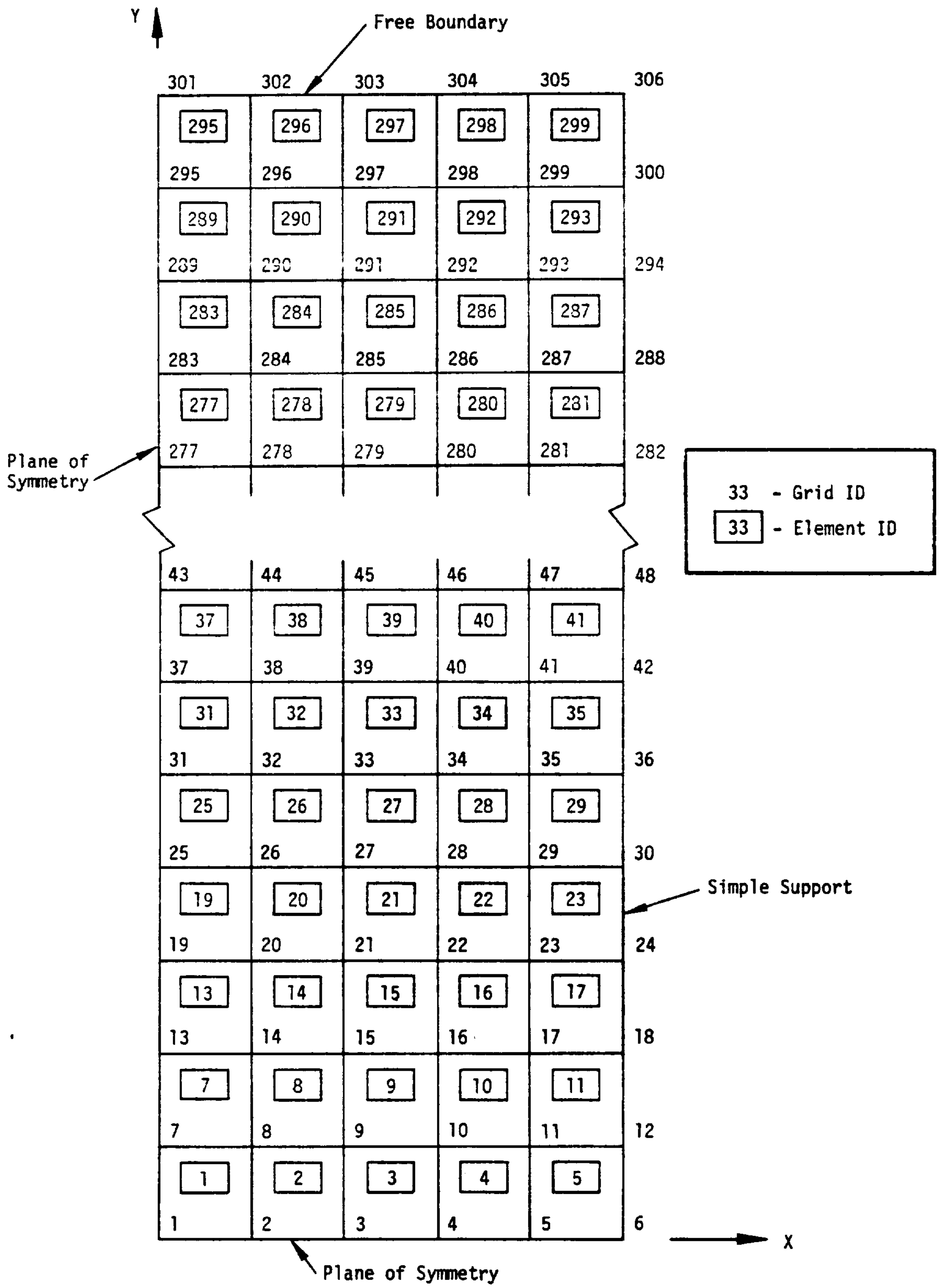


Figure 2. 5 x 50 Long, narrow, orthotropic plate model.

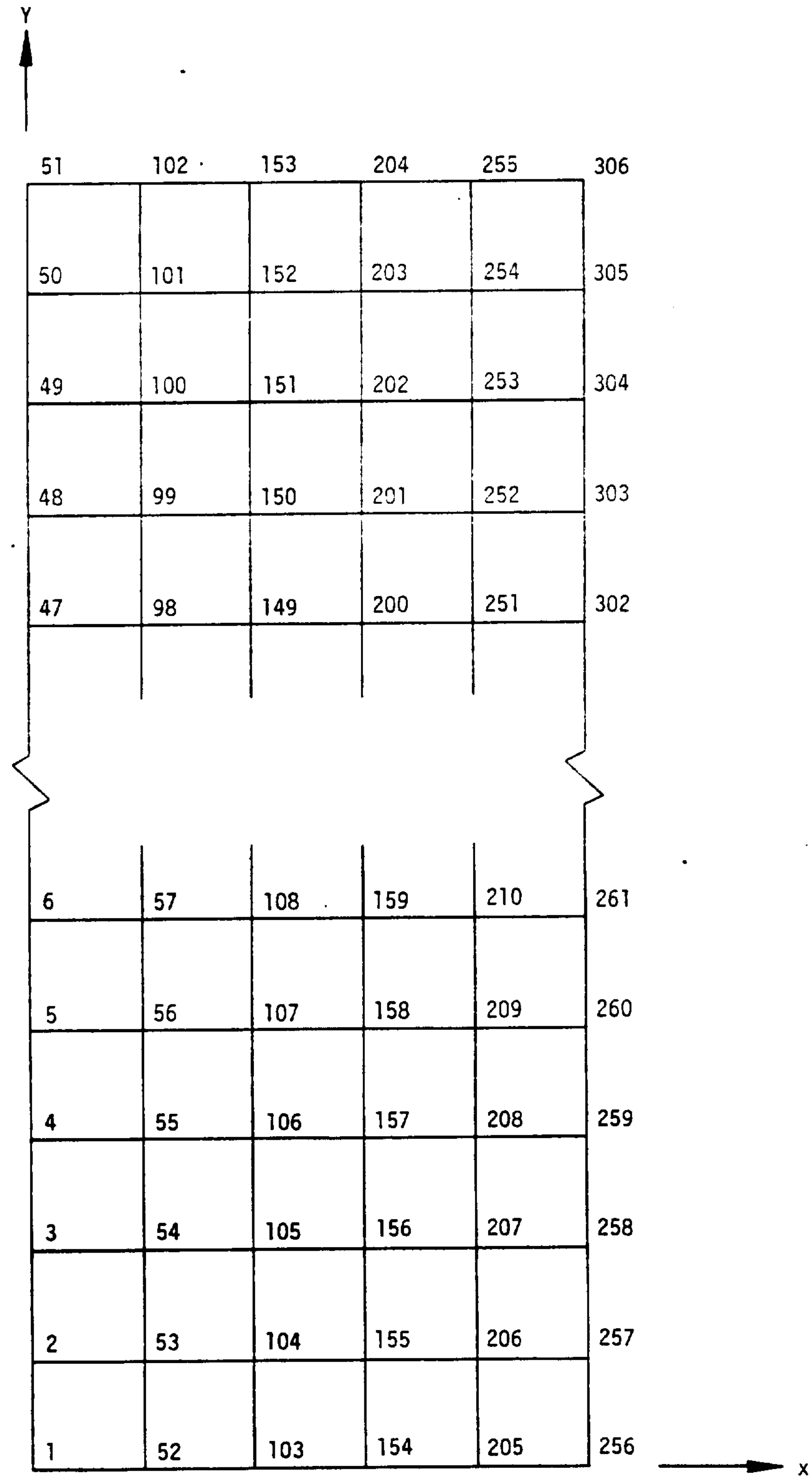


Figure 3. Long, narrow, orthotropic plate model resequenced for wide band.

RIGID FORMAT No. 1, Static Analysis

Nonsymmetric Bending of a Cylinder of Revolution (1-5-1)

A. Description

This problem illustrates the application of the conical shell element and its related special data. This element uses the Fourier components of displacement around an axisymmetric structure as the solution coordinates. The geometry of the structure is defined by rings instead of grid points. Its constraints must be defined by the particular Fourier harmonics, and the loads must be defined either with special data or in a harmonic form. This element may not be used in conjunction with any of the other structural elements.

The structure to be solved is described in Reference 6 and illustrated in Figure 1. It consists of a short, wide cylinder with a moderate thickness ratio. The applied loads and the output stresses are pure uncoupled harmonics. The basic purpose of this problem is to check the harmonic deflections, element stresses, and forces. Figures 2 and 3 compare the NASTRAN results with the results given in Reference 6.

B. Input

The Fourier coefficients of the applied moment per length are:

$$m_n = \cos(n\theta) \quad . \tag{1}$$

The applied input loads are defined as:

$$M_n = \int_0^{2\pi} m_n \cos(n\theta)\, R\, d\theta \quad . \tag{2}$$

The values of applied moment on the MØMAX cards are:

$$M_{n\phi} = \pi R \qquad n > 0 \quad , \tag{3}$$

and

$$M_{o\phi} = 2\pi R \qquad n = 0 \quad . \tag{4}$$

The applied moments for each harmonic are shown in Figure 1. The bending moments in the elements are defined as:

$$M_v = \text{Moment about } u_\phi \quad , \tag{5}$$

and

$$M_u = \text{Moment about } u_z \quad . \tag{6}$$

Positive bending moments indicate compression on the outer side.

1. Parameters:

   $R = 50$ Radius

   $s = 50$ Height

   $t = 1.0$ Thickness

   $E = 91.0$ Modulus of Elasticity

   $\nu = 0.3$ Poisson's Ratio

2. Loads:

   $M_{n(100)} = 157.0796$ Force·Length

   $M_{n(50)} = -157.0796$ Force·Length

3. Single Point Constraints:

Ring ID	Harmonic	Coordinates	
50	all	u_r, u_ϕ, u_z	Radial, tangential and axial translations
100	all	u_r, u_ϕ, u_z	Radial, tangential and axial translations
all	all	θ_r	Rotation normal to surface

The AXISYM = CØSINE statement in case control defines the motions to be symmetric with respect to the x-z plane.

C. Results

Theoretical and NASTRAN results for element bending moments and radial deflections for 4 of the 20 harmonics used are given in Figure 2 and 3. Notice that for higher harmonics the effect of the load is limited to the edges. A smaller element size at the edges and a relatively large size in the center would have given the same accuracy with fewer degrees of freedom.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID       DEM1051,NASTRAN
  2   UMF      1977     10510
  3   TIME     24
  4   APP      DISP
  5   SØL      1,1
  6   CEND

  7   TITLE = NØNSYMMETRIC BENDING ØF A CYLINDER ØF REVØLUTIØN
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-5-1
  9       LØAD = 15
 10   AXISYM = CØSINE
 11   ØUTPUT
 12       SET 1 = 5,10,15,20,25,30,35,40,45,50,100
 13       SET 2 = 1,6,11,16,21,26,31,36,41,46,50
 14       DISP = 1
 15       ELFØRCE = 2
 16   HARMØNICS = ALL
 17   BEGIN BULK
 18   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
AXIC	20								
CCØNEAX	1	15	100	1					
MAT1	15	91.0		.3	.5				
MØMAX	15	50	0	157.0796		2.0			
PCØNEAX	15	15	1.0	15	.0833333	15	1.0	.5	+PC
+PC	.0	.5	.0	90.	180.				
PØINTAX	200	100							
RINGAX	1		50.0	1.0			4		

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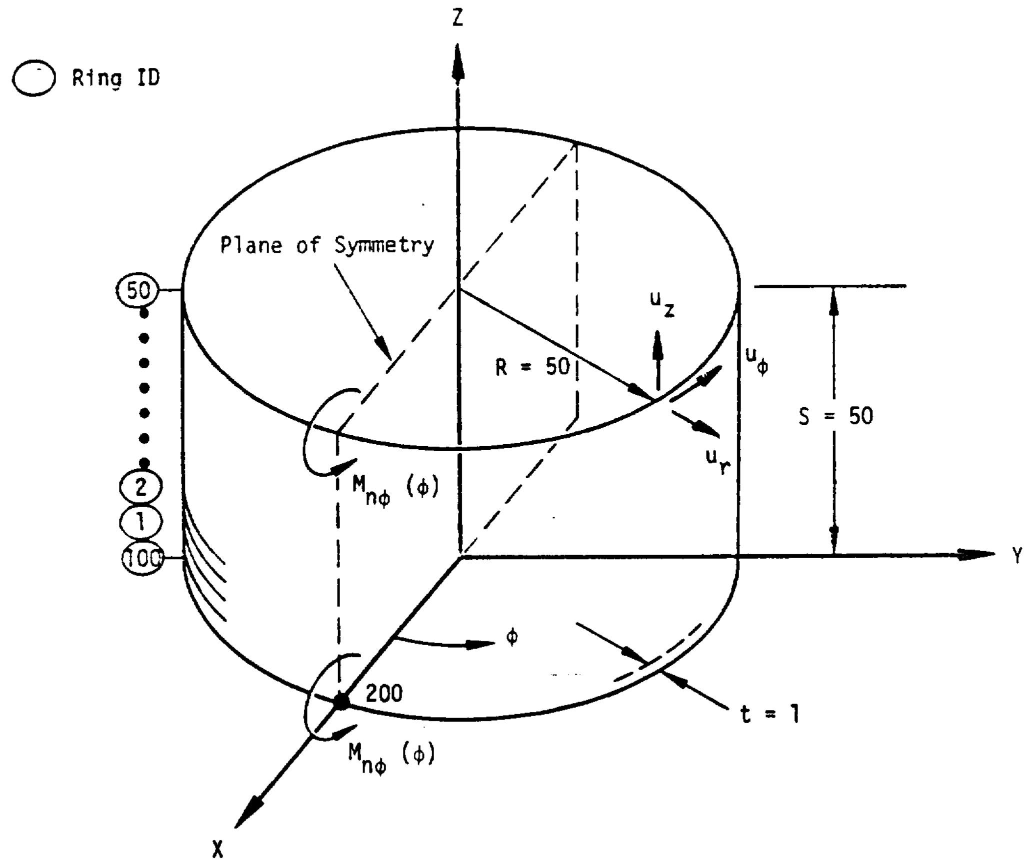


Figure 1. Cylinder under harmonic loads.

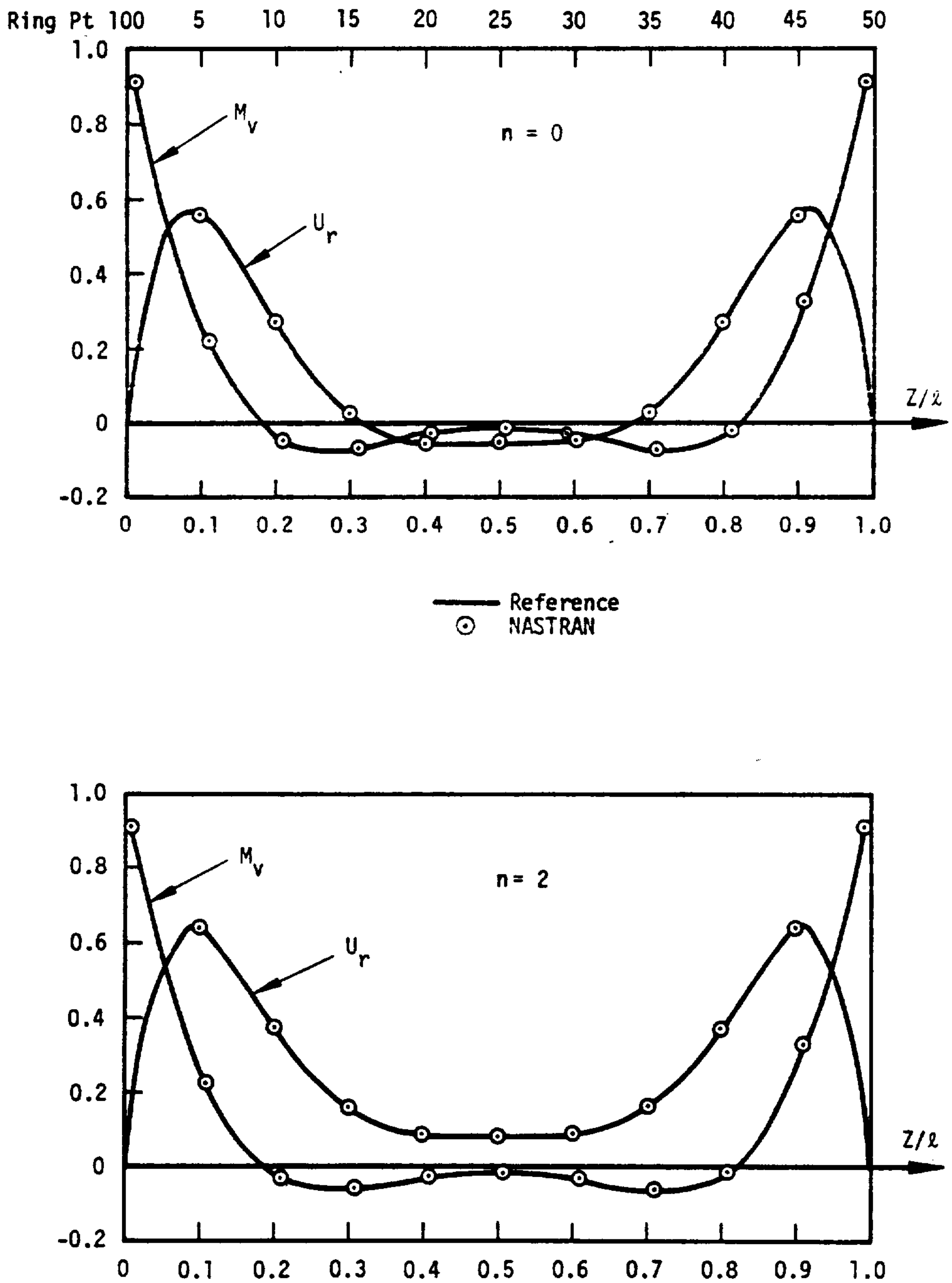
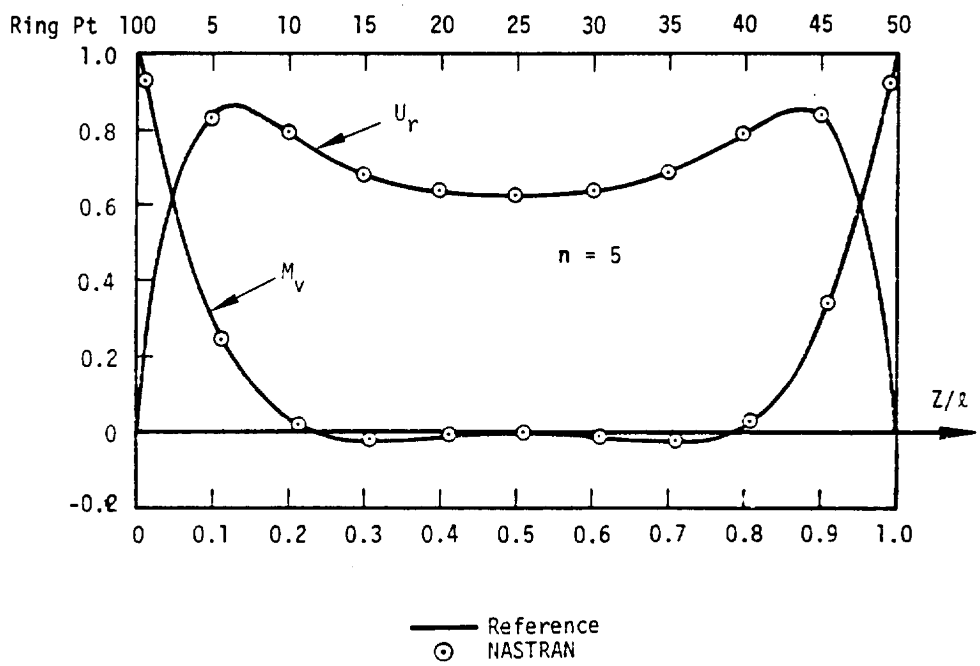


Figure 2. Element bending moments and radial deflections along length of cylinder.



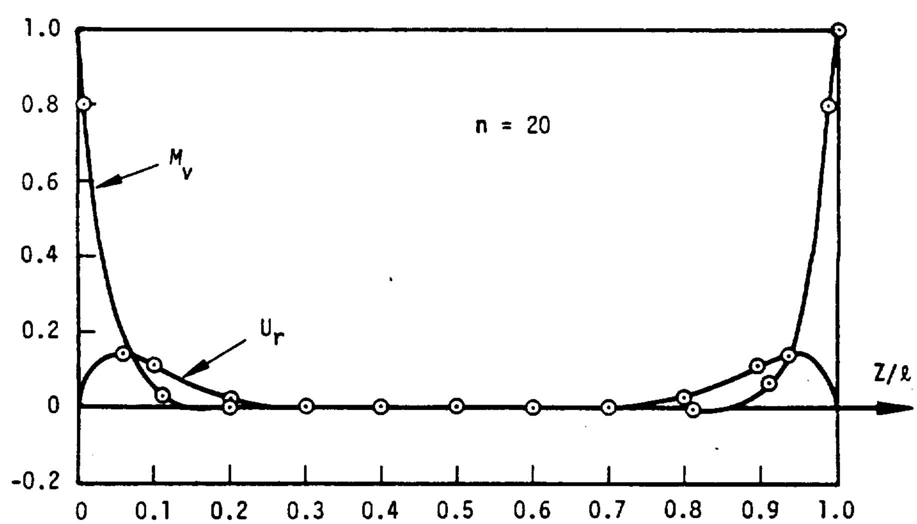


Figure 3. Element bending moments and radial deflections along length of cylinder.

Solid Disk with Radially Varying Thermal Load (1-6-1)

A. Description

This problem demonstrates the use of the NASTRAN axisymmetric solid element, the trapezoidal ring. The trapezoidal ring elements are used to model a solid circular disk which is subjected to a radially varying thermal load of the form

$$T = 100\left(1 - \frac{r^2}{b^2}\right) \quad , \qquad (1)$$

where

r = the radius at any point in the disk,

and b = the outside radius = 0.10 inches.

B. Input

The structure is shown in Figure 1 along with its associated material properties and pertinent dimensions. The finite element idealization employed for this structure is shown in Figure 2. The thermal loading on the solid disk is established via an internally generated thermal load vector derived from specified grid point temperature values.

1. Parameters

$R = 0.10$ in	(radius)
$t = 0.01$ in	(thickness)
$E = 1.0 \times 10^7$ lb/in^2	(modulus of elasticity)
$\upsilon = 0.3$	(Poisson's ratio)
$\alpha = 0.1 \times 10^{-6}$ in/in/°F	(thermal expansion coefficient)

2. Constraints

$u_2 = u_4 = u_5 = u_6 = 0.0$	at all Grids (required by use of the axisymmetric solid element)
$u_1 = u_3 = 0.0$	at Grid 1
$u_1 = 0.0$	at Grid 2

3. Loads

The thermal load is shown in Figure 2 and is specified on TEMP Bulk Data cards.

C. Results

Figure 3 displays the radial displacement utilizing the idealization shown in Figure 2. Figure 4 presents radial and circumferential stress values which result from the thermal loading. Reference 14 provides an analytical solution to this problem which is based on the theory of elasticity. Note that the solid lines represent the analytical solution while the circles and squares represent the solution obtained utilizing the finite element solution.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

 0   NASTRAN FILES=UMF
 1   ID         DEM1061,NASTRAN
 2   UMF        1977     10610
 3   APP        DISP
 4   SØL        1,1
 5   TIME       5
 6   CEND

 7   TITLE = SØLID DISC WITH RADIALLY VARYING THERMAL LØAD
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-6-1
 9   LABEL = TRAPEZØIDAL RING ELEMENTS
10   SPC = 16
11   TEMPERATURE(LØAD) = 16
12       ØUTPUT
13       SET 1 = 1,3,5,7,9,11,13,15,17,19,21,23,25,26
14       DISP = 1
15       ELSTRESS = ALL
16   BEGIN BULK
17   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CTRAPRG	1	1	3	4	2	.0	12		
GRDSET							2456		
GRID	1		.0						
MAT1	12	1.0+7		.3	.2587-3	1.0-7	.0		
SPC	16	1	13	.0	2	1	.0		
TEMP	16	1	100.	2	100.	3	99.75		

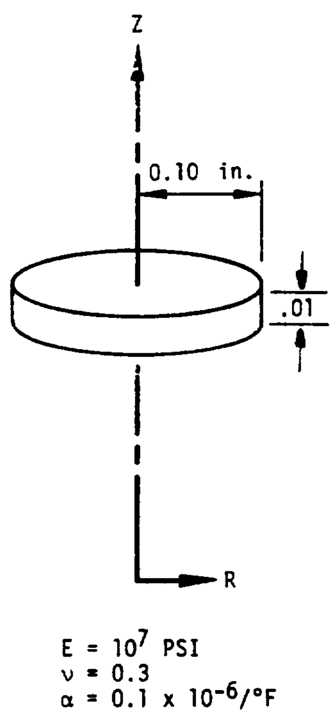


Figure 1. Solid circular disk.

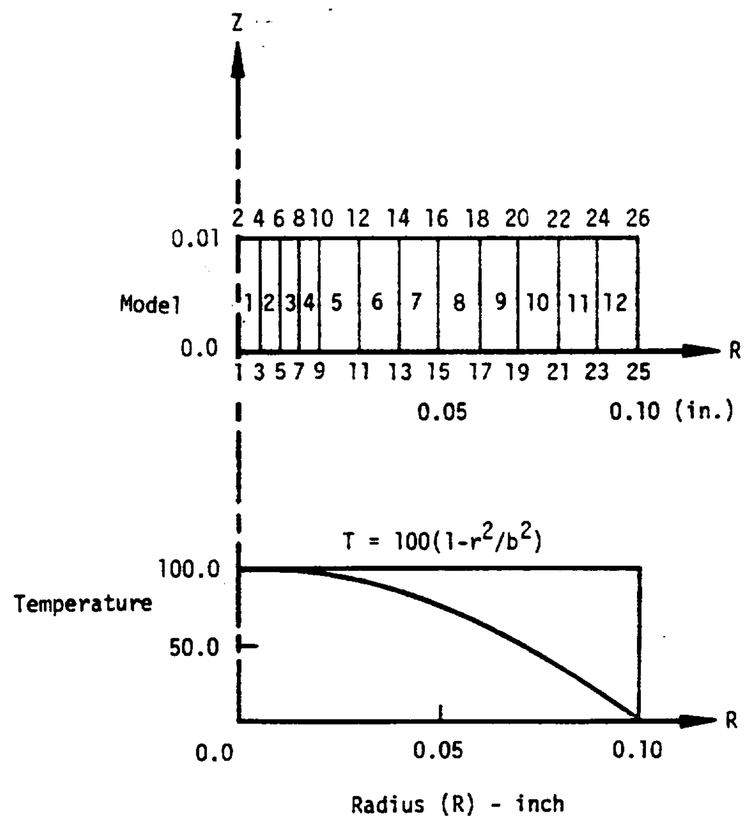


Figure 2. Finite element idealization and temperature distribution.

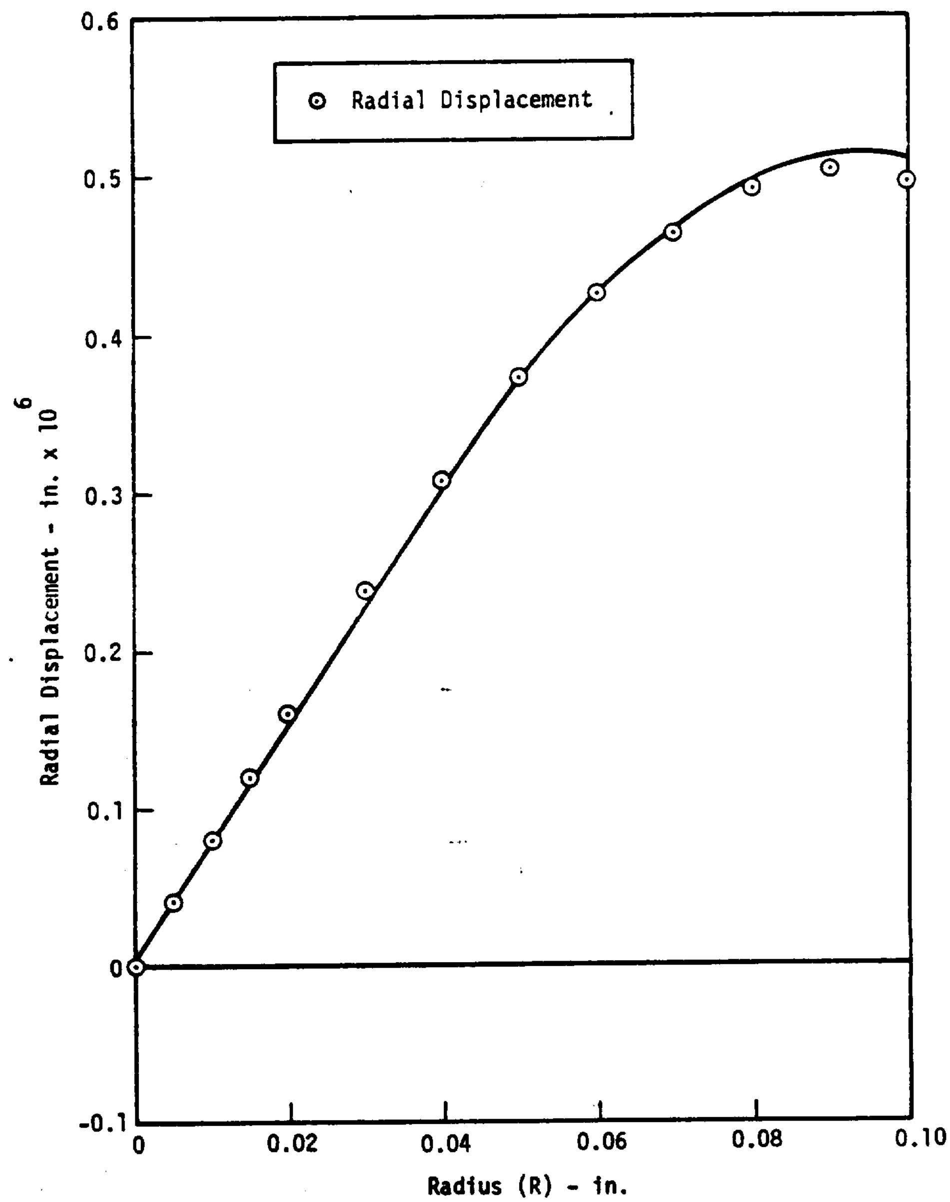


Figure 3. Radial displacement, solid disk with radially varying thermal load.

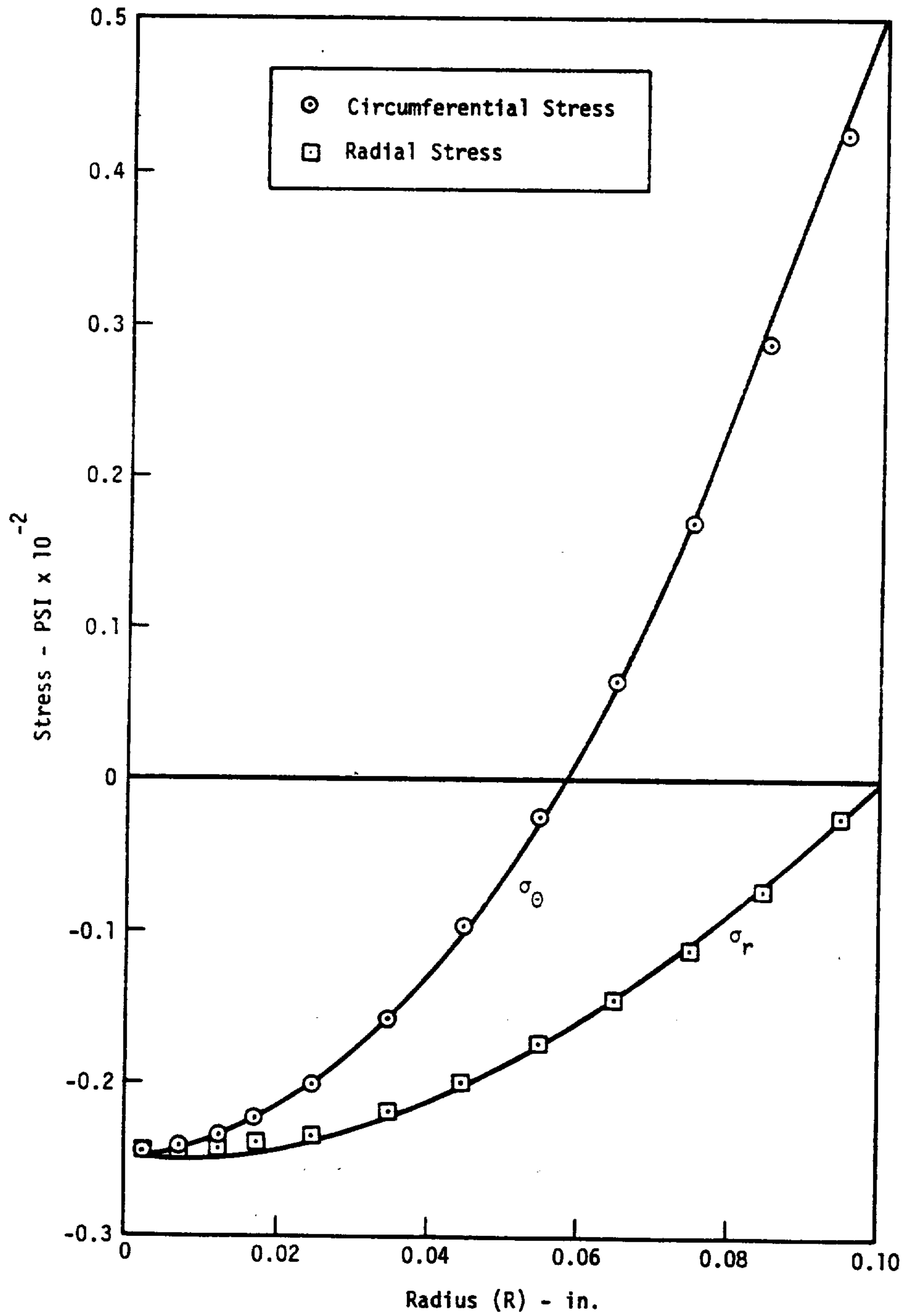


Figure 4. Radial and circumferential stress in solid disk at the centroid of the elements with radially varying thermal load.

RIGID FORMAT No. 1, Static Analysis

Shallow Spherical Shell Subjected to External Pressure Loading (1-7-1)

A. Description

The shallow spherical shell problem (see Problem 1-2-1) is again solved to demonstrate the applicability of the shell cap generalization of the toroidal ring element to this type of problem.

B. Input

The shallow spherical shell with a built-in edge is subjected to an external pressure loading of 1 psi. The shell is shown in Figure 1 along with its pertinent dimensions and associated material properties. The finite element idealization for the shell is displayed in Figure 2. Due to symmetry, only one half of the shell was analyzed.

1. Parameters

$r = 90.0$ in	(radius)
$t = 3.0$ in	(thickness)
$E = 3.0 \times 10^6$ lb/in^2	(modulus of elasticity)
$\nu = .1666$	(Poisson's ratio)

2. Constraints

$u_2 = 0.0$	all Grids
$u_1 = u_4 = 0.0$	Grid 1
$u_1 = u_3 = u_4 = 0.0$	Grid 14

3. Loads

Forces and moments are applied to the grid points to simulate an external pressure load of 1 psi.

C. Results

The meridional bending moment is taken to characterize the behavior predicted for this structure. The exact solution from Reference 4 and the 13-element NASTRAN model solution is presented in Figure 3. The reference solution is designated by the solid line while the finite element solution is designated by the circles. Figure 4 displays the radial displacement obtained utilizing this idealization and the theoretical solution from Reference 4.

D. Driver Decks and Sample Bulk Data

Card
No.

```
 0   NASTRAN FILES=UMF
 1   ID        DEM1071,NASTRAN
 2   UMF 1977          10710
 3   APP       DISPLACEMENT
 4   SØL       1,1
 5   TIME      5
 6   CEND

 7   TITLE = SPHERICAL SHELL WITH TØRØIDAL RING ELEMENT
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-7-1
 9   LABEL = EXTERNAL PRESSURE LØADING
10   SPC = 1
11   LØAD = 1
12   ØUTPUT
13       DISP = ALL
14       ØLØAD = ALL
15       ELFØRCE = ALL
16            STRESSES = ALL
17   BEGIN BULK
18   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CTØRDRG	1	1	1	2	.0	2.0			
FØRCE	1	1	0	1.0	.0	.0	-8.85885		
GRDSET							2		
GRID	1	0	.0	.0	90.00				
MAT1	12	3.0E6		.1667		12.5E-6	.0		CMAT11
MØMENT	1	2	0	1.0	14.83917	.0	-10.1998		
PTØRDRG	1	12	3.0	3.0					
SPC	1	1	14	.0	14	134	.0		

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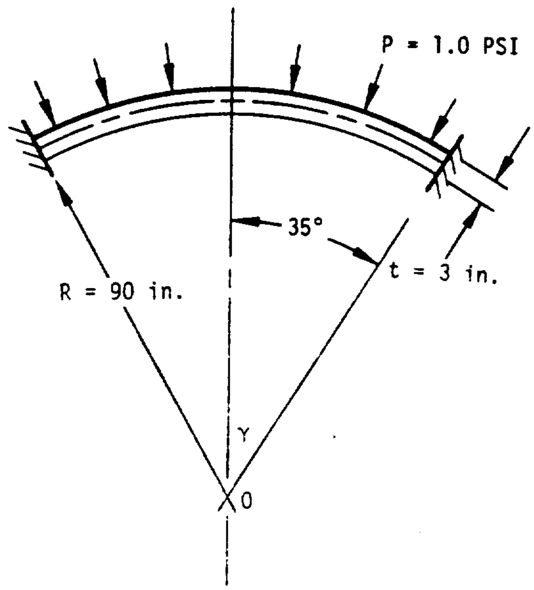

$E = 3.0 \times 10^6$ PSI
$\nu = 0.167$

Figure 1. Shallow spherical shell.

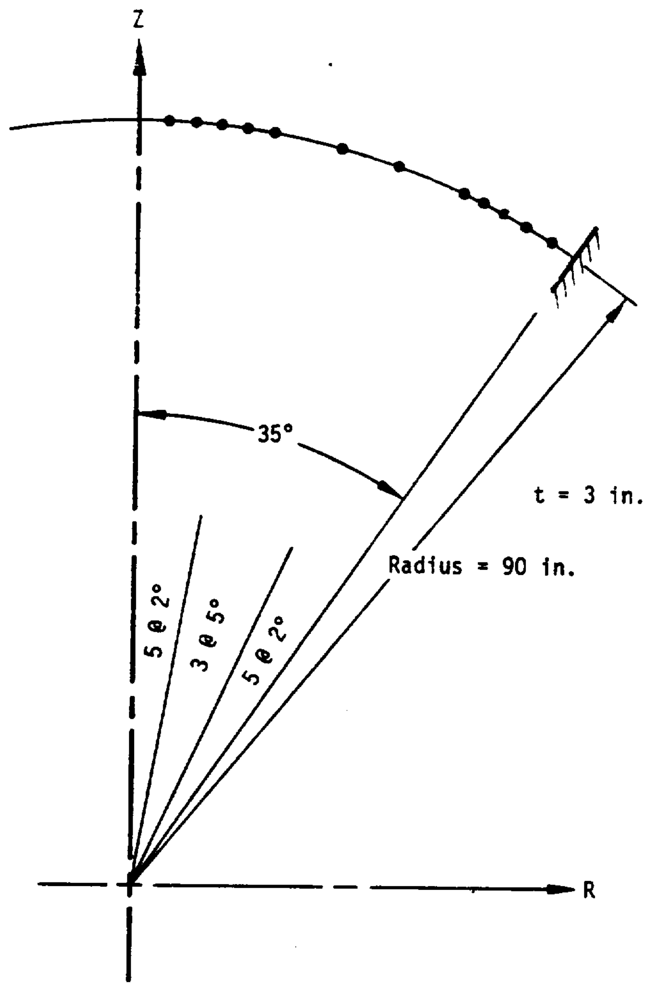

Figure 2. Finite element idealization.

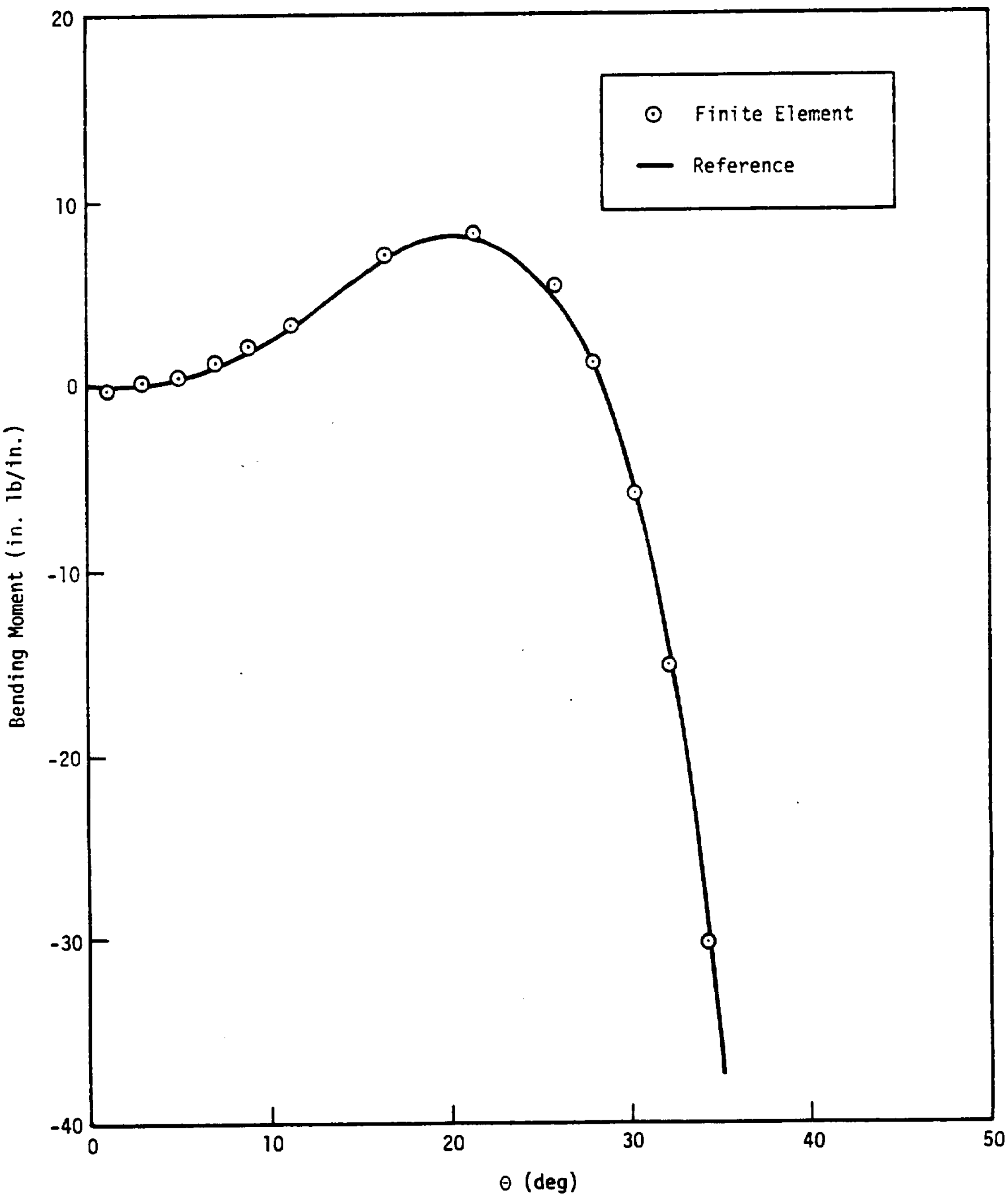

Figure 3. Meridional moment, shallow spherical shell.

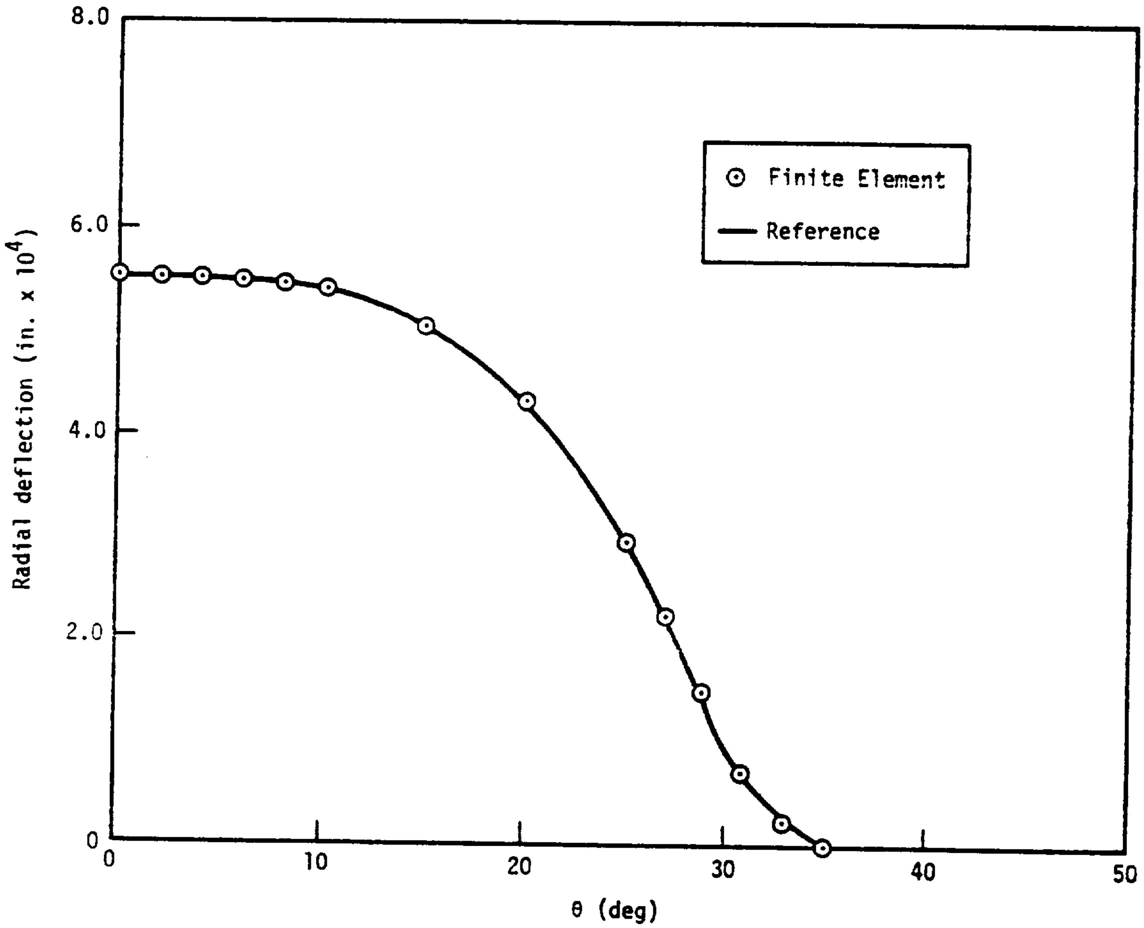

Figure 4. Radial displacement, shallow spherical shell.

RIGID FORMAT No. 1, Static Analysis

Bending of a Beam Fabricated from HEXA1 Solid Elements (1-8-1)

A. Description

The properties of solid bodies may be modeled with the NASTRAN tetrahedra, wedge, or hexahedron finite elements. This problem demonstrates the analysis of a solid fabricated from the six-sided HEXA1 solid elements. The problem consists of a rectangular parallelepiped subdivided into forty cubic subelements as shown in Figure 1.

The loads were chosen to approximate the stress distribution due to a moment on one end of a beam; the other end is constrained to resist the moment. Two planes of symmetry were used to simulate an actual problem having twice the width and twice the height.

B. Input

1. Parameters:

   ℓ = 20.0 (length)

   w = 4.0 (width of full section)

   h = 16.0 (height of full section)

   $E = 3.0 \times 10^6$ (modulus of elasticity)

   ν = 0.2 (Poisson's ratio)

2. Boundary Constraints:

   on y = 0 plane, $u_x = u_z = 0$ (antisymmetry)

   on z = 0 plane, $u_z = 0$ (symmetry)

   on x = 0 plane, $u_x = 0$ (symmetry)

3. Loads:

   Total Moment: $M_y = 2.048 \times 10^3$

   This moment will produce bending about the z axis. It is modeled by a set of axial loads at $x = \ell$ which, in turn, represent an axial stress distribution:

$$\sigma_{xx} = 1.5y \quad . \tag{1}$$

C. Theory

A prismatic beam with an axial stress which varies linearly over the cross section has an exact solution. The theoretical stress distribution is

$$\sigma_{xx} = -\frac{M}{I} y \quad , \tag{2}$$

and

$$\sigma_{yz} = \sigma_{zz} = \tau_{xy} = \tau_{xz} = \tau_{yz} = 0 \ , \tag{3}$$

where $I = \frac{1}{12} wh^3$.

The displacements are:

$$u_x = -\frac{M}{EI} xy \quad , \tag{4}$$

$$u_y = \frac{M}{2EI} (x^2 - \nu y^2 - \nu z^2) \quad , \tag{5}$$

and

$$u_z = \nu \frac{M}{EI} yz. \tag{6}$$

D. Results

Tables 1 and 2 are comparisons of displacements and stresses for the theoretical case and the NASTRAN model.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1081,NASTRAN
  2   UMF       1977    10810
  3   APP       DISPLACEMENT
  4   SØL       1,3
  5   TIME      4
  6   CEND
  7   TITLE = 1 X 4 X 10   CANTILEVER BEAM USING CUBIC CHEXA1 ELEMENTS.
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-8-1
  9   LABEL = TWØ PLANES ØF SYMMETRY, PURE BENDING MØMENT
 10       SPC = 10
 11       LØAD = 10
 12   ØUTPUT
 13       DISPLACEMENT = ALL
 14       SPCFØRCE = ALL
 15       ØLØAD = ALL
 16       STRESS = ALL
 17   BEGIN BULK
 18   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CHEXA1	1	1	2	1	3	4	12	11	+HEX 1
+HEX 1	13	14							
CNGRNT	1	2	THRU	40					
FØRCE	10	103		5.818182	-1.0	.0	.0		
GRID	1		.00	.00	.00		456		
MAT1	1	3.0+6		.2	1.0	.001	10.0		+MAT1
SPC	10	1	123	.0	2	13	.0		
SPC1	10	1	3	4	5	6	7	8	+3
+3	9	10							

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POINT/DIRECTION	DISPLACEMENT x 10^{-4} THEORY	NASTRAN
21/y	.0400	.0417
41/y	.1600	.1607
61/y	.360	.366
81/y	.640	.651
101/y	1.000	1.016
109/x	0.800	0.844
110/z	.016	0.007

Table 1. Comparisons of Displacement

ELEMENT	THEORY			NASTRAN		
	σ_{xx}	σ_{yy}	τ_{xy}	σ_{xx}	σ_{yy}	τ_{xy}
1	-1.5	0	0	-1.56	.02	-.01
2	-4.5	0	0	-4.53	.036	-.05
3	-7.5	0	0	-7.39	.06	-.06
4	-10.5	0	0	-9.95	-.11	.12

NOTE: NASTRAN stresses are average; theoretical stresses are calculated at the center of the element.

Table 2. Comparisons of Stress

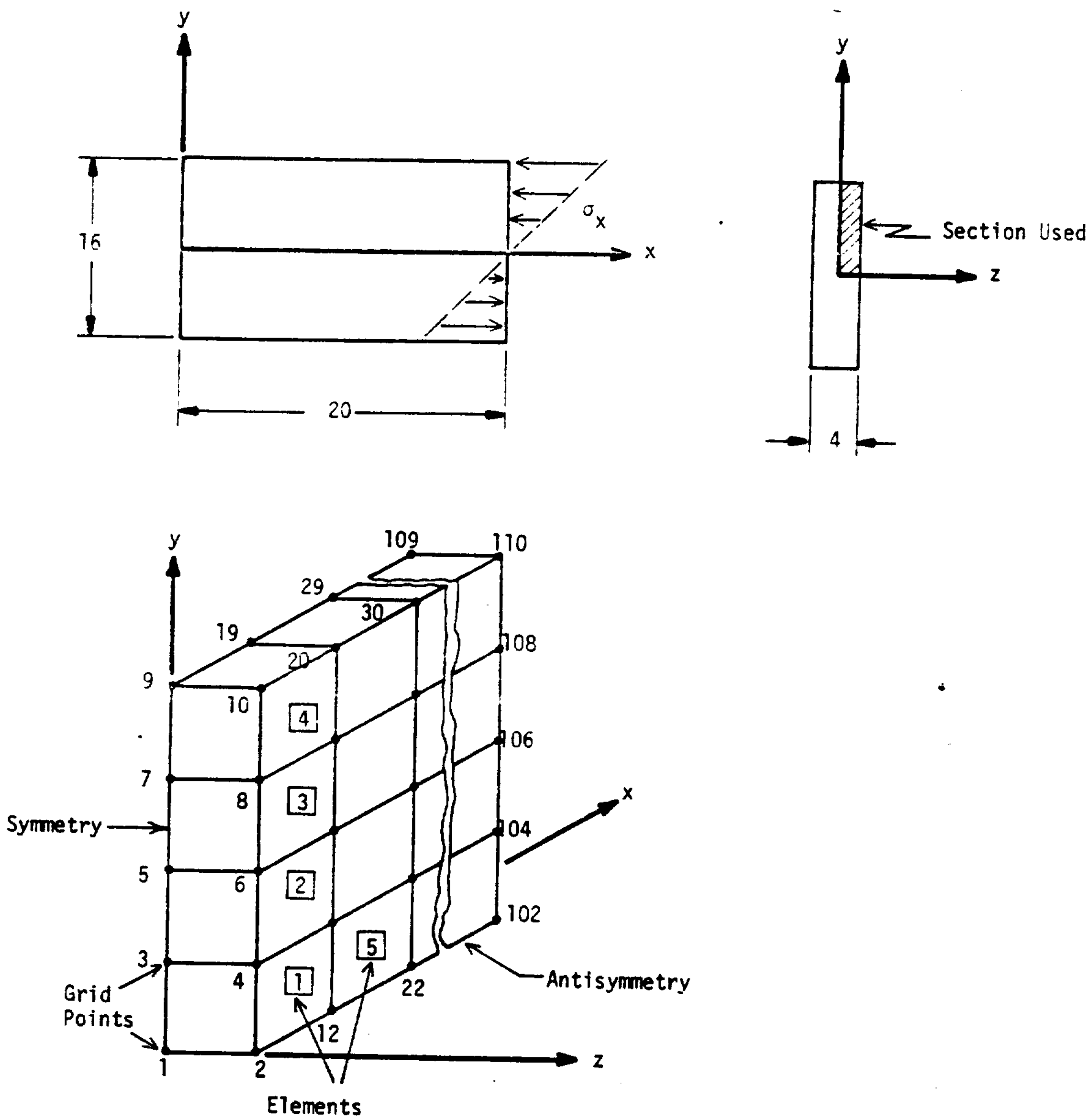


Figure 1. Model of solid using hexahedrons.

RIGID FORMAT No. 1, Static Analysis

Thermal and Applied Loads on HEXA2 Solid Elements (1-9-1)

Thermal and Applied Loads on TRIM6 Membrane Elements (1-9-2)

A. Description

This problem demonstrates a static analysis of a cantilevered beam under two loading conditions: axial stress and thermal expansion. The analysis is performed twice, once with a model consisting of HEXA2 solid hexahedron elements (Problem 1-9-1) and once with a model built using the TRIM6 higher order triangular membrane element (Problem 1-9-2).

Forty HEXA2 elements are used to model a symmetric quarter of the 4 x 4 x 20 beam as shown in Figure 1. Symmetric boundary conditions are used on both the vertical and the horizontal planes of symmetry.

Ten TRIM6 elements are used to model one half of the 4 x 4 x 20 beam as shown in Figure 2. Symmetry boundary conditions are used on the vertical plane of symmetry (see Reference 31, pp. 168-172).

B. Input

1. Parameters:

ℓ = 20.0	(length)
w = 4.0	(width)
h = 4.0	(height)
$E = 3.0 \times 10^6$	(modulus of elasticity)
$\nu = 0.2$	(Poisson's ratio)
$\alpha = .001$	(thermal expansion coefficient)
$T_o = 10°$	(reference temperature)

2. Support Boundary Constraints:

HEXA2 Model	TRIM6 Model
$u_x = 0$ at $x = 0$	$u_x = u_y = 0$ at $x = 0$
$u_y = 0$ at $y = 0$	$u_y = 0$ at $y = 0$
$u_z = 0$ at $z = 0$	$u_z = 0$ at all grid points

3. Loads

Subcase 1 (HEXA2 Model):

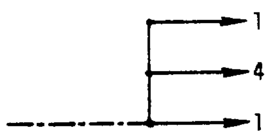
An axial force F_x distributed for uniform pressure over the end of the beam where

$$F_x = 24 \times 10^3 \text{ (total axial force)}.$$

Subcase 1 (TRIM6 Model):

$$F_x = 24 \times 10^3 \text{ (total axial force)}.$$

$$\text{Total force on symmetric part} = \frac{24}{2} = 12 .$$



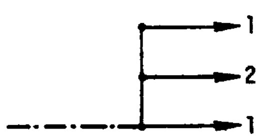
Force divided into the ratio of 1:4:1, i.e., $\frac{1 \times 12}{6}$, $\frac{4 \times 12}{6}$, $\frac{1 \times 12}{6}$

Subcase 2 (Both Models):

$$T = 60° \text{ (uniform temperature field)}.$$

$$T_o = 10° \text{ (reference temperature)}.$$

Subcase 3 (TRIM6 Model Only):



Force divided into the ratio of 1:2:1, i.e., $\frac{1 \times 12}{4}$, $\frac{2 \times 12}{4}$, $\frac{1 \times 12}{4}$

C. <u>Theory</u>

1. Subcase 1 and Subcase 3

The distributed axial load is equivalent to a stress field of:

$$\sigma_{xx} = 1.5 \times 10^3 , \tag{1}$$

and

$$\sigma_{yy} = \sigma_{zz} = \tau_{xy} = \tau_{xz} = \tau_{yz} = 0 . \tag{2}$$

The displacement field is

$$u_x = \frac{\sigma_{xx}}{E} x = 0.5 \times 10^{-3} x \,, \tag{3}$$

$$u_y = \frac{-\nu\sigma_{xx}}{E} y = -0.1 \times 10^{-3} y \,, \tag{4}$$

and

$$u_z = \frac{-\nu\sigma_{xx}}{E} z = -0.1 \times 10^{-3} z \,. \tag{5}$$

2. Subcase 2

The uniform expansion due to temperature will not cause any stress. The strains, however, are uniform and equal. Therefore, the displacements are

$$u_x = \sigma(T-T_o)x = .05x \,, \tag{6}$$

$$u_y = \sigma(T-T_o)y = .05y \,, \tag{7}$$

and

$$u_z = \sigma(T-T_o)z = .05z \,. \tag{8}$$

where T is the uniform temperature and T_o is the reference temperature.

D. Results

The results of both subcases are exact to the single precision limits of the particular computer used. Table 1 presents the theoretical solutions and the results of the TRIM6 finite element model analysis.

E. Driver Decks and Sample Bulk Data

```
Card
No.

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 1   ID          DEM1091,NASTRAN
 2   UMF         1977     10910
 3   APP         DISP
 4   SØL         1,3
 5   TIME        4
 6   CEND

 7   TITLE = 2 X 2 X 10  FIXED-FREE BEAM USING RECTANGULAR CHEXA2 ELEMENTS.
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-9-1
 9   LABEL = TWØ PLANES ØF SYMMETRY
10   SPC=2
11       ØUTPUT
12             DISPLACEMENTS = ALL
13             ØLØAD = ALL
14   SUBCASE 1
15             LØAD = 20
16             LABEL = UNIFØRM STRESS.
17             SPCFØRCE = ALL
18             STRESS = ALL
19   SUBCASE 2
20             TEMPERATURE(LØAD) = 30
21             LABEL = UNIFØRM TEMPERATURE LØAD
22   BEGIN BULK
23   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CHEXA2	1	1	1	2	5	4	10	11	+HEX 1
+HEX 1	14	13							
CNGRNT	1	2	THRU	40					
FØRCE1	20	91	.375+3	82	91				
GRID	1		0.0	0.0	0.0		456		
MAT1	1	3.0+6		.2	1.0	.001	10.0		+MAT1
SPC1	100	1	1	2	3	4	5	6	
SPCADD	2	100	104	103					
TEMPD	30	60.0							

```
Card
 No.

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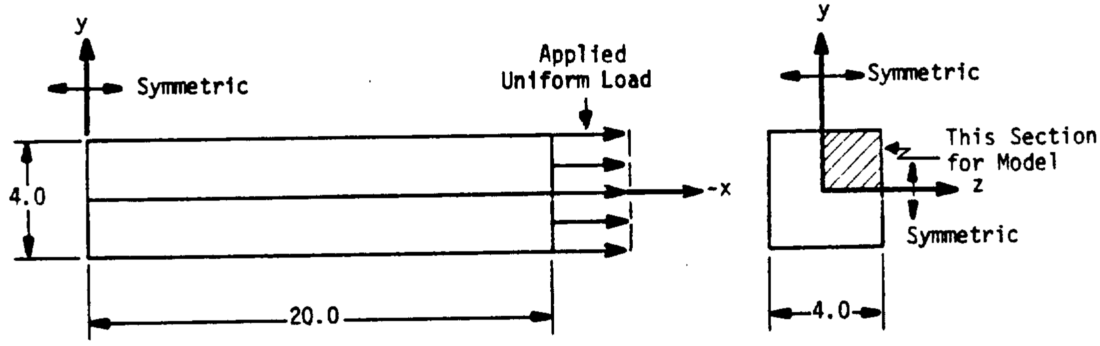
  1    ID       DEM1092,NASTRAN
  2    UMF      1977  10920
  3    APP      DISPLACEMENT
  4    SØL      1,0
  5    TIME     10
  6    CEND

  7    TITLE = 2 X 1 X 10  FIXED-FREE BEAM USING CTRIM6 ELEMENTS
  8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-9-2
  9    SPC = 1
 10    ØUTPUT
 11    DISP = ALL
 12    SPCFØRCE = ALL
 13    STRESS = ALL
 14    SUBCASE 1
 15    LABEL = CØNSISTENT LØADING (FØRCE RATIØ 1 TØ 4 TØ 1)
 16    LØAD = 1
 17    ØLØAD = ALL
 18    SUBCASE 2
 19    LABEL = UNIFØRM TEMPERATURE LØAD
 20    TEMPERATURE(LØAD) = 30
 21    SUBCASE 3
 22    LABEL = LUMPED STRESS LØADING (FØRCE RATIØ 1 TØ 2 TØ 1)
 23    LØAD = 40
 24    ØLØAD = ALL
 25    BEGIN BULK
 26    ENDDATA
```

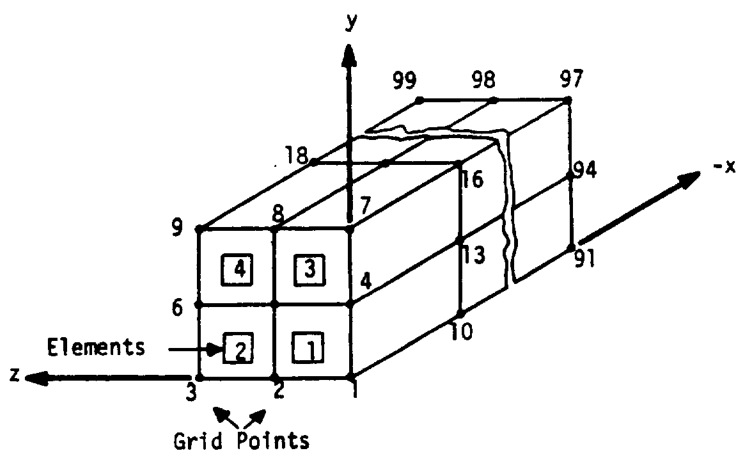
1	2	3	4	5	6	7	8	9	10
CTRIM6	1	80	9	6	3	2	1	5	+TE1
+TE1									
FØRCE1	20	31	2.0+3	28	31				
GRDSET							3456		
GRID	1		.0	.0	.0				
MAT1	90	3.0+6		.2	1.	.001	10.		
PTRIM6	80	90	4.	.0	.0				
SPC1	1	2	4	7	10	13	16	19	+GJD
+GJD	22	25	28	31					
TEMPD	30	60.							

Table 1. TRIM6 and Theoretical Solutions

X	Pressure Load			Temperature Load	
	Exact Sol. (10^{-3})	Subcase 1 (10^{-3})	Subcase 3 (10^{-3})	Exact Sol.	Subcase 2
0	0	0	0	0	0.
2	1	0.98	0.98	0.1	0.109
4	2	1.98	1.98	0.2	0.2093
6	3	2.98	2.981	0.3	0.3093
8	4	3.98	3.98	0.4	0.4093
10	5	4.98	4.981	0.5	0.5093
12	6	5.98	5.981	0.6	0.6093
14	7	6.98	6.98	0.7	0.7093
16	8	7.98	7.98	0.8	0.8093
18	9	8.98	8.99	0.9	0.9093
20	10	9.98	10.026	1.0	1.00937



Problem



Model

Figure 1. Model of cantilevered beam using HEXA2 elements.

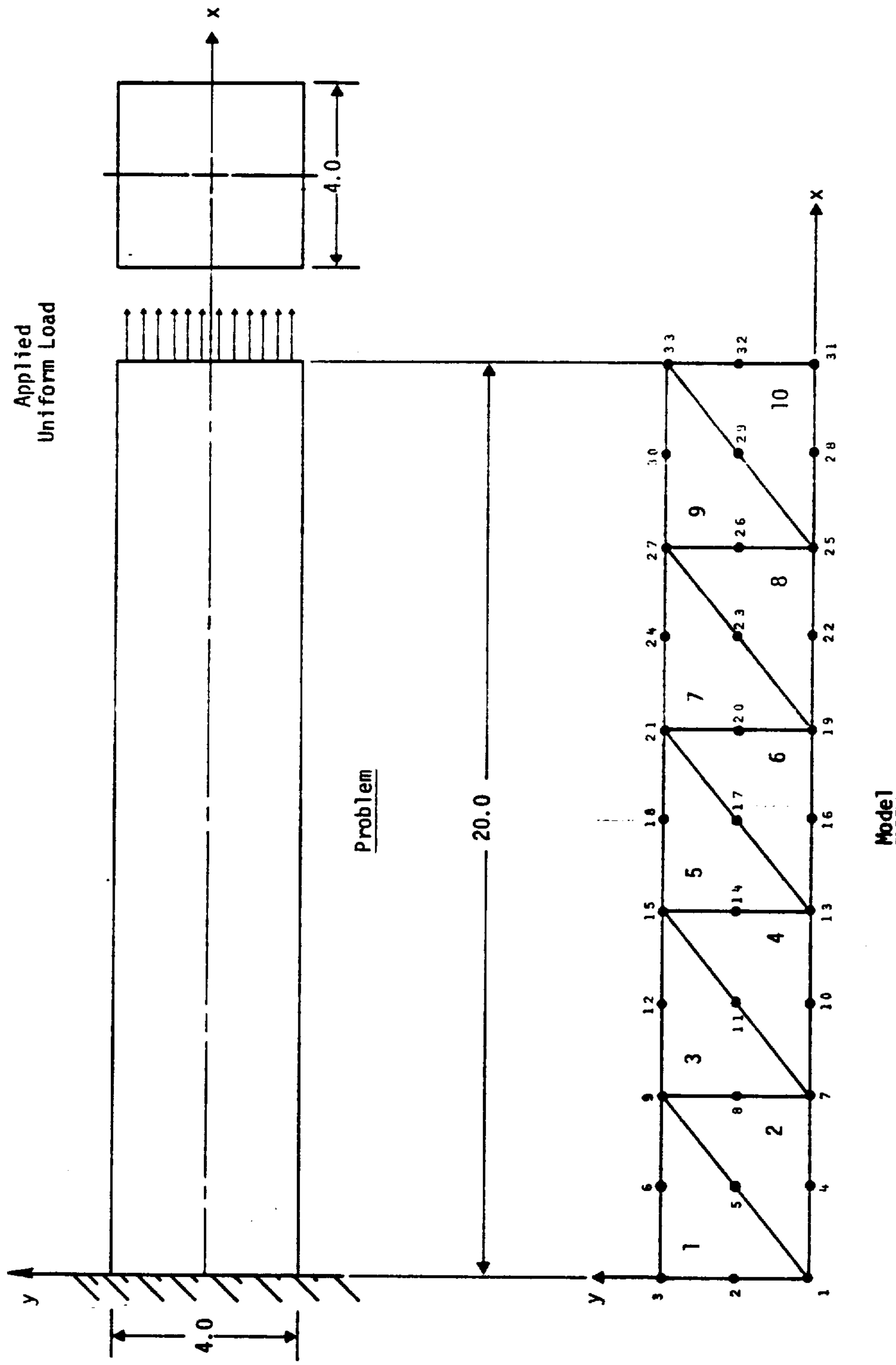


Figure 2. Model of cantilevered beam using TRIM6 elements.

RIGID FORMAT No. 1, Static Analysis

Thermal Bending of a Beam (1-10-1)

A. Description

This problem demonstrates the solution of a beam subjected to a thermal gradient over the cross-section. Two end conditions are solved, clamped-free and clamped-pinned end conditions.

An equivalent linear gradient in the normal direction was used for the input data. However, the actual temperatures at points on the cross-section were input on the TEMPRB card in order to produce correct stresses. The beam was subdivided into 14 variable lengths for maximum efficiency.

B. Input

Figure 1 describes the beam and the thermal field to be analyzed and Figure 2 shows the finite element model.

C. Theory

For subcase 1, the effective temperature gradient, T', (see NASTRAN Theoretical Manual) is:

$$T'(x) = \frac{1}{I} \int_z \int_y T(x,y,z) y \, dy \, dz \quad , \tag{1}$$

where

$$I = \int_z \int_y y^2 \, dy \, dz \quad . \tag{2}$$

Using the given temperature distribution the effective gradient is:

$$T' = T_c \, x^3 \quad , \tag{3}$$

where T_c is calculated to be 0.170054°/in^4 by substituting the temperature distribution into Equation 1 and evaluating the expression:

$$T_c = \frac{1}{I} \int_z \int_y C y^4 \, dy \, dz \tag{4}$$

Since the bar is not redundantly constrained the curvature at the center line is:

$$\frac{d^2v}{dx^2} = -\alpha T' = -\alpha T_c x^3 \quad . \tag{5}$$

The slope is:

$$\frac{dv}{dx} = \int_0^x \frac{d^2v}{dx^2}\,dx = -\frac{\alpha}{4} T_c x^4 \quad . \tag{6}$$

The deflection is:

$$v(x) = \int_0^x \frac{dv}{dx}\,dx = -\frac{\alpha}{20} T_c x^5 \quad . \tag{7}$$

The moment, M, shear, V, and axial stress, σ_x, are:

$$\left.\begin{aligned} M &= EI\left(\frac{d^2v}{dx^2} + \alpha T'\right) = 0 \quad , \\ V &= \frac{dM}{dx} = 0 \quad , \\ \sigma_x(x,y) &= E(\varepsilon_x - \alpha T) = E(\sigma y T' - \alpha T) = E\alpha(T_c y - Cy^3)\,x^3 \quad , \end{aligned}\right\} \tag{8}$$

where C = 1 has dimensions of degrees/length^6.

For subcase 2, with a simple support at x = 10.0, we calculate the deflection due to subcase 1 and apply a constraint load P_L to remove the deflection at the end.

$$P_L = -\frac{3EI}{L^3} v(L) = 3EI \frac{\alpha T_c}{20} L^2 \quad . \tag{9}$$

Note: Transverse shear deflection is neglected.

The deflections and slopes are the sum of the results for the two independent loads as follows

deflection: $$v(x) = \frac{P_L}{6EI}(3Lx^2 - x^3) - \frac{\alpha T_c}{20} x^5 = \frac{\alpha T_c}{40}(3L^3 - L^2x - 2x^3)\,x^2 \quad , \tag{10}$$

slope: $$\theta_z(x) = \frac{\partial v}{\partial x} = \frac{\alpha T_c}{40}(6L^3 - 3L^2x - 10x^3)x \quad . \tag{11}$$

The net stress is the sum of the stress due to each load:

$$\sigma_x(x,y) = E\alpha(T_c y - Cy^3)x^3 - \frac{M_L y}{I} = E\alpha\left[\left(T_c y - Cy^3\right)x^3 - \frac{3}{20} T_c L^2(L - x)\ y\right] \tag{12}$$

where M_L is the moment due to the constraint load.

D. Results

Tables 1 and 2 compare the analytical maximum value of displacement, constraint force, element force, and stress to the maximum deviation of NASTRAN in each category. All results are within 2.66%.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

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  1   ID         DEM1101,NASTRAN
  2   UMF        1977    11010
  3   SØL        1,0
  4   TIME       9
  5   APP        DISPLACEMENT
  6   CEND
  7   TITLE = THERMAL BENDING ØF A BAR.
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-10-1
  9              TEMPERATURE(LØAD) = 20
 10       ØUTPUT
 11              DISPLACEMENT = ALL
 12              SPCFØRCE = ALL
 13              ØLØAD = ALL
 14              ELFØRCE = ALL
 15              STRESS = ALL
 16   SUBCASE 1
 17              LABEL = CØNSTRAINTS ARE - FIXED AND FREE ENDS.
 18              SPC = 1
 19   SUBCASE 2
 20              LABEL = CØNSTRAINTS ARE - FIXED AND SIMPLY SUPPØRTED ENDS.
 21              SPC = 2
 22   BEGIN BULK
 23   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
BARØR					.0	1.00	.0	1	
CBAR	101	10	1	2					
GRDSET							345		
GRID	1		.0	.0	.0				
MAT1	10	1.0+7		.3		1.3-5	.0		
PBAR	10	10	.52	.0689333	.0337333				+BAR
+BAR	.0		.3		.5		-0.5		
SPC	1	1	126	.0					
TEMPRB	20	101	.0	.0	.0	2.35083	.0	.0	+1T
+1T	.0	.0	.0	.0	.0	.373248	1.728	-1.728	

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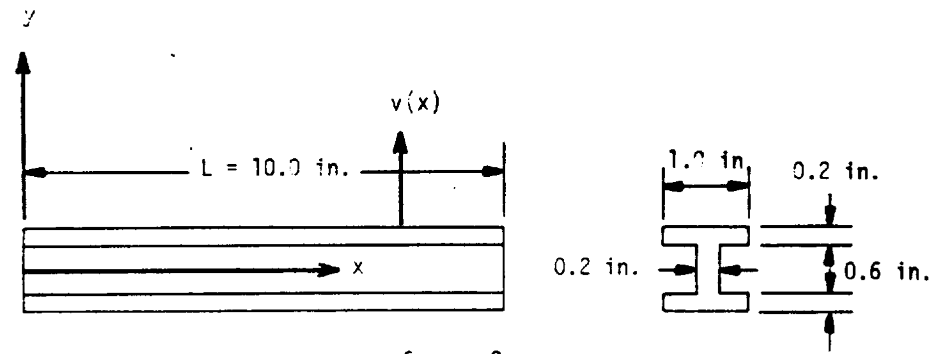


$E = 10.0 \times 10^6\ lb/in^2$ (Young's modulus)

$\nu = 0.3$ (Poisson's ratio)

$\alpha = 13.0 \times 10^{-6}$ in/in/°F (Thermal expansion coefficient)

$T_0 = 0.0°F$ (Reference temperature)

The beam is loaded by the temperature distribution:

$$T(°F) = Cx^3y^3$$

where $C = 1.0\ °F/in^6$

Figure 1. Thermal loading of a beam.

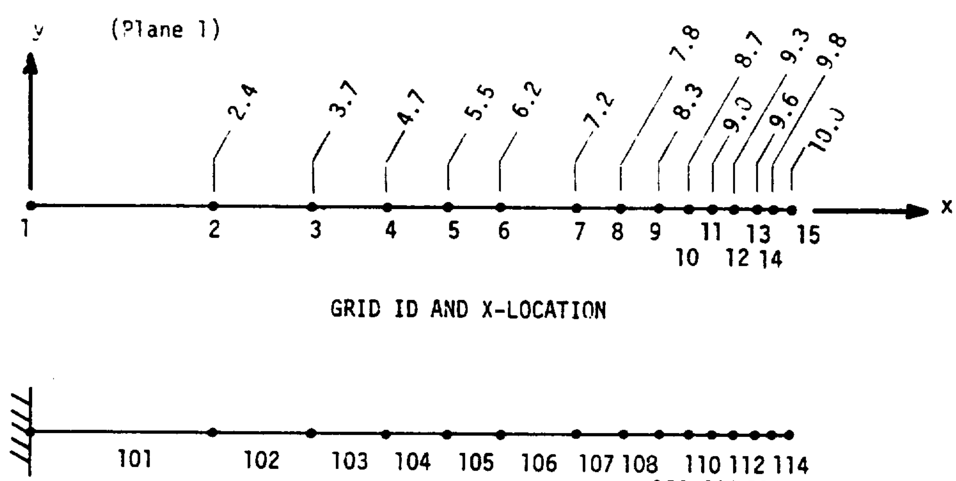


Figure 2. Finite element model.

Table 1. Comparison of NASTRAN and analytical results, clamped-free ends (subcase 1).

CATEGORY	MAXIMUM ANALYTICAL VALUE	MAXIMUM NASTRAN DIFFERENCE	PER CENT ERROR
Displacement	-1.1054×10^{-2}	2.9424×10^{-4}	2.66
Constraint Force	0	*	*
Element Force	0	*	*
Element Stress	$5.1965 \times 10^{+3}$	0.671	0.01

*These results vary with the computer. The very small numbers are essentially zero when compared to subcase 2 results.

Table 2. Comparison of NASTRAN and analytical results, clamped-pinned ends (subcase 2).

CATEGORY	MAXIMUM ANALYTICAL VALUE	MAXIMUM NASTRAN DIFFERENCE	PER CENT ERROR
Displacement	4.3936×10^{-3}	8.024×10^{-6}	0.18
Constraint Force	$-2.2859 \times 10^{+2}$	6.0841	2.66
Element Force	$2.2859 \times 10^{+2}$	6.0846	2.66
Element Stress	$5.1965 \times 10^{+3}$	4.4136×10	0.85

Simply-Supported Rectangular Plate with a Thermal Gradient (1-11-1)
Simply-Supported Rectangular Plate with a Thermal Gradient (INPUT, 1-11-2)

A. Description

This problem illustrates the solution of a general thermal load on a plate with the use of an equivalent linear thermal gradient. The thermal field is a function of three dimensions, demonstrated by the TEMPP1 card. The plate is modeled with the general quadrilateral, QUAD1, elements as shown in Figure 1. Two planes of symmetry are used. This problem is repeated via the INPUT module to generate the QUAD1 elements.

B. Input

E	=	3.0×10^5 pounds/inch^2	(Youngs modulus)
ν	=	0.3	(Poisson's ratio)
ρ	=	1.0 pound-sec.^{2}/inch^4	(Mass density)
α	=	0.01 inch/°F/inch	(Thermal expansion coefficient)
T_R	=	0.0 °F	(Reference temperature)
T_o	=	2.5 °F	(Temperature difference)
a	=	10.0 inch	(Width)
b	=	20.0 inch	(Length)
t	=	0.5 inch	(Thickness)

The thermal field is

$$T = T_o(\cos \frac{\pi x}{a})\ (\cos \frac{\pi y}{b}) \left(\frac{2z}{t}\right)^3 \quad ,$$

anc

$$= 160.0(\cos \frac{\pi x}{10})\ (\cos \frac{\pi y}{20})\ z^3 \text{ °F} \quad .$$

C. Theory

The plate was solved using a minimum energy solution. The net moments, $\{M_N\}$, in the plate are equal to the sum of the elastic moments, $\{M_e\}$, and the thermal moments, $\{M_t\}$.

$$\{M_N\} = \{M_t\} + \{M_e\} \quad , \tag{1}$$

where the thermal moment is

$$\{M_t\} = \alpha T_o' \, D(1+\nu) \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \quad , \tag{2}$$

and

$$D = \frac{Et^3}{12(1-\nu^2)} \quad .$$

and $T_o' = 6T_o/5t$ is the effective thermal gradient.

The elastic moment is defined by the curvatures, χ, with the equation:

$$\{M_e\} = D \begin{Bmatrix} \chi_x + \nu\chi_y \\ \chi_y + \nu\chi_x \\ \frac{(1-\nu)}{2}\chi_{xy} \end{Bmatrix} \quad . \tag{3}$$

Assuming a normal displacement function, W, of

$$W = \sum_n \sum_m W_{nm} \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} \quad , \tag{4}$$

then

$$\left.\begin{aligned} \chi_x &= \frac{\partial^2 W}{\partial x^2} = -\sum_n \sum_m \pi^2 W_{nm} \left(\frac{n}{a}\right)^2 \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} \quad , \\ \chi_y &= \frac{\partial^2 W}{\partial y^2} = -\sum_n \sum_m \pi^2 W_{nm} \left(\frac{m}{a}\right)^2 \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} \quad , \\ \chi_{xy} &= 2\frac{\partial^2 W}{\partial x \partial y} = 2\sum_n \sum_m \pi^2 W_{nm} \left(\frac{nm}{ab}\right) \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b} \quad . \end{aligned}\right\} \tag{5}$$

The work done by the thermal load is:

$$U = \int_A \{\chi\}^T \{M_t\} \, dA + \frac{1}{2} \int_A \{\chi\}^T \{M_e\} \, dA \quad , \tag{6}$$

where A is the surface area. Performing the substitution and integrating results in the energy expression:

$$U = -\frac{\alpha T_o' D(1+\nu)\pi^2 (a^2+b^2)}{4ab} W_{11} + \frac{D}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\pi^4 ab}{4} \left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)^2 W_{nm}^2 \ . \tag{7}$$

The static solution exists at a minimum energy:

$$\frac{\partial U}{\partial W_{nm}} = 0 \ . \tag{8}$$

This results in all but W_{11} equal to zero. The displacement function is therefore:

$$W(x,y) = \frac{\alpha T_o' (1+\nu)a^2 b^2}{\pi^2 (a^2 + b^2)} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \ . \tag{9}$$

Solving for moments by differentiating W and using equation (3) results in the equations for element moments:

$$M_x = \alpha T_o' D(1+\nu) \left[1 - \frac{b^2+\nu a^2}{a^2+b^2}\right] \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \ , \tag{10}$$

$$M_y = \alpha T_o' D(1+\nu) \left[1 - \frac{a^2+\nu b^2}{a^2+b^2}\right] \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \ , \tag{11}$$

$$M_{xy} = \frac{\alpha T_o' D(1-\nu^2)ab}{a^2 + b^2} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \ . \tag{12}$$

D. Results

Figure 2 compares the element forces given by the above equation and the NASTRAN results. Figure 3 compares the normal displacements. The maximum errors for displacements, constraint forces, element forces and element stresses are listed in Table 1.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1111,NASTRAN
  2   UMF       1977     11110
  3   APP       DISPLACEMENT
  4   SØL       1,3
  5   TIME      9
  6   CEND

  7   TITLE = SIMPLY SUPPØRTED RECTANGULAR PLATE WITH A THERMAL GRADIENT
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-11-1
  9             SPC = 1
 10             TEMP(LØAD) = 20
 11       ØUTPUT
 12             DISPLACEMENT = ALL
 13             SPCFØRCE = ALL
 14             ELFØRCE = ALL
 15             STRESSES = ALL
 16   BEGIN BULK
 17   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	2	THRU	59					
CQUAD1	1	101	1	2	8	7			
GRDSET							6		
GRID	1		.00	.00	.00				
MAT1	1	3.0+5		.3	1.0	.01	.0		
PARAM	IRES	1							
PQUAD1	101	1	.5	1	.0104167				+PQUAD1
+PQUAD1	.25	-0.25							
SPC1	1	34	6	12	18	24	30	36	+SPC-34
+SPC-34	42	48	54	60	66				
TEMPP1	20	1	.0	5.90786	2.46161	-2.46161			

Card
No.

```
 0   NASTRAN FILES=UMF
 1   ID        DEM1112,NASTRAN
 2   UMF       1977     11120
 3   APP       DISPLACEMENT
 4   TIME      9
 5   SØL       1,3
 6   DIAG      14
 7   ALTER     1
 8   PARAM     //C,N,NØP/N,N,TRUE=-1  $
 9   INPUT,    ,,,GEØM4,/G1,G2,,G4,/C,N,3/C,N,1  $   QUAD1 ELEMENT
10   EQUIV     G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE  $
11   ENDALTER
12   CEND

13   TITLE = SIMPLY-SUPPØRTED RECTANGULAR PLATE WITH THERMAL GRADIENT
14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-11-2
15             SPC = 5010
16             TEMP(LØAD) = 20
17       ØUTPUT
18             DISPLACEMENT = ALL
19             SPCFØRCE = ALL
20             ELFØRCE = ALL
21             STRESSES = ALL
22   BEGIN BULK
23   ENDDATA

24       5       10      1.0      1.0       6      0.0      0.0
25     421      125       53       34       0        0
```

1	2	3	4	5	6	7	8	9	10
MAT1	1	3.0+5		.3	1.0	.01	.0		
PQUAD1	101	1	.5	1	.0104167				+PQUAD1
+PQUAD1	.25	-0.25							
TEMPP1	20	1	.0	5.90786	2.46161	-2.46161			
ENDDATA									

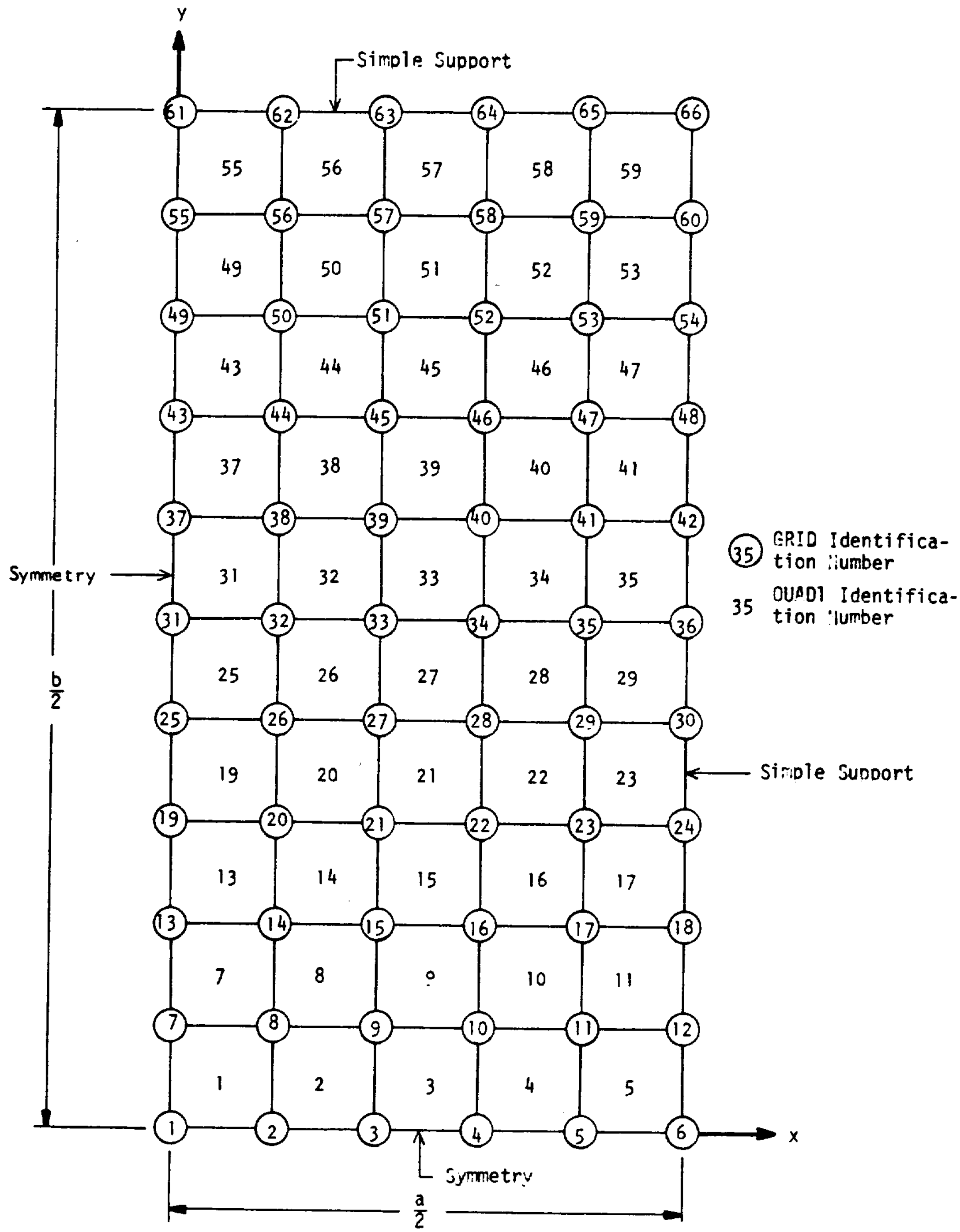


Figure 1. Simply-supported rectangular plate with a thermal gradient.

Table 1. Comparison of analytical and NASTRAN results.

CATEGORY	MAXIMUM ANALYTICAL	MAXIMUM DIFFERENCE	PER CENT ERROR
Displacement	6.2898×10^{-1}	-1.5464×10^{-3}	-0.25
Constraint Force	150.0	-.9594	-0.65
Element Mom., M_x	1.4770×10^{2}	-1.1767	-0.80
Element Stress	7.764618×10^{3}	-90.33275	-1.16

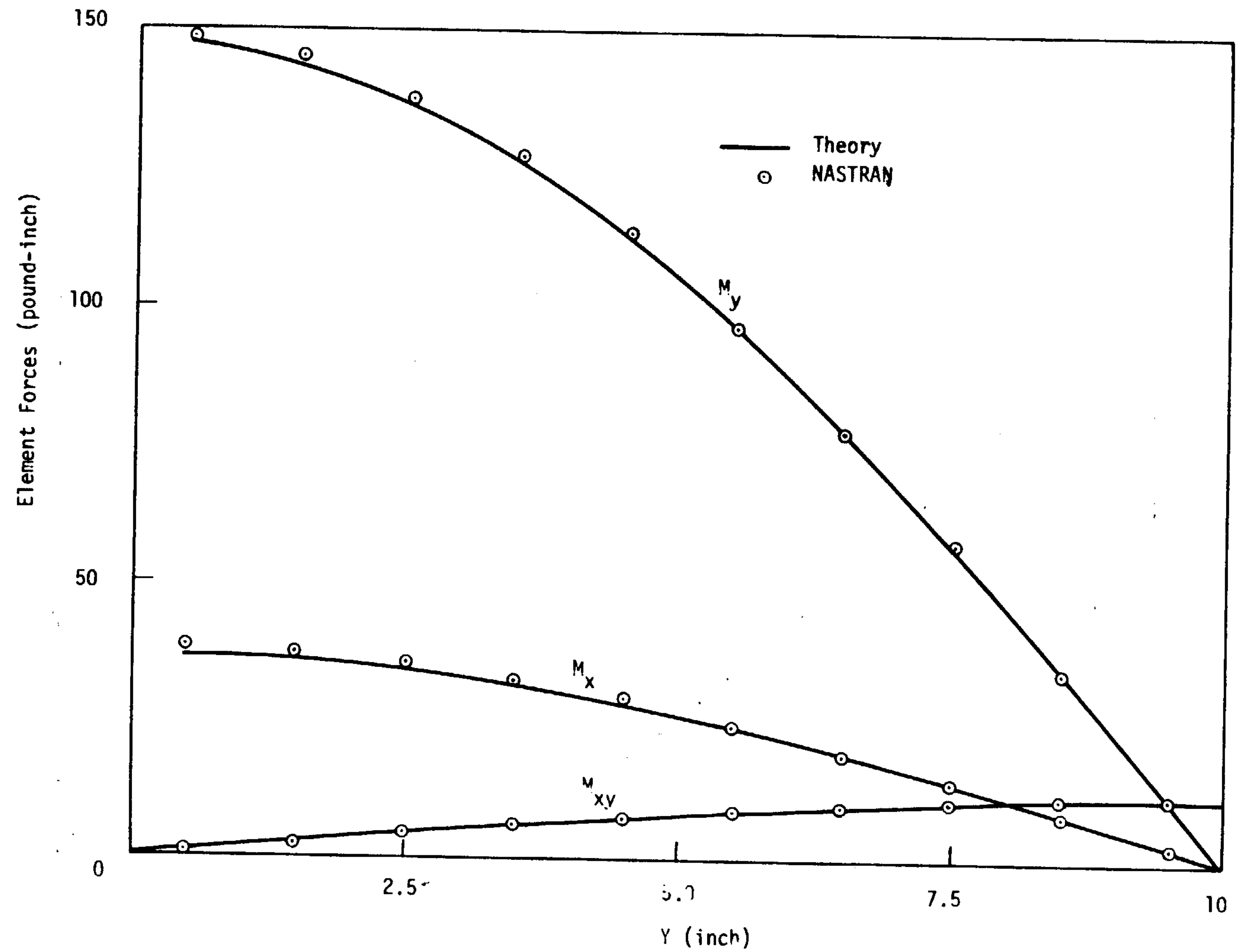


Figure 2. Element forces at x = 0.5.

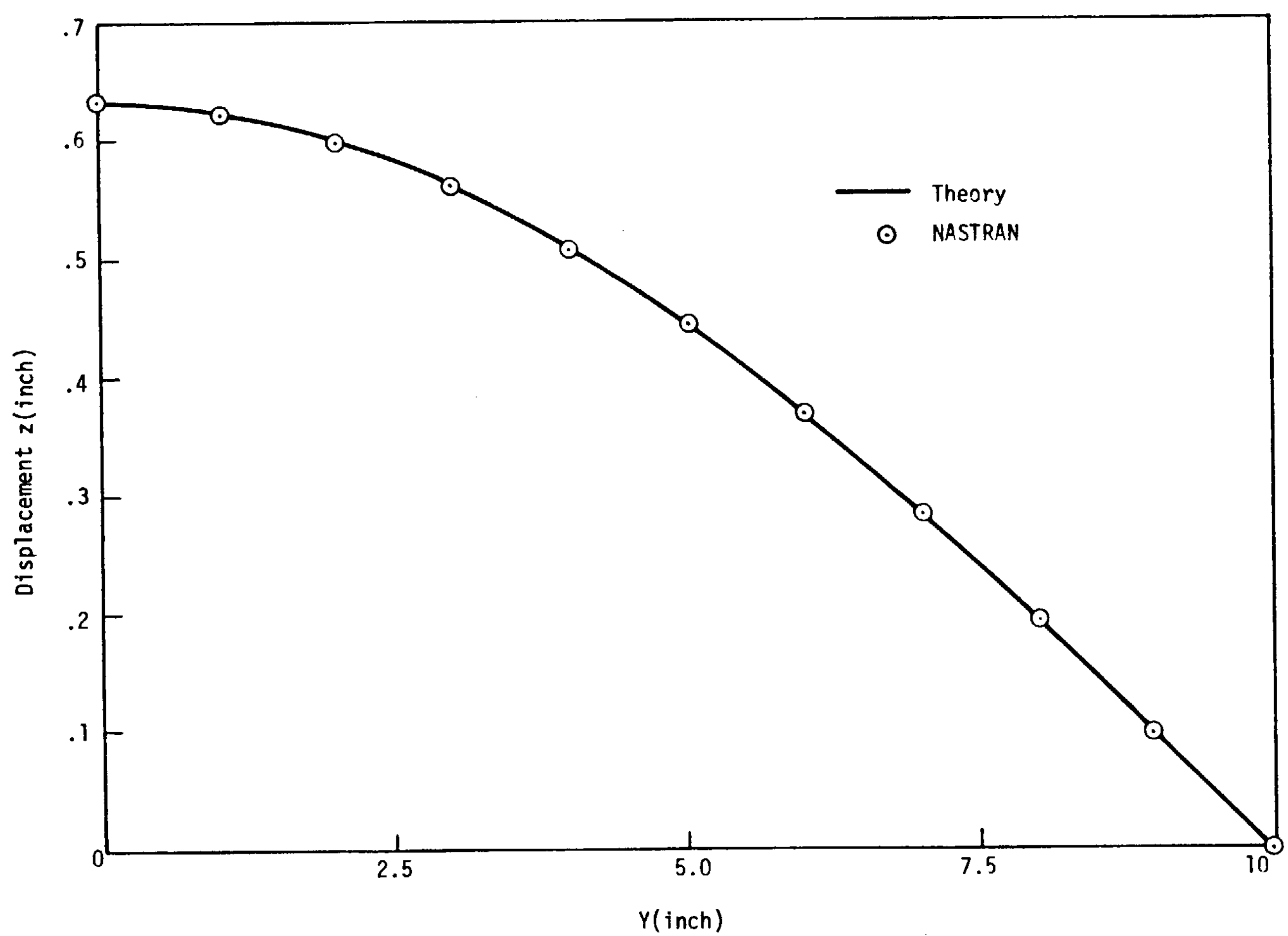


Figure 3. Displacement at x = 5.0.

RIGID FORMAT No. 1 (APP HEAT), Heat Conduction Analysis
Linear Steady State Heat Conduction Through a Washer
Using Solid Elements (1-12-1)
Linear Steady State Heat Conduction Through a Washer
Using Ring Elements (1-12-2)

A. Description

This problem illustrates the capability of NASTRAN to solve heat conduction problems. The washer, shown in Figure 1, has a radial heat conduction with the temperature specified at the outside and a film heat transfer condition at the inner edge. Due to symmetry about the axis and the assumption of negligible axial gradients, the temperature depends only upon the radius.

B. Input

The first NASTRAN model is shown in Figure 2. The solid elements (HEXA1, HEXA2, WEDGE and TETRA) and boundary condition element (HBDY, type AREA4) are used. The conductivity of the material is specified on a MAT4 card. Temperatures are specified at the outer boundary with SPC cards. Punched temperature output is placed on TEMP bulk data cards suitable for static analysis.

Another variation of the problem is shown in Figure 3. Solid of revolution elements (TRIARG and TRAPRG) and boundary condition element (HBDY, type REV) are used. The conductivity of the material and the convective film coefficient are specified on a MAT4 card. The CHBDY card references a scalar point at which the ambient temperature is specified using an SPC card. An SPC1 card is used to constrain the temperature to zero degrees at gridpoints on the outer surface.

C. Theory

The mathematical theory for the continuum is simple, and can be solved in closed form. The differential equation is

$$\frac{1}{r}\ \frac{\partial}{\partial r}\left(rk\frac{\partial U}{\partial r}\right) = 0 \quad . \tag{1}$$

The boundary conditions are

$$-k\frac{\partial U}{\partial r} = H(U_a - U) \text{ at } r = r_1 \quad , \tag{2}$$

and

$$U = 0 \qquad \text{at } r = r_2 \quad . \tag{3}$$

The solution is

$$U(r) = \frac{HU_a}{(k/r_1) + H\,\ell n(r_2/r_1)}\ \ell n(r_2/r)$$

$$= 288.516\ \ell n(2/r)$$

D. Results

A comparison with the NASTRAN results is shown in Table 1.

Table 1. Comparison of Theoretical and NASTRAN Temperatures for Heat Conduction in a Washer.

r(radius)	Theoretical Temperatures	NASTRAN Temperatures (Solids)*	NASTRAN Temperatures (Rings)*
1.0	199.984	202.396	199.932
1.1	172.486	173.904	172.448
1.2	147.381	148.833	147.355
1.3	124.288	124.783	124.269
1.4	102.906	102.852	102.894
1.5	83.001	82.913	82.992
1.6	64.380	64.306	64.375
1.7	46.889	46.832	46.886
1.8	30.398	30.356	30.397
1.9	14.799	14.773	14.798
2.0	0.000	0.000	0.000

*These are the average temperatures at a radius.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1121,NASTRAN
  2   UMF       1977     11210
  3   TIME      1
  4   APP       HEAT
  5   SØL       1,1
  6   CEND

  7   TITLE = LINEAR STEADY STATE HEAT CØNDUCTIØN THRØUGH A WASHER
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-12-1
  9   LABEL = SØLID ELEMENTS,SURFACE FILM HEAT TRANSFER
 10   ØLØAD = ALL
 11   SPCFØRCES = ALL
 12   THERMAL(PRINT,PUNCH) = ALL
 13   ELFØRCE = ALL
 14   SUBCASE 123
 15   LABEL = TEMPERATURE SPECIFIED AT ØUTER BØUNDARY
 16   SPC = 351
 17   LØAD = 251
 18   BEGIN BULK
 19   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CHBDY	701	702	AREA4	1	12	112	101		
CHEXA1	1	200	1	2	13	12	101	102	+SØL1
+SØL1	113	112							
CHEXA2	2	200	2	3	14	13	102	103	+SØL2
+SØL2	114	113							
CØRD2C	111	0	.0	.0	.0	.0	.0	100.0	+CØRD111
+CØRD111	100.0	.0	.0						
CTETRA	3	200	104	114	3	103			
CWEDGE	8	200	4	5	15	104	105	115	
GRDSET						111			
GRID	1	111	1.0	.0	.0				
MAT4	200	1.0							
PARAM	IRES	1							
PHBDY	702								
QBDY1	251	288.5	701						
SEQGP	12	1.1	13	2.1	14	3.1	15	4.1	
SPC	351	11	1	.0	22	1	.0		

Card No.

```
 0  NASTRAN FILES=UMF
 1  ID        DEM1122,NASTRAN
 2  UMF       1977     11220
 3  APP       HEAT
 4  SØL       1,0
 5  TIME      10
 6  CEND

 7  TITLE = LINEAR STEADY STATE CØNDITIØN THRØUGH A WASHER
 8  SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-12-2
 9  LABEL = RING ELEMENTS, FILM HEAT TRANSFER
10  ØUTPUT
11  ØLØAD = ALL
12  SPCFØRCE = ALL
13  THERMAL (PRINT,PUNCH) = ALL
14  ELFØRCE = ALL
15  SPC = 350
16  BEGIN BULK
17  ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CHBDY	14	100	REV	1	12				+HBDY14
+HBDY14	23	23							
CTRAPRG	7	4	5	16	15	.0	200		
GRID	1		1.0	.0	.0				
MAT4	200	1.0							
PHBDY	100	300							
SEQGP	12	1.1	13	2.1	14	3.1	15	4.1	
SPC	352	23		488.5					
SPC1	351	1	11	22					
SPCADD	350	351	352						
SPØINT	23								
TEMPD	201	.0							

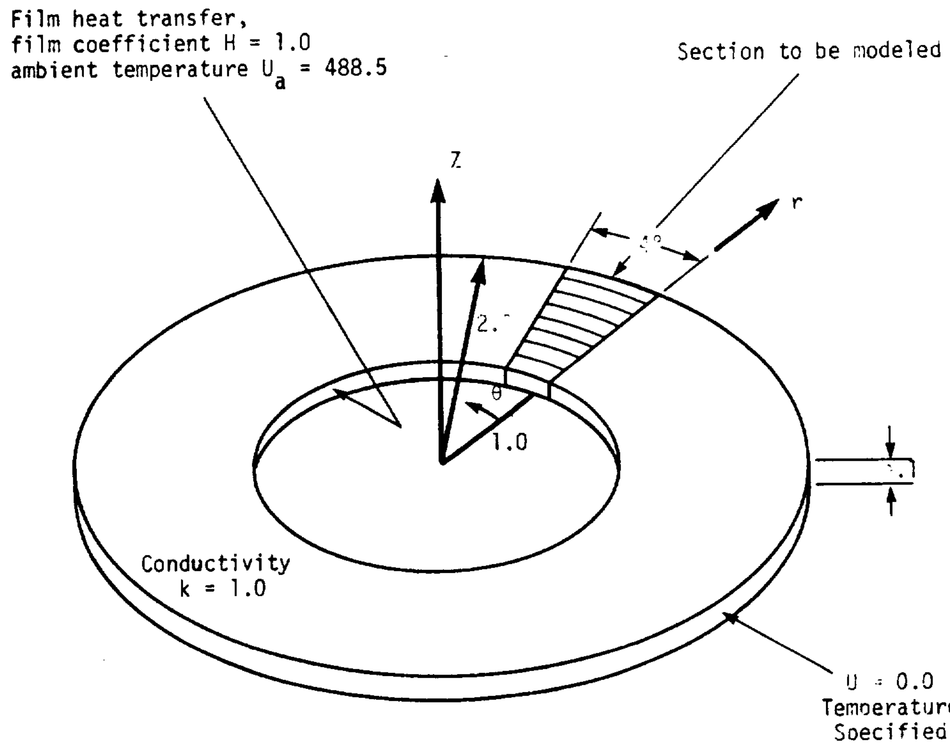
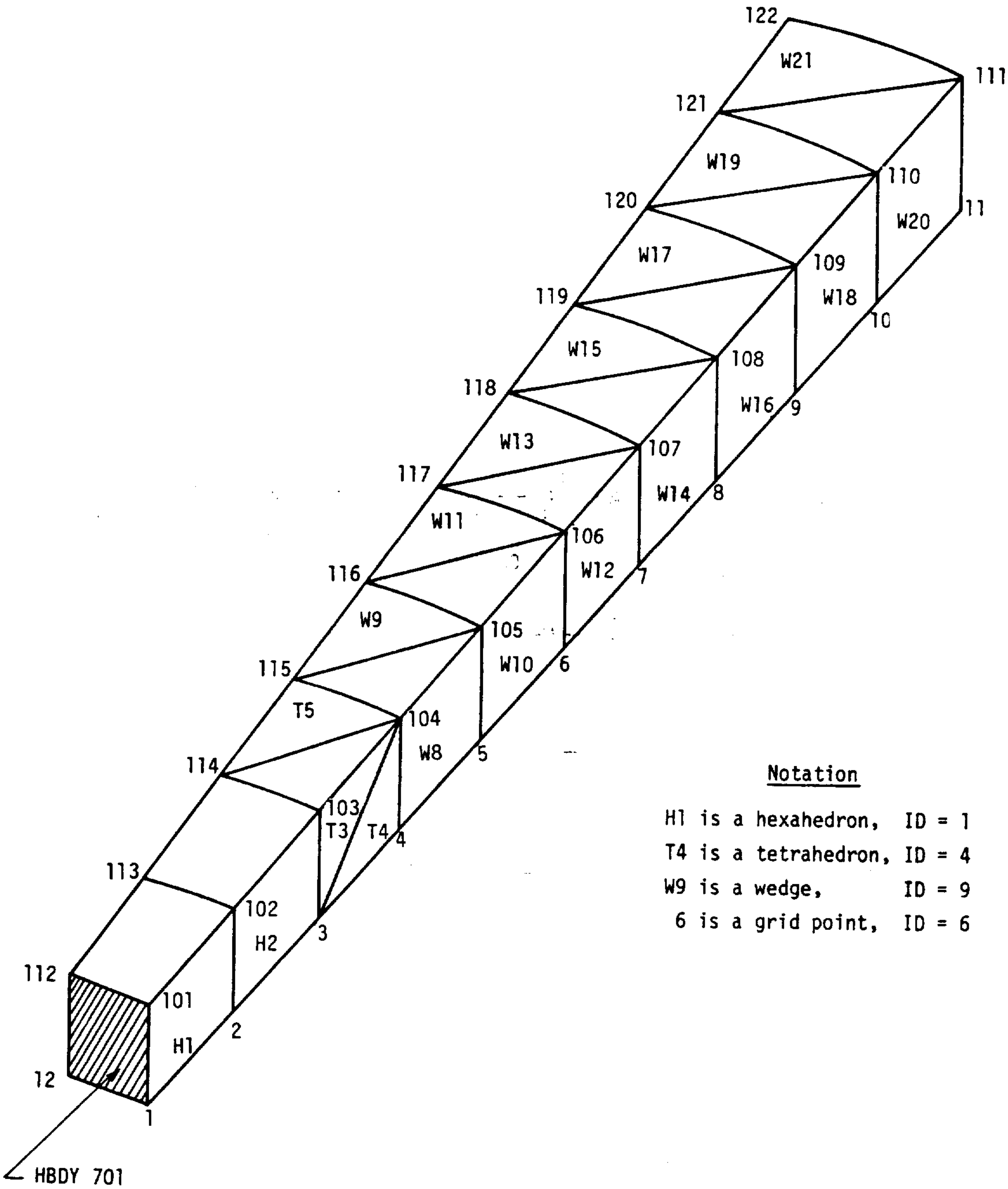


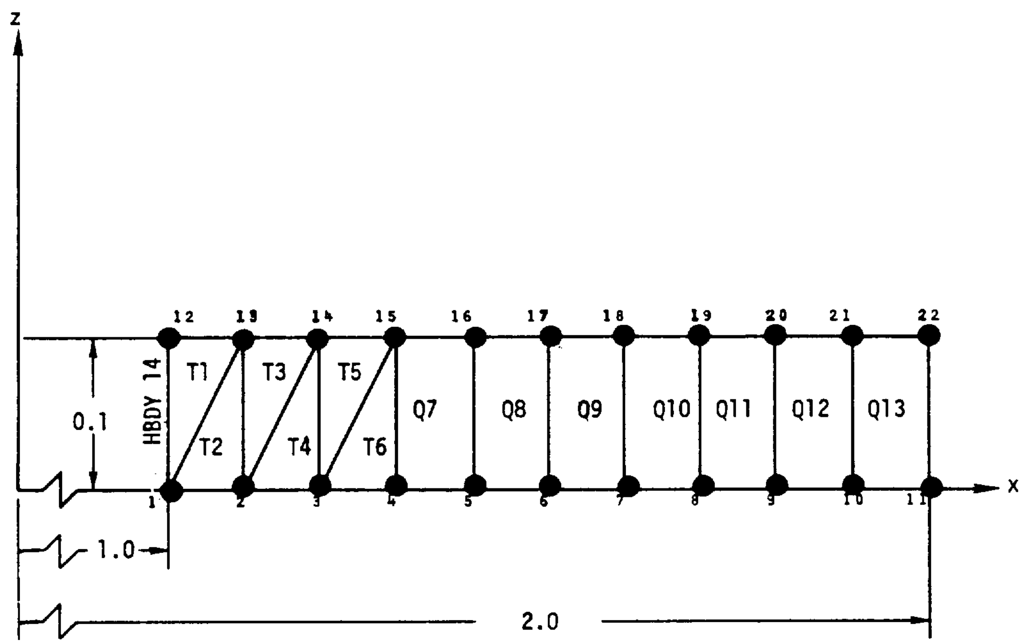
Figure 1. Washer Analyzed in Heat Conduction Demonstration Problem



<u>Notation</u>

H1 is a hexahedron, ID = 1
T4 is a tetrahedron, ID = 4
W9 is a wedge, ID = 9
6 is a grid point, ID = 6

Figure 2. Elements and Grid Points



T TRIARG elements

Q TRAPRG elements

U_a = 488.5 at left end

U_a = 0.0 at right end

Figure 3. Section of a pipe, modeled with ring elements

RIGID FORMAT No. 1, Static Analysis

Thermal and Pressure Loads on a Long Pipe Using Linear Isoparametric Elements (1-13-1)
Thermal and Pressure Loads on a Long Pipe Using Quadratic Isoparametric Elements (1-13-2)
Thermal and Pressure Loads on a Long Pipe Using Cubic Isoparametric Elements (1-13-3)

A. Description

These problems demonstrate the use of the linear, quadratic and cubic isoparametric solid elements, IHEX1, IHEX2 and IHEX3, respectively. A long pipe, assumed to be in a state of plane strain, is subjected to an internal pressure and a thermal gradient in the radial direction. The structure modeled is shown in Figure 1. The finite element NASTRAN models for each of the elements are shown in Figures 2, 3 and 4.

B. Input

1. Parameters:

$r_{inner} = a = 4$ in.	(radius to the inner surface)
$r_{outer} = b = 5$ in.	(radius to the outer surface)
$E = 30.\text{x}10^6$ psi	(Young's Modulus)
$\nu = 0.3$	(Poisson's Ratio)
$\alpha = 1.428 \times 10^{-5}$	(thermal expansion coefficient)
$\rho = 7.535 \times 10^{-4} \dfrac{\text{lb-sec}^2}{\text{in}^4}$	(mass density)
$p = 10$ psi	(inner surface pressure)
$T_i = 100.0°F$	(inner surface temperature)
$T_o = 0.0°F$	(outer surface temperature)

2. Boundary Conditions:

$u_\theta = 0$ at all points on the right side

$u_\theta = 0$ at all points on the left side

$u_z = 0$ at all points on the bottom surface

$u_z = 0$ at all points on the top surface

3. Loads:

Subcase 1,

$p = 10$ psi (internal pressure)

Subcase 2,

$$T_r = \frac{(T_i - T_o)}{\ln(\frac{b}{a})} \ln(\frac{b}{r}) = \frac{100}{\ln(1.25)} \ln(\frac{5}{r}) \text{ , where r is any radius.}$$

C. Theory

1. Subcase 1

The normal stresses due to the pressure load (Reference 24) are obtained by

$$\sigma_r = -\frac{a^2b^2}{(b^2-a^2)} \frac{p}{r^2} + \frac{pa^2}{(b^2-a^2)} \quad , \tag{1}$$

$$\sigma_\theta = \frac{a^2b^2}{(b^2-a^2)} \frac{p}{r^2} + \frac{pa^2}{(b^2-a^2)} \quad , \tag{2}$$

and

$$\sigma_z = 2\nu \frac{pa^2}{(b^2-a^2)} \quad . \tag{3}$$

where r is the radius and all shearing stresses are zero.

The displacement in the radial direction is

$$u_r = \frac{(1-2\nu)(1+\nu)}{E} r \frac{pa^2}{(b^2-a^2)} + \frac{(1+\nu)}{E} \frac{1}{r} \frac{pa^2b^2}{(b^2-a^2)} \quad , \tag{4}$$

and all other displacements are zero.

2. Subcase 2

The stresses in the radial and tangential directions due to the thermal load (Reference 24) are given by

$$\sigma_r = \frac{\alpha E T_i}{2(1-\nu)\ln(\frac{b}{a})} \left[-\ln(\frac{b}{r}) - \frac{a^2}{(b^2-a^2)} (1 - \frac{b^2}{r^2}) \ln(\frac{b}{a}) \right] \quad , \tag{5}$$

and

$$\sigma_\theta = \frac{\alpha E T_i}{2(1-\nu)\ln(\frac{b}{a})} \left[1 - \ln(\frac{b}{r}) - \frac{a^2}{(b^2-a^2)} (1 + \frac{b^2}{r^2}) \ln(\frac{b}{a}) \right] \quad . \tag{6}$$

The stress in the axial direction is obtained via the procedure contained in the reference as

$$\sigma_z = \frac{\alpha E T_i}{2(1-\nu)\ln(\frac{b}{a})} \left[\nu - \frac{2a^2\nu}{(b^2-a^2)} \ln(\frac{b}{a}) - 2 \ln(\frac{b}{r}) \right] . \tag{7}$$

All shearing stresses are zero.

The displacement in the radial direction is

$$u_r = \frac{(1 + \nu)}{(1 + \nu)} \alpha \frac{T_i}{\ln(\frac{b}{a})} \left\{ - \frac{1}{r} \left[\frac{a^2 b^2}{2(b^2-a^2)} \ln(\frac{b}{a}) \right] \right.$$

$$\left. + \frac{r}{4} \left[2 \ln(\frac{b}{r}) + 1 + (1-2\nu) \left(1 - \frac{2a^2}{(b^2-a^2)} \ln(\frac{b}{a}) \right) \right] \right\} . \tag{8}$$

D. Results

Representative displacements and stresses for the finite element results compared to theoretical predictions are plotted in Figures 5 and 6. Note that five IHEX1 elements were used along the radial thickness whereas one element was used for each of the IHEX2 and IHEX3 cases. Two values for the stress occur at the boundary of two adjacent IHEX1 elements resulting in a sawtooth pattern.

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E. Driver Decks and Sample Bulk Data

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Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1131,NASTRAN
  2   UMF       1977     11310
  3   APP       DISPLACEMENT
  4   SØL       1,0
  5   TIME      5
  6   CEND

  7   TITLE = LØADS ØN A LØNG PIPE USING LINEAR ISØPARAMETRIC ELEMENTS
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-13-1
  9   DISP = ALL
 10   STRESS = ALL
 11   SPC = 100
 12   SUBCASE 1
 13   LABEL = PRESSURE LØAD
 14   LØAD = 400
 15   SUBCASE 2
 16   LABEL = THERMAL LØAD
 17   TEMP(LØAD) = 500
 18   BEGIN BULK
 19   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CIHEX1	1	200	1	2	20	19	7	8	+HEX1-1
+HEX1-1	26	25							
CNGRNT	1	6	11	16	21	26	31	36	
CØRD2C	1	0	.0	.0	.0	.0	.0	100.0	+CØRD2-1
+CØRD2-1	100.0	.0	.0						
GRDSET		1				1	456		
GRID	1		4.0	-14.0					
MAT1	300	3.+7		.3	7.535-4	1.428-5	.0		
PIHEX	200	300		4	4.5	10.0			
PLØAD3	400	-10.0	1	1	25	21	7	31	
SPC1	100	2	1	THRU	18				
TEMP	500	1	100.0	7	100.0	13	100.0		
TEMPD	500	.0							

Card
No.

```
 0   NASTRAN FILES=UMF
 1   ID         DEM1132,NASTRAN
 2   UMF        1977     11320
 3   APP        DISPLACEMENT
 4   SØL        1,0
 5   TIME       5
 6   CEND

 7   TITLE = LØADS ØN A LØNG PIPE USING QUADRATIC ISØPARAMETRIC ELEMENTS
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-13-2
 9   DISP = ALL
10   STRESS = ALL
11   SPC = 200
12   SUBCASE 1
13   LABEL = PRESSURE LØAD
14   LØAD = 400
15   SUBCASE 2
16   LABEL = THERMAL LØAD
17   TEMP(LØAD) = 500
18   BEGIN BULK
19   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CIHEX2	1	200	1	2	3	10	15	14	+HEX-1
+HEX-1	13	9	4	5	17	16	6	7	+HEX-11
+HEX-11	8	12	20	19	18	11			
CNGRNT	1	2							
CØRD2C	10	0	.0	.0	.0	.0	.0	100.0	+CRD-1
+CRD-1	100.0	.0	.0						
GRDSET		10				10	456		
GRID	1		4.0	-14.0	.0				
MAT1	300	3.+7		.3	7.535-4	1.428-5	.0		
PIHEX	200	300		4					
PLØAD3	400	-10.0	1	13	6	2	25	18	
SPC1	200	2	1	THRU	8				
TEMP	500	1	100.0	4	100.0	6	100.0		
TEMPD	500	.0							

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Card
 No.

 0   NASTRAN FILES=UMF
 1   ID         DEM1133,NASTRAN
 2   UMF        1977    11330
 3   APP        DISPLACEMENT
 4   SØL        1,0
 5   TIME       3
 6   CEND

 7   TITLE = LØADS ØN A LØNG PIPE USING CUBIC ISØPARAMETRIC ELEMENTS
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-13-3
 9   DISPLACEMENT = ALL
10   STRESS = ALL
11   SPC = 200
12   SUBCASE 1
13   LABEL = PRESSURE LØAD
14   LØAD = 80
15   SUBCASE 2
16   LABEL = THERMAL LØAD
17   TEMP(LØAD) = 90
18   BEGIN BULK
19   ENDDATA
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1	2	3	4	5	6	7	8	9	10
CIHEX3	10	60	1	2	3	4	5	6	+HEX-31
+HEX-31	7	8	9	10	11	12	13	14	+HEX-32
CØRD2C	111	0	.0	.0	.0	.0	.0	50.0	+COR1
+CØR1	50.0	.0	.0						
GRDSET		111				111	456		
GRID	1		4.0	.0	.0				
MAT1	70	3.+7		.3	7.535-4	1.428-5	.0		
PIHEX	60	70		4					
PLØAD3	80	-10.0	10	30	1				
SPC1	200	2	1	2	3	4	13	14	+SPC-A2
+SPC-A2	17	18	21	22	23	24	7	8	+SPC-B2
TEMP	90	1	100.0	12	100.0	11	100.0		
TEMPD	90	.0							

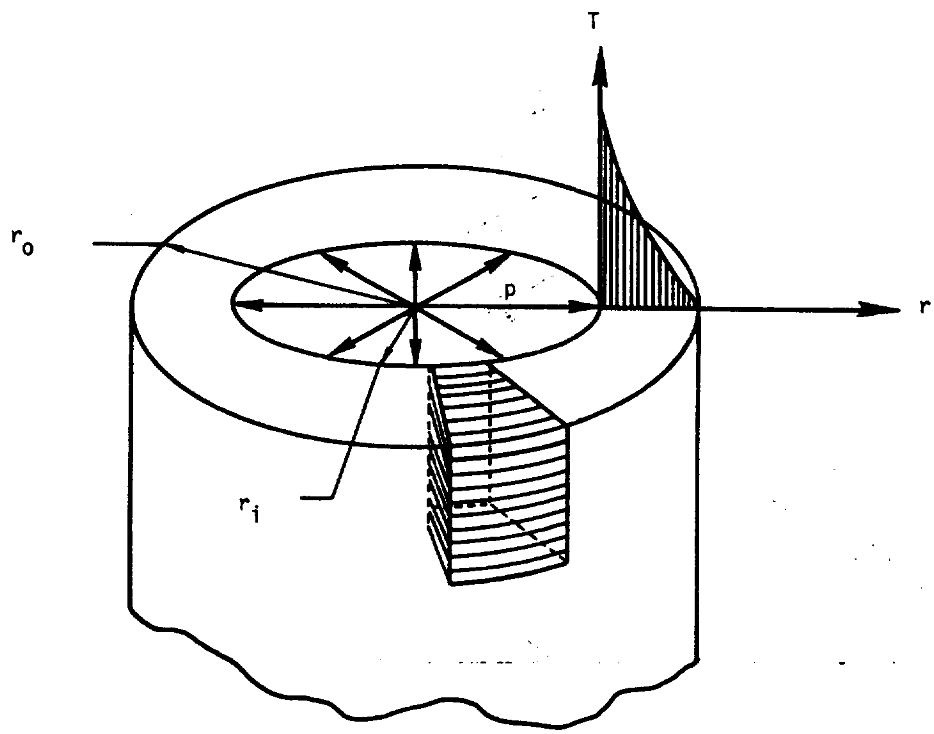


Figure 1. Long pipe with pressure and thermal loads.

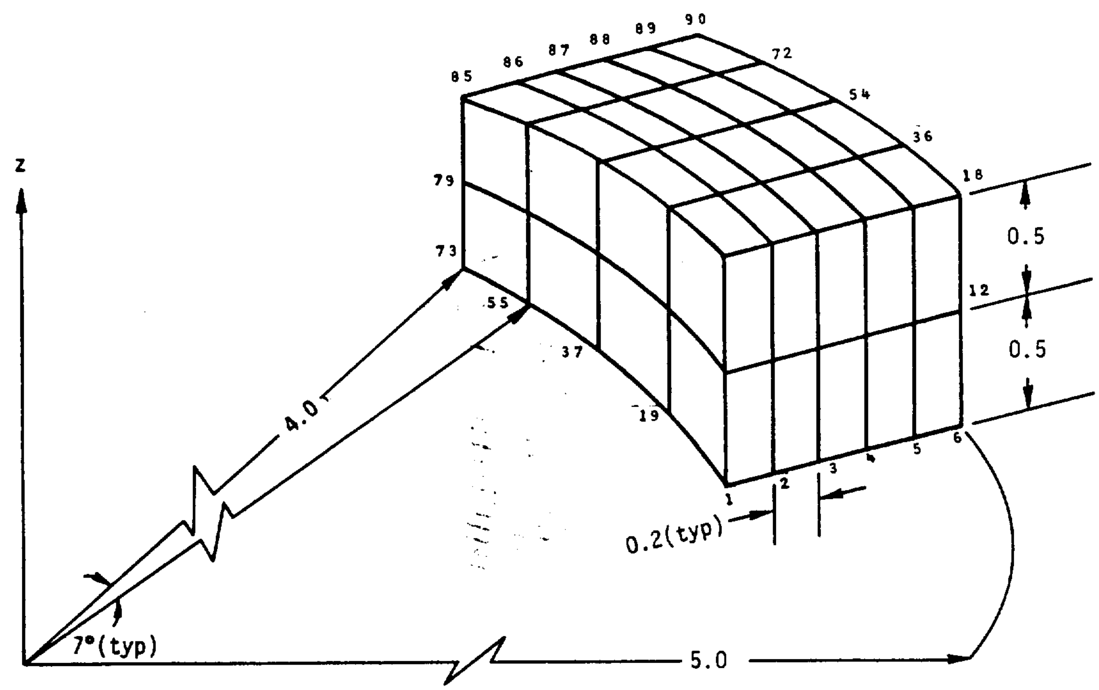


Figure 2. Model of section using forty IHEX1 elements.

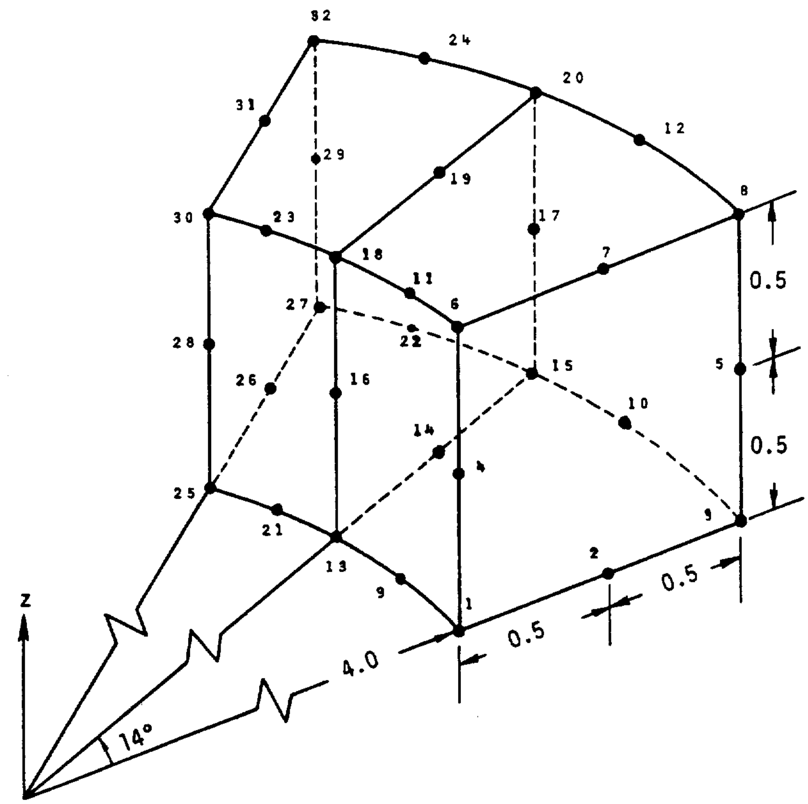


Figure 3. Model of section using two IHEX2 elements.

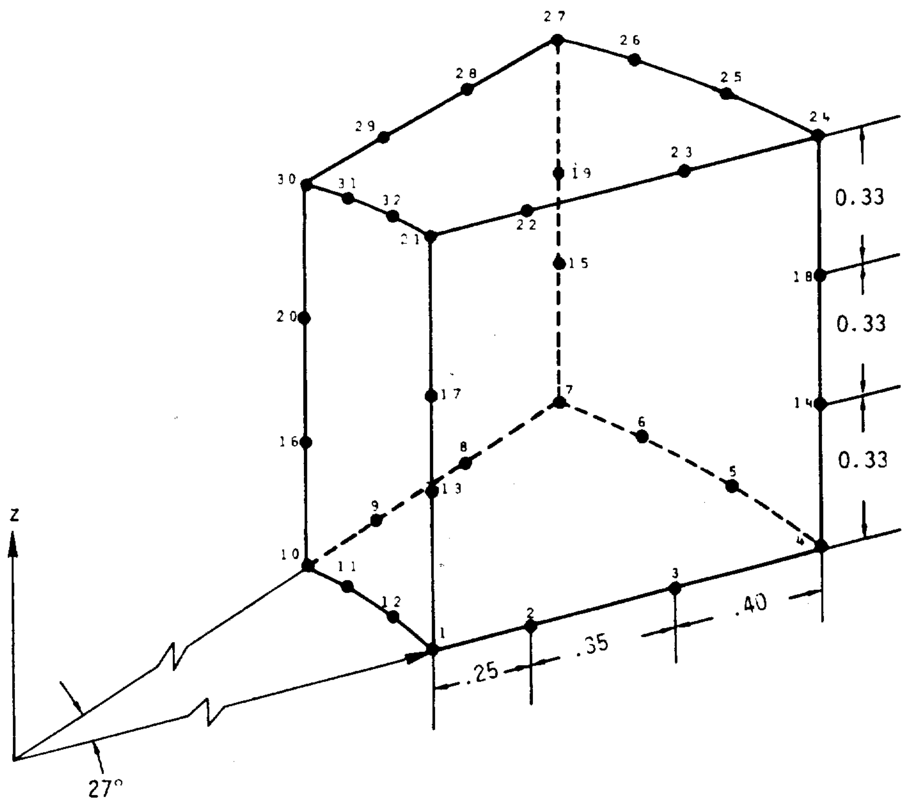
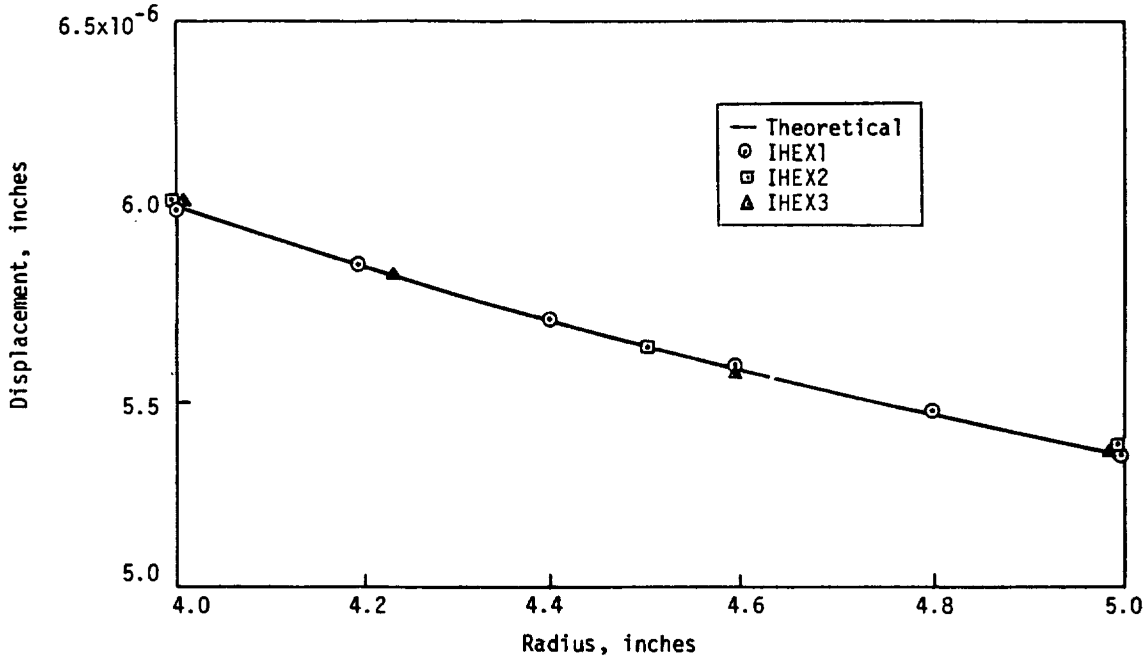
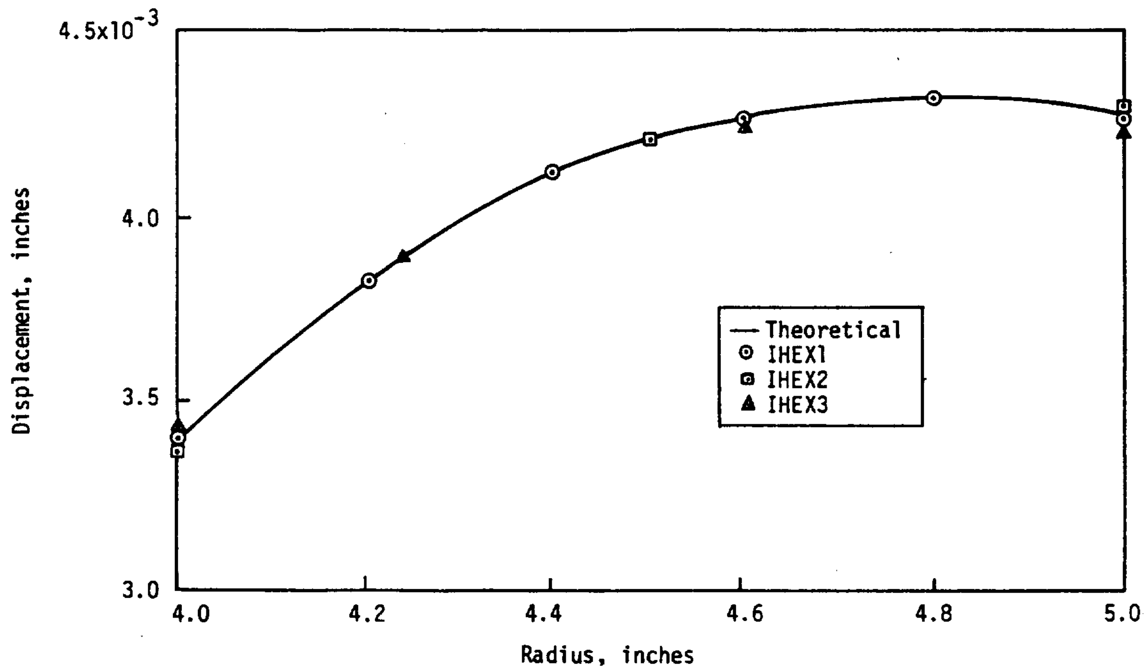


Figure 4. Model of section using one IHEX3 element.

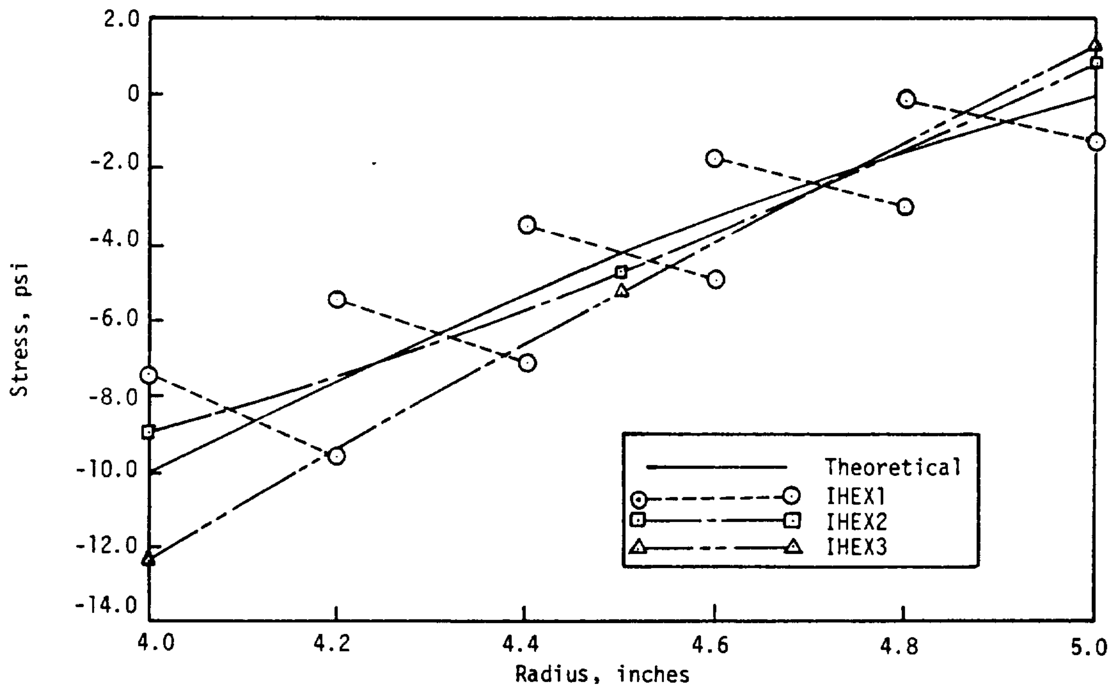


(a) Radial deflections, pressure load.

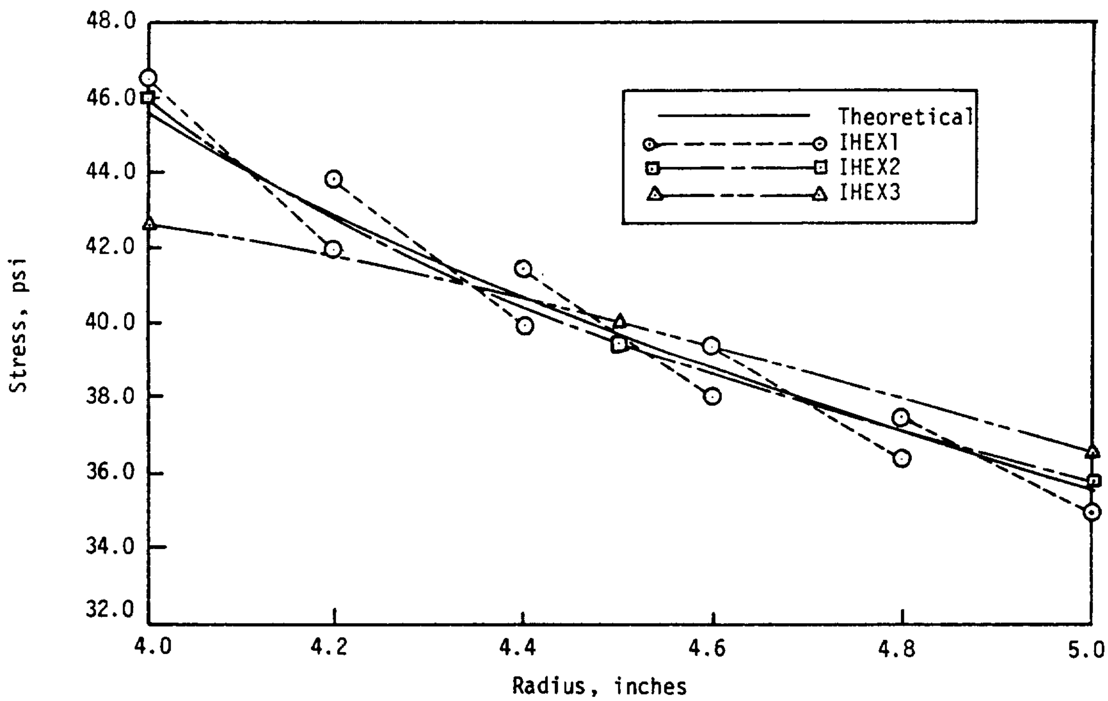


(b) Radial deflections, thermal load.

Figure 5. Deflection comparisons.



(a) Radial stress, pressure load.



(b) Circumferential stress, pressure load.

Figure 6. Stress comparisons.

Static Analysis of a Beam Using General Elements (1-14-1)

A. Description

This problem demonstrates the use of general GENEL elements having various types of input format in the static analysis of a cantilever beam subjected to tension and bending. The beam consists of five GENEL elements and one BAR element as shown in Figure 1.

The GENEL elements are constructed as follows:

GENEL Element	Approach	Matrix Size	$\{u_d\}$	[S]
1	Flexibility	3	No	No
2	Stiffness	6	No	No
3	Stiffness	3	Yes	Yes
4	Stiffness	3	Yes	No
5	Flexibility	3	Yes	No

B. Input

1. Parameters

ℓ = 6.0 m (length)

E = 6.0 N/m^2 (modulus of elasticity)

V = 0.3 (Poisson's ratio)

A = 1.0 m^2 (cross-sectional area)

I = .083 m^4 (bending moment of inertia)

F_x = 1.0 N (axial load)

P_y = 1.0 N (transverse load)

C. Theory

The stiffness matrix for the BAR element in its general form is given in section 8 of the NASTRAN Programmer's Manual.

Define [Z] as the matrix of deflection influence coefficients (flexibility matrix) whose terms are $\{u_i\}$ when $\{u_d\}$ is rigidly constrained,

[K] as the stiffness matrix,

[S] as a rigid body matrix whose terms are $\{u_i\}$ due to unit motions of $\{u_d\}$, when all $\{f_i\}$ = 0,

$\{f_i\}$ as the vector of forces applied to the element at $\{u_i\}$, and $\{f_d\}$ as the vector of forces applied to the element at $\{u_d\}$. They are assumed to be statically related to the $\{f_i\}$ forces, i.e., they constitute a nonredundant set of reactions for the element. If transverse shear is neglected and the beam is confined to motion in the X-Y plane, then

$$\{f_i\} = [K]\ \{u_i\} \ ,$$

where

$$\{f_i\} = \begin{Bmatrix} F \\ V_2 \\ M_1 \end{Bmatrix} \qquad \{u_i\} = \begin{Bmatrix} \delta x \\ \delta y \\ \theta z \end{Bmatrix} \ ,$$

$$[K] = \begin{bmatrix} \frac{AE}{\ell} & 0 & 0 \\ 0 & \frac{12EI}{\ell^3} & \frac{6EI}{\ell^2} \\ 0 & \frac{6EI}{\ell^2} & \frac{4EI}{\ell} \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 3 \\ 0 & 3 & 2 \end{bmatrix} \ ,$$

$$[F] = [K]^{-1} \begin{bmatrix} \frac{1}{6} & 0 & 0 \\ 0 & \frac{2}{3} & -1 \\ 0 & -1 & 2 \end{bmatrix} \ ,$$

and

$$[S] = \begin{bmatrix} 1 & 0 & \Delta u_y \\ 0 & 1 & \Delta u_x \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \ ,$$

where $\Delta u = u_d - u_i$, i.e., the difference between the dependent displacement degree of freedom $\{u_d\}$ and the independent displacement degree of freedom $\{u_i\}$.

D. <u>Results</u>

The theoretical maximum deflection of the cantilever beam subjected to tension and bending (for the input values) are

$$\delta x = \frac{F\ell}{AE} = 1.0 \text{ m} \quad \text{(tension)}$$

and

$$\delta y = \frac{P\ell^3}{3EI} = 144.0 \text{ m} \quad \text{(bending)}$$

These results are obtained by NASTRAN.

E. Driver Decks and Sample Bulk Data

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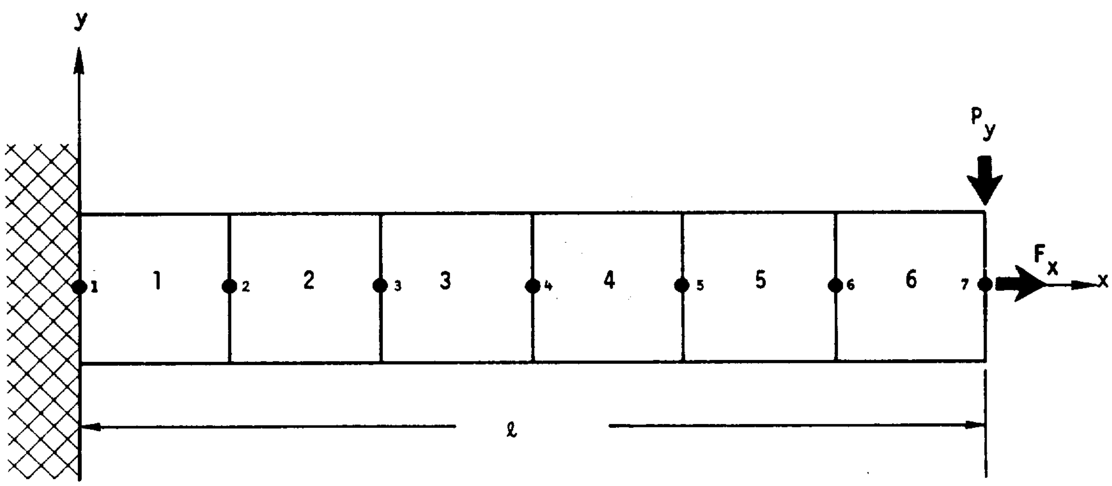
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID       DEM1141,NASTRAN
  2   UMF      1977    11410
  3   APP      DISPLACEMENT
  4   SØL      1,0
  5   TIME     5
  6   CEND

  7   TITLE = STATIC ANALYSIS ØF A BEAM USING GENERAL ELEMENTS
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-14-1
  9       DISPLACEMENT = ALL
 10       ELFØRCE = ALL
 11   SUBCASE 1
 12   LABEL = AXIAL LØAD
 13       LØAD = 1
 14   SUBCASE 2
 15   LABEL = BENDING LØAD
 16       LØAD = 2
 17   BEGIN BULK
 18   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CBAR	6	1	6	7	.0	1.0	.0	1	
FØRCE	1	7		1.	1.				
GENEL	1		2	1	2	2	2	6	+G11
+G11	Z	.1666667	.0	.0	.6666667	1.0	2.0		
GRDSET							345		
GRID	1		.0	.0	.0		123456		
MAT1	1	6.		.3					
PBAR	1	1	1.	.083333					



GENEL elements 1 thru 5
RØD element 6

Figure 1. NASTRAN General Element model.

RIGID FORMAT No. 1, Static Analysis

Axisymmetric Cylinderical Thick Shell Subjected to Asymmetric Pressure Loading (1-15-1)

A. Description

This problem demonstrates the use of elements TRAPAX and TRIAAX in the analysis of asymmetrically loaded solids of revolution. The structure, illustrated in Figure 1, consists of a circular cylindrical shell loaded with a uniform external pressure over a small square area.

The cylindrical shell wall is assumed to be simply supported, i.e., the radial and circumferential deflections and the bending moments are zero at the ends.

The upper half of the structure is modeled as shown in Figure 2. Trapezoidal elements having small and large dimensions, are used in the vicinity of the load and away from the load, respectively. A transition area, between the two trapezoidal configurations, is modeled with triangular elements to illustrate their use.

The loads and deflections, not required to be axisymmetric, are expanded in Fourier series with respect to the azimuthal coordinate. Due to the one plane of symmetry of this problem with respect to the ϕ = 0 plane, the deflections are represented by a cosine series selected by the AXISYM Case Control card. The highest harmonic used, 10, is defined on the AXIC Bulk Data card. The pressure load is defined using PRESAX bulk Data cards.

B. Input

1. Parameters:

r_a	=	15 in.	(Average radius)
t	=	1 in.	(Thickness)
ℓ	=	45 in.	(Length)
$2c$	=	3.75 in.	(Load Length)
β	=	0.125 radians	(Load Arc ($\beta = c/r_a$))
E	=	66666.7 psi	(Modulus of Elasticity)
ν	=	0.3	(Poisson's Ratio)
n	=	10	(Harmonics)

2. Loads:

p	=	7.11111 psi	(Pressure)
A	=	14.063 in^2	(Area of Load ($A = 4c^2$))

3. Supports:

Simply supported at the ends: $u_r = 0$, $u_\phi = 0$

Symmetry at the midplane: $u_z = 0$

C. Theory

Theoretical results for this problem are taken from Reference 20, p. 568. The following theoretical values occur at the center of the load ($z = \frac{\ell}{2}$, $\phi = 0$):

$u_r = 272 \frac{pA}{Er_a} = 0.0272$ in. (Radial Deflection (inward))

$M_\phi = 0.1324\ pA = 13.24$ in-lb/in (Circumferential Bending Moment)

$M_z = 0.1057\ pA = 10.57$ in-lb/in (Longitudinal Bending Moment)

$F_\phi = -2.6125 \frac{pA}{r_a} = -17.42$ lb/in (Circumferential Membrane Force)

$F_z = -2.320 \frac{pA}{r_a} = -15.47$ lb/in (Longitudinal Membrane Force)

Theoretical stresses on the inside and outside walls at this point ($z = \frac{\ell}{2}$, $\phi = 0$) are computed as follows:

$\sigma_z = \frac{F_z}{t} \pm \frac{6M_z}{t^2} =$ 47.95 psi (Inside Wall Longitudinal Stress)

-78.89 psi (Outside Wall Longitudinal Stress)

$\sigma_\phi = \frac{F_\phi}{t} \pm \frac{6M_\phi}{t^2} =$ 62.02 psi (Inside Wall Circumferential Stress)

-96.86 psi (Outside Wall Circumferential Stress)

D. Results

Figure 3 shows the NASTRAN radial deflection at the center of the load as a function of the number of harmonics selected for the solution. As can be seen, the solution is near convergence with ten harmonics.

Figure 4 shows stresses, σ_z and σ_ϕ, through the shell wall, at the center of the load. Ten harmonics shows very good convergence to nearly the theoretical values computed above. However, seven harmonics would result in relatively poor convergence even through Figure 3 indicates the displacement was close to convergence. Thus, displacement convergence alone may be an invalid indicator of an adequate solution.

E. Driver Decks and Sample Bulk Data

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  2    UMF       1977    11510
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  4    SØL       1,1
  5    TIME      90
  6    CEND

  7    TITLE = ASYMMETRIC PRESSURE LØADING ØF AN AXISYMMETRIC CYLINDRICAL SHELL
  8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-15-1
  9    AXISYM = CØSINE
 10    LØAD = 20
 11        SET 10 = 11 THRU 34, 111 THRU 231, 235, 241, 245, 251, 255, 261,
 12                 265, 271, 275, 281, 285, 291, 295, 301, 305, 311, 315
 13                 321, 325, 331, 335, 341, 345, 351, 355, 361, 365, 371,
 14                 375, 381, 385, 391, 395, 401, 405, 411 THRU 415
 15        SET 9  = 111 THRU 227, 231, 234, 241, 244, 251, 254, 261, 264, 271,
 16                 274, 281, 284, 291, 294, 301, 304, 311, 314, 321, 324, 331,
 17                 334, 341, 344, 351, 354, 361, 364, 371, 374, 381, 384, 391,
 18                 394, 401  THRU 404
 19    HARMØNICS = ALL
 20    DISPLACEMENT = 10
 21    ØLØAD = ALL
 22    STRESS = 9
 23    ELFØRCE = 9
 24    PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-15-1
 25    ØUTPUT(PLØT)
 26    PLØTTER SC
 27    SET 1 = ALL
 28    $
 29    $  CØNVERT IDS TØ NASTRAN IDS FØR ELEMENTS 111 THRU 227 (ID*1000+N)
 30    $
 31    SET 2  INCLUDE ELEMENTS 111001 THRU 227001
 32    AXES Z, X, Y
 33    VIEW 0.0, 0.0, 0.0
 34    FIND SCALE, ØRIGIN 1, SET 1
 35    PTITLE = FULL MØDEL
 36    PLØT SET 1, ØRIGIN 1
 37    FIND SCALE, ØRIGIN 2, SET 2
 38    PTITLE = LØADED SECTIØN (TRAPAX) AND TRANSITIØN SECTIØN (TRIAAX)
 39    PLØT SET 2, ØRIGIN 2
 40    BEGIN BULK
 41    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
AXIC	10								
CTRAPAX	111	5	111	112	122	121			
CTRIAAX	181	10	181	192	191				
MAT1	15	66666.7		.3					
PØINTAX	11	111	.0						
PRESAX	20	-7.11111	114	124	-7.162	7.162			
PTRAPAX	5		15	.0	7.1				
PTRIAAX	10		15	.0	3.581	7.162			
RINGAX	111		14.5	.0			3456		

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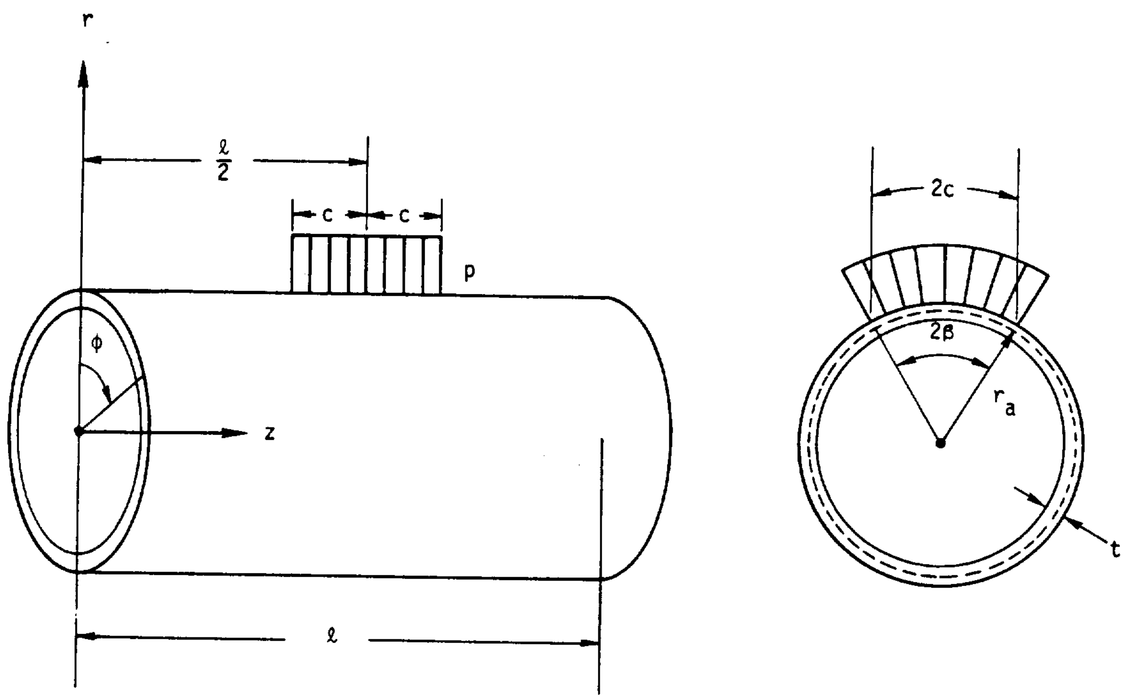


Figure 1. Cylindrical shell loaded by a uniformly distributed load

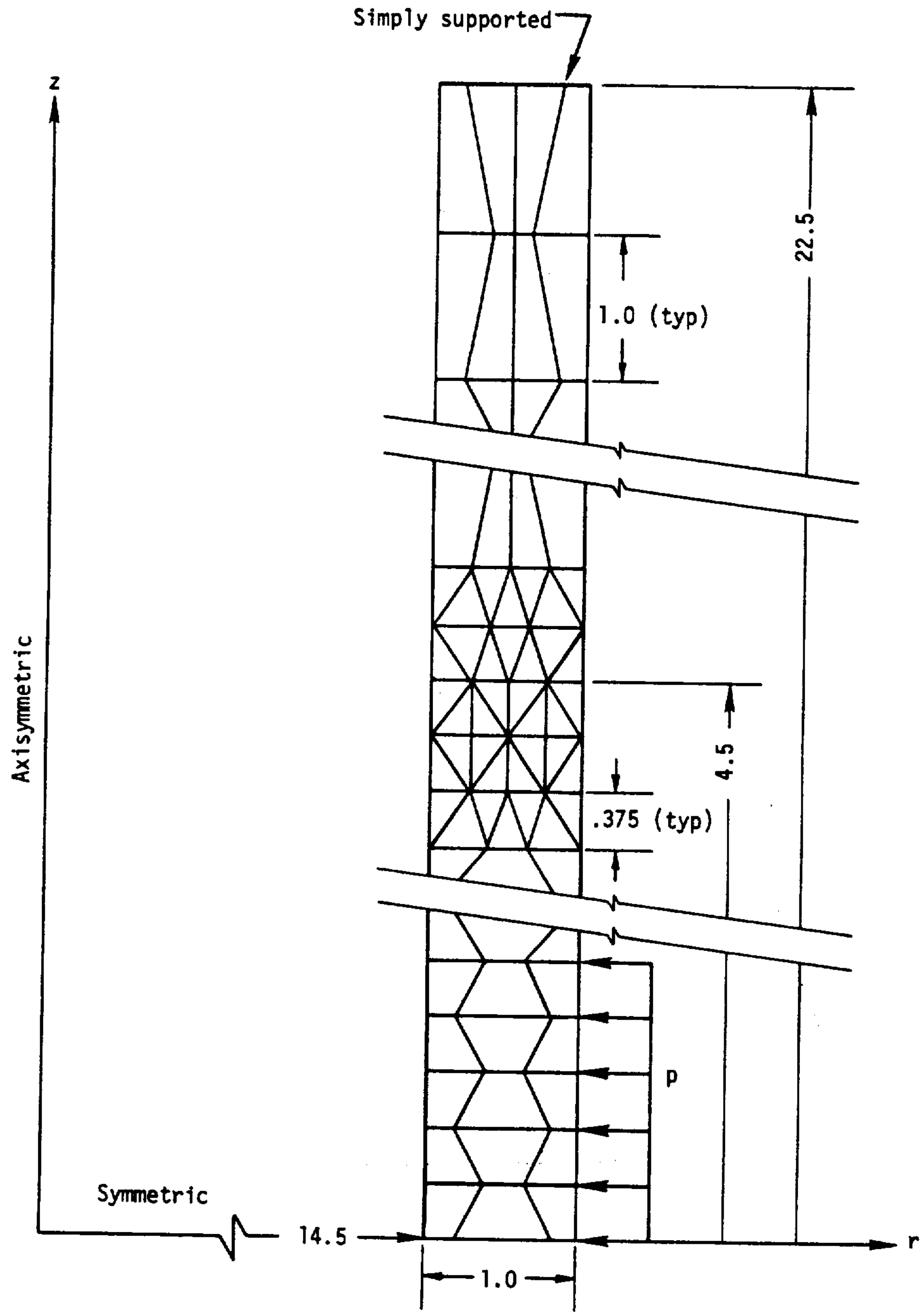


Figure 2. NASTRAN shell model.

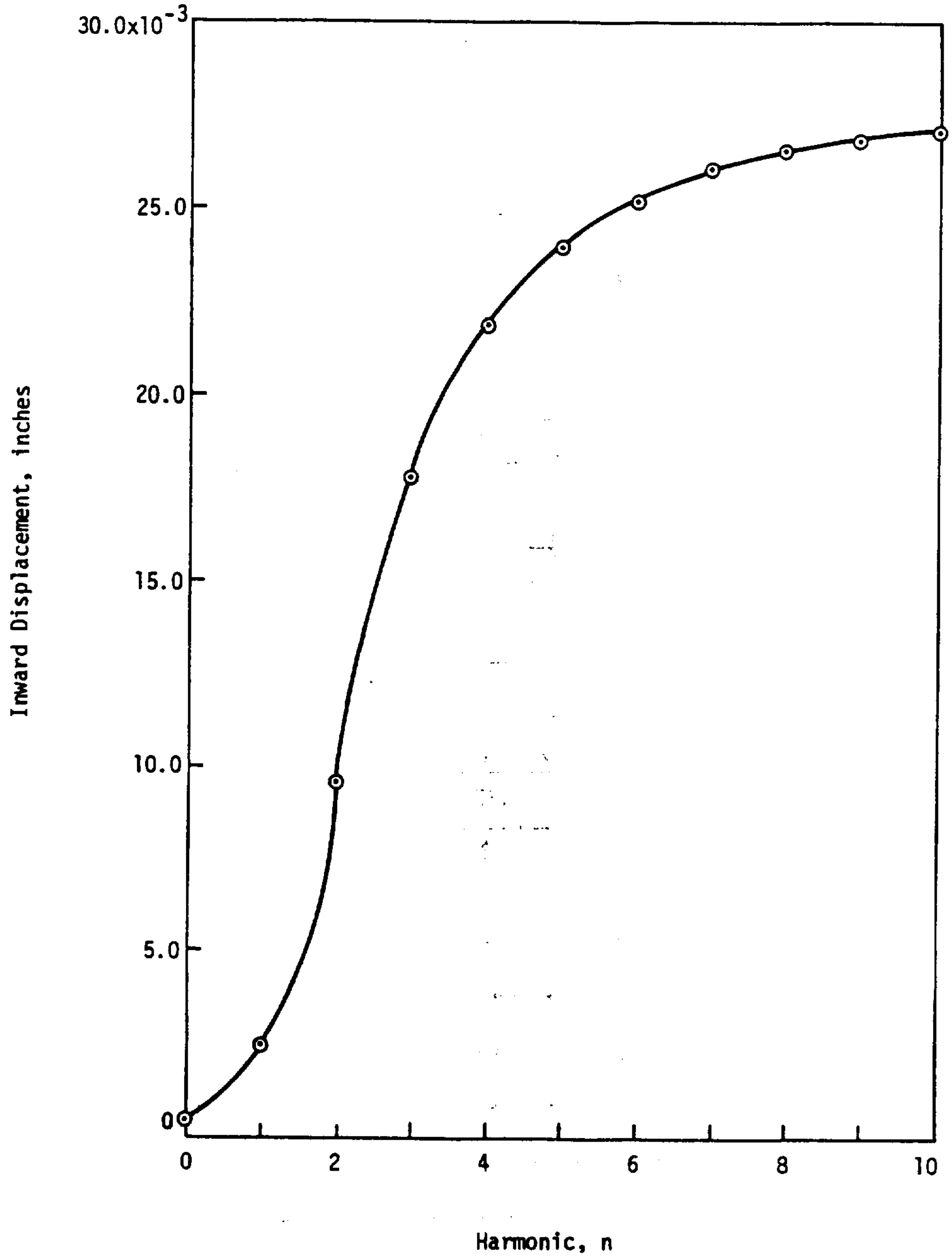
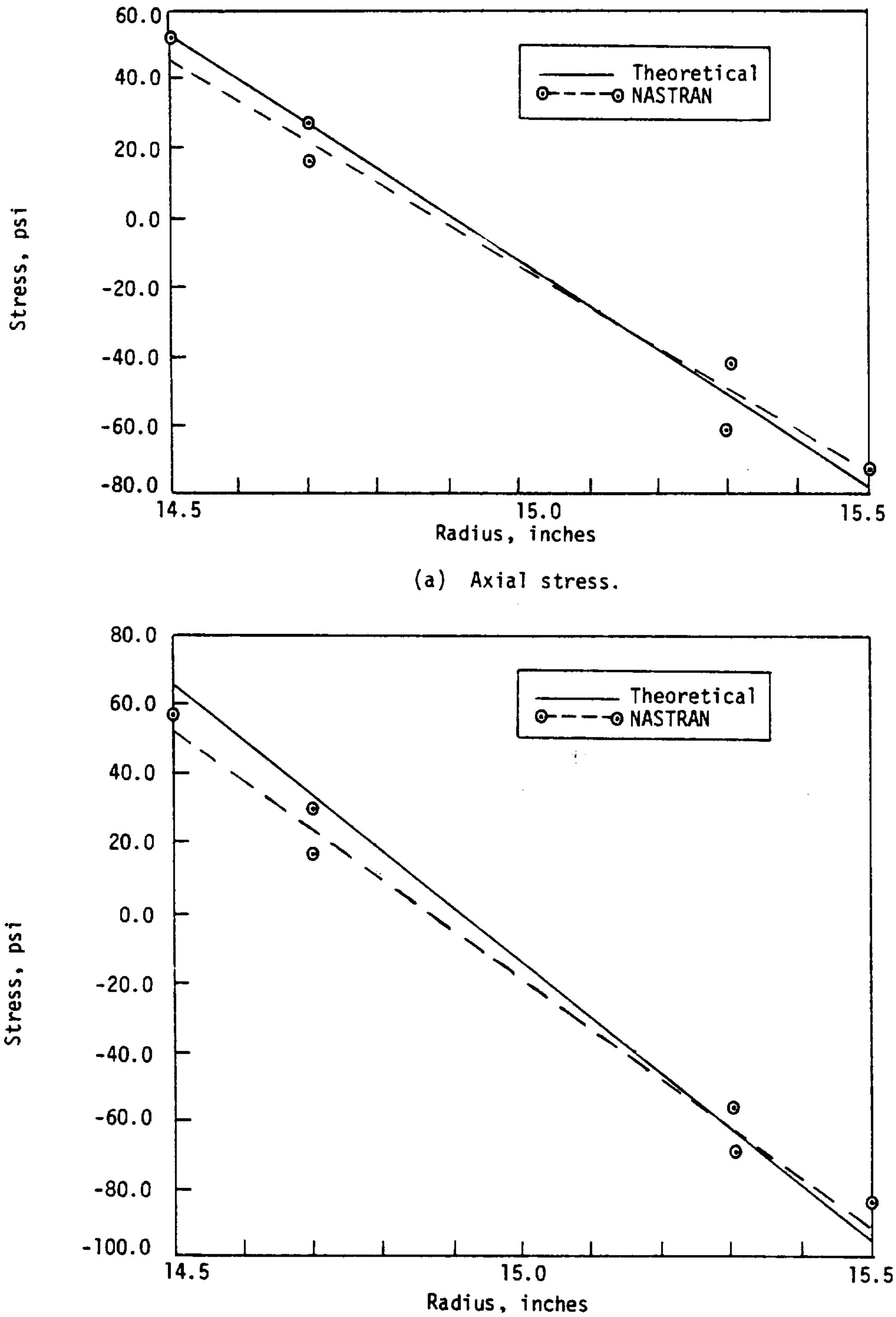


Figure 3. Radial deflection at center of load.



(a) Axial stress.

(b) Circumferential stress.

Figure 4. Stresses at center of load.

A. Description

A flat plate with a reinforced hole in the center is optimized for stresses due to a uniform end load. Restrictions on the minimum thickness are maintained. The plate is shown in Figure 1 and the finite element idealization is illustrated in Figure 2. This problem has been investigated by G. G. Pope (Reference 21).

Due to symmetry, only one quadrant is modeled. Due to the membrane load all rotations and normal displacements are constrained. The QDMEM and TRMEM elements are used for the plate and RØD elements for the reinforcement around the hole.

The problem demonstrates several features unique to fully stressed design capability in NASTRAN. These features are:

1. Elements with no limits on the range of the property change, i.e., the RØD has no PLIMIT data.
2. Elements with a lower limit on the property optimization card. All membrane elements are required to have a resultant thickness which must not be less than a minimum thickness. This minimum is determined from the thickness obtained when the plate without a hole is subjected to an end load at a prescribed stress limit.
3. Elements whose stress is not inspected but being in an area of nearly uniform stress have their properties changed due to another element's stress. Element 7 has no stress request but does have the same property identification number as element 17. This type of optimization can save computer time at the expense of a design that may not be truely optimized.
4. A property whose value depends on the maximum stress of two elements. Elements 5 and 15 have the same property card. This option may be necessary if insufficient core is allocated.
5. Temperature dependent stress limits for material 3.
6. Using one stress limit only. The membrane elements use the maximum principle shear only. This is 1/2 the major principle stress allowed. This stress limit was chosen to better model the octahedral limit in Reference 21.

   The rod elements use only the tension and compression stress appropriate to the given property, namely area.
7. An additional load case that was not included in the fully stressed design because a stress request was not made. The second subcase may be considered a displacement verification of this load case.

B. Input

1. Parameters

   ℓ = 30.0 in (total length)

   w = 20.0 in (total width)

   d = 10.0 in (hole diameter)

   t_o = 3.348 in (initial plate thickness)

   A_o = 1.674 in^2 (initial rod cross sectional area)

   E = $30. \times 10^6$ psi (modulus of elasticity)

   ν = 0.3 (Poisson's ratio)

   t_e = 1.0 in (lower limit for plate thickness corresponding to a 25.0×10^3 maximum principle stress)

2. Boundary conditions:

   on y = 0 plane, $u_y = 0$ (symmetry)

   on x = 0 plane, $u_x = 0$ (symmetry)

   all points $u_z = \theta_x = \theta_y = \theta_z = 0$ (permanent constraints)

3. Loads:

   First subcase: uniform load, $F_{10} = 25.0 \times 10^3$ lb/in

   Second subcase: at grid points 69 and 79, $F_{12} = -1000.0$ lb

   at grid point 78, $F_{12} = -2000.0$ lb

   (contact load on rim of hole - displacement check only)

C. Theory

The theoretical approach developed for the property optimization technique in NASTRAN is contained in the NASTRAN Theoretical Manual, Section 4.4. This technique is a fully stressed design approach. A mathematical programming technique is used in reference 21 from which the example problem was taken.

The two techniques might be expected to give similar results when the same model is used. However, reference 21 employs elements which allow varying properties and stresses while NASTRAN elements allow only constant properties and constant stresses. Somewhat different geometry is used in the NASTRAN model, i.e., the use of quadrilateral elements for illustration. Additional features of the NASTRAN model are discussed in items 3, 4 and 5 of Part A.

D. Results

The optimization process in this problem is terminated at 5 iterations. The initial weight to final weight ratio is 2.70 compared to Pope's results of 2.63. Tables 1 and 2 show the optimized nondimensional properties of the elements around the arch. Note that the results from reference 21 are averaged to provide an equivalent constant property element for comparison.

Table 1. Optimized Nondimensional Thickness Comparisons.

Element	Original t/t_e	Reference 21 Average t/t_e	NASTRAN t/t_e
37	3.348	1.24	1.00
38	3.348	1.00	1.04
39	3.348	1.00	1.00
46	3.348	2.10	1.14
47	3.348	1.34	2.00
57	3.348	3.32	1.34
59	3.348	3.19	4.40
67	3.348	4.58	5.47
68	3.348	3.26	1.00
69	3.348	4.52	5.49

Table 2. Optimized Nondimensional Area Comparisons.

Element	Original A/dt_e	Reference 21 Average A/dt_e	NASTRAN A/dt_e
101	.1674	.0249	.00716
102	.1674	.0238	0.0 effective
103	.1674	.0636	.05019
104	.1674	.1880	.1839
105	.1674	.3540	.3287

E. Driver Decks and Sample Bulk Data

```
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  2    UMF       1977     11610
  3    APP       DISPLACEMENT
  4    SØL       1,0
  5    TIME      9
  6    CEND

  7    TITLE = FULLY STRESSED DESIGN ØF A PLATE WITH A REINFØRCED HØLE
  8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-16-1
  9    LABEL = TEMPERATURE DEPENDENT MATERIALS.
 10        TEMPERATURE(MATERIALS) = 3000
 11        SPC = 11
 12        DISPLACEMENT = ALL
 13    SUBCASE 10
 14        LABEL = DESIGN CASE - UNIFØRM END LØAD
 15        SET 111 = 1 THRU 105 EXCEPT 7
 16        STRESS = 111
 17        LØAD = 10
 18    SUBCASE 12
 19        LABEL = CHECK CASE - CØNTACT LØAD AT NØZZLE.
 20        LØAD = 12
 21    PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-16-1
 22    ØUTPUT(PLØT)
 23    PLØTTER SC
 24        SET 1 = 1, 7, 38, 61, 69
 25        SET 2 = INCLUDE ELEMENTS QDMEM, TRMEM
 26             MAXIMUM DEFØRMATIØN 0.8
 27             AXES Z, X, Y
 28             VIEW  0.0, 0.0, 0.0
 29        FIND SCALE, ØRIGIN 12, SET 1
 30    PTITLE = ARCH MØDEL
 31    PLØT SET 2, ØRIGIN 12 LABEL
 32    PTITLE = DEFLECTIØN VECTØRS FØR BØTH LØADS AND EACH ITERATIØN
 33    PLØT STATIC DEFØRMATIØN SET 2, ØRIGIN 12, VECTØR RXY, SYMBØL 7
 34        FIND SCALE, ØRIGIN 12, SET 1, REGIØN 0.0, 0.0, 0.6, 1.0
 35    PTITLE = ARCH MØDEL REFLECTED ABØUT VERTICAL AXIS
 36    PLØT SET 2, ØRIGIN 12, SYMMETRY X
 37    BEGIN BULK
 38    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CQDMEM	1		11	13	3	1			
CRØD	101		48	49	102	102	49	59	
CTRMEM	11		13	11	21				
FØRCE	10	1		.3125E5	.0	1.0	.0		
GRDSET							3456		
GRID	1		-10.	15.					
MAT1	1	30.E06		.3	.283		70.0		+CØNST
+CØNST			12.5E3						
MATT1	2								+MATT1
+MATT1			222						
PARAM	GRDPNT	0							
PLIMIT	QDMEM	.2986858		1	THRU	61			FSD
PØPT	5	.04	.95	2	YES				FSD
PQDMEM	1	1	3.348						
PRØD	101	3	1.674						
PTRMEM	11	1	3.348						
SPC1	11	1	9	19	29	39	48		
TABLEM1	222								+TAB-M1
+TAB-M1	1.	12.5E3	10.	12.5E3	ENDT				
TEMPD	3000	80.							

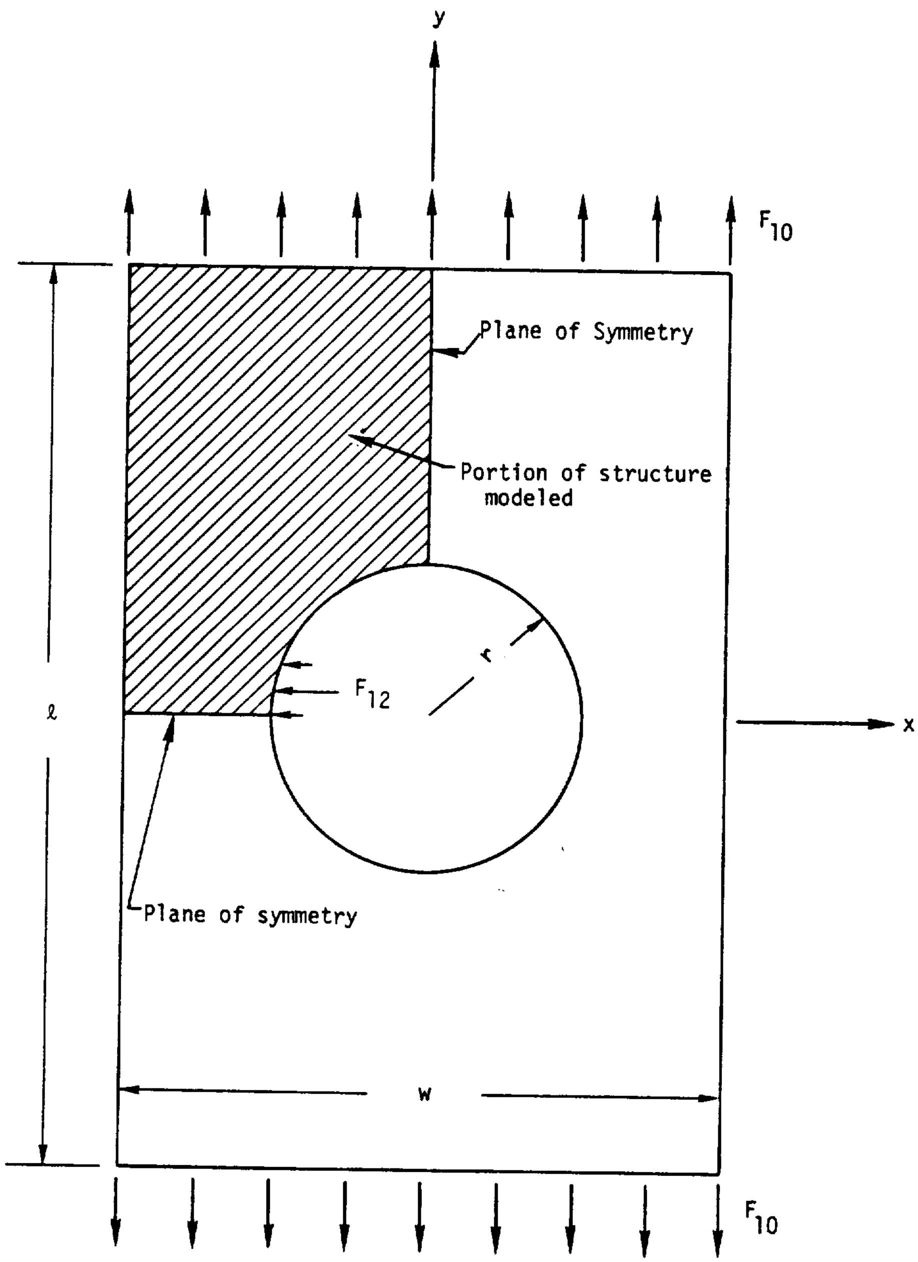

Figure 1. Plate with reinforced hole.

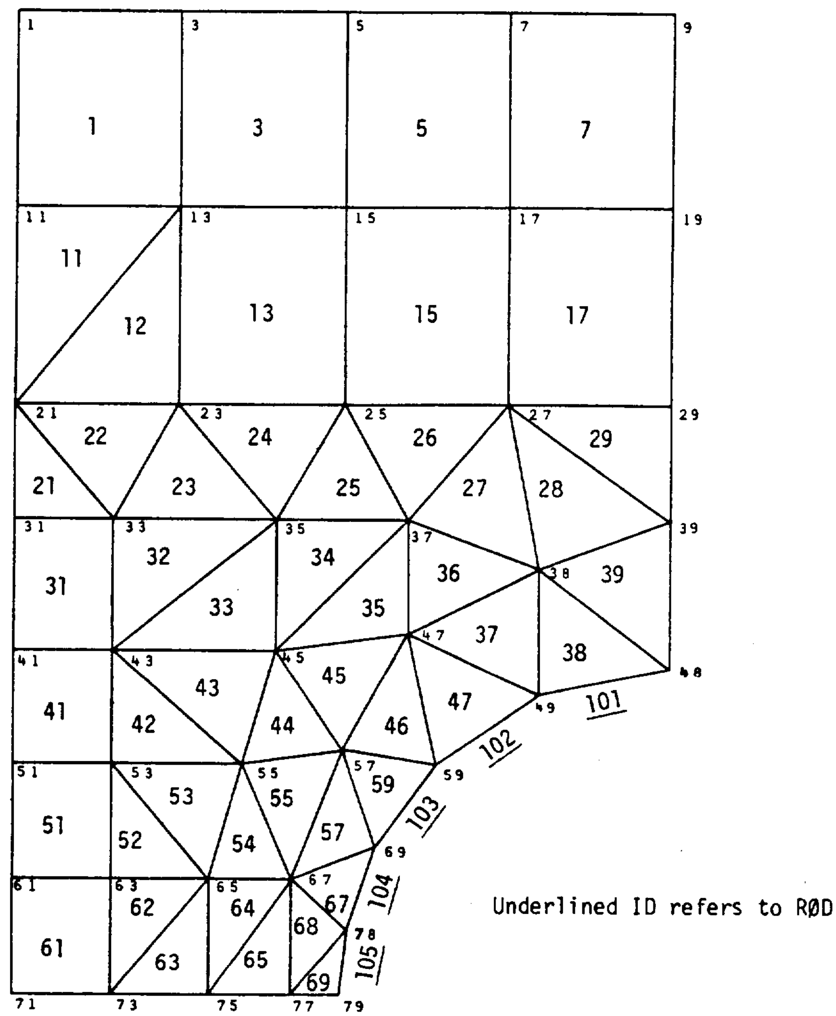

Figure 2. Finite element model.

RIGID FORMAT No. 1, Static Analysis

Rectangular Plate With Variable Moduli of Elasticity (1-17-1)

A. Description

This problem illustrates the use of the element stress precision check feature, NCHECK. A rectangular plate is modeled using CQUAD2 elements. The thickness is constant, but the modulus of elasticity is varied versus distance along the plate length. Concentrated forces and thermal loads are applied so as to produce uniform stress distribution in selected directions. The problem is designed so that stress calculations for certain elements will involve operations with small differences between large numbers to produce a loss of precision in the calculations.

B. Input

The model is shown in Figure 1. The relevant data are listed below.

1. Parameters:

$t = 1.0$ inch	(Plate thickness)
E = (see Figure 1)	(Modulus of elasticity)
$\nu = 0.0$	(Poisson's ratio)
$\alpha = 1.0 \times 10^{-6}$ in/in/°F	(Thermal expansion coefficient)
$T = 170°F$	(Applied temperature, uniform)
$T_0 = 70°F$	(Reference temperature)

2. Constraints:

Subcases 1, 2, and 3

$u_6 = 0$	at all Grid points
$u_2 = u_3 = 0$	at Grids 11 and 13
$u_1 = u_2 = u_3 = u_4 = u_5 = 0$	at Grid 12

Subcase 4

$u_6 = 0$	at all Grid points
$u_1 = u_2 = u_3 = u_4 = u_5 = 0$	at Grid points 11, 12, 13, 51, 52, and 53

3. Loads:

Subcase 1 $F_y = 100.$ at Grids 51 and 53

$F_y = 400.$ at Grid 52

Subcase 2 F_x = 1000. at Grid 52

Subcase 3 F_z = 100. at Grid 52

Subcase 4 T = 170.^{o}F at all Grids

4. Output Requests:

   DISP = ALL

   ELSTRESS = ALL

   NCHECK = 12

C. Results

A summary of stress precision in the number of significant digits is presented in Table 1. The quantities shown in the table are indicative of the general trends observed in all stress precision output for this problem. The trend shows that elements with higher moduli of elasticity provide less precise stresses relative to the other elements under the same loading.

D. Driver Decks and Sample Bulk Data

```
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 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1171,NASTRAN
  2   UMF 1977 11710
  3   APP     DISP
  4   SOL     1,0
  5   TIME     10
  6   CEND

  7   TITLE = RECTANGULAR PLATE WITH VARIABLE MØDULI ØF ELASTICITY
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-17-1
  9   LABEL = ELEMENT STRESS PRECISIØN CHECKS
 10   SPC = 10
 11   OUTPUT
 12   DISPLACEMENT = ALL
 13   ELSTRESS = ALL
 14      NCHECK = 12
 15   SUBCASE 1
 16   LABEL = LØAD IN LØNGITUDINAL DIRECTIØN
 17   LØAD = 1
 18   SUBCASE 2
 19   LABEL = LØAD IN TRANSVERSE DIRECTIØN
 20   LØAD = 2
 21   SUBCASE 3
 22   LABEL = LØAD NØRMAL TØ SURFACE
 23   LØAD = 3
 24   SUBCASE 4
 25   LABEL = THERMAL LØAD
 26   TEMP(LØAD) = 4
 27   SPC = 20
 28   BEGIN BULK
 29   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CQUAD2	11	10	11	12	22	21	.0		
FØRCE	1	51		100.0	.0	1.0	.0		
GRDSET							6		
GRID	11		.0	.0	.0				
MAT1	10	1.0E3		.0		1.0E-6	70.0		
PQUAD2	10	10	1.0	.0	20	20	1.0	.0	
SPC1	10	23	11	13					
TEMPD	4	170.0							

Table 1. Stress Precision Summary

Case (CDC)	Modulus of Elasticity	Subcase 1	Subcase 2	Subcase 3	Subcase 4
Significant Load or Stress		σ_y	τ_{xy}	M_y	σ_y
Elements 11, 12	10^3	14.5	>12	>12	>12
Elements 21, 22	10^5	12.1	11.4	11.9	>12
Elements 31, 32	10^7	10.1	9.2	9.7	10.6
Elements 41, 42	10^9	8.1	7.1	7.2	9.0

Case (IBM)	Modulus of Elasticity	Subcase 1	Subcase 2	Subcase 3	Subcase 4
Significant Load or Stress		σ_y	τ_{xy}	M_y	σ_y
Elements 11, 12	10^3	7.2	>12	>12	>12
Elements 21, 22	10^5	4.9	4.2	4.7	>12
Elements 31, 32	10^7	2.9	2.0	2.5	3.3
Elements 41, 42	10^9	1.0	0.5	1.7	2.0

Case (UNIVAC)	Modulus of Elasticity	Subcase 1	Subcase 2	Subcase 3	Subcase 4
Significant Load or Stress		σ_y	τ_{xy}	M_y	σ_y
Elements 11, 12	10^3	8.1	>12	>12	>12
Elements 21, 22	10^5	5.8	5.1	5.6	>12
Elements 31, 32	10^7	3.8	2.9	3.4	4.3
Elements 41, 42	10^9	1.0	0.8	0.7	2.7

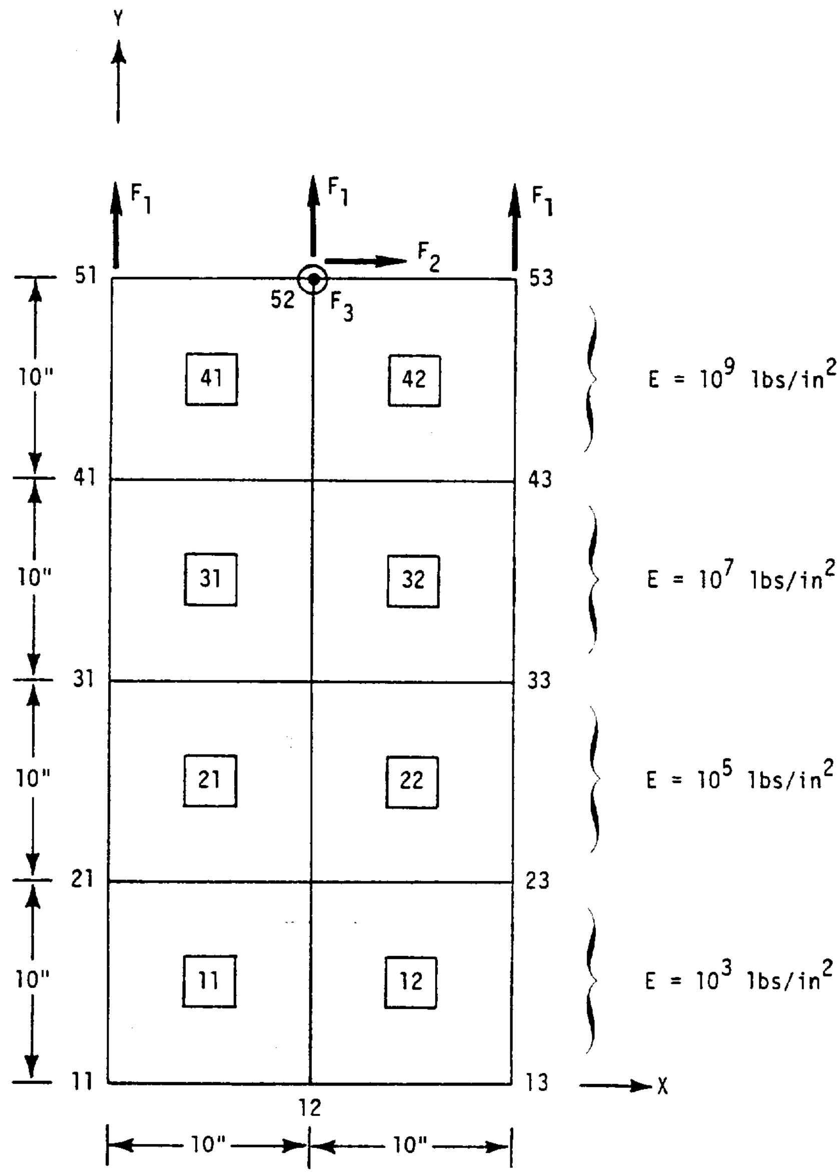

Figure 1. Finite element model.

Inertia Relief Analysis of a Circular Ring Under Concentrated and Centrifugal Loads (2-1-1)

A. Description

This problem illustrates the use of inertia relief analysis to solve a free-body problem. In inertia relief the structure is under constant acceleration due to the applied loads; the reactions to the applied load are due to the masses of the structure. Ficticious, nonredundant, support points must be provided to define a reference system attached to the body. The displacements of the body are measured relative to the supported coordinates.

The basic problem is illustrated in Figure 1. The structure consists of a spinning ring with a constant radial load applied to one point. The rotational velocity creates centrifugal loads and the point load causes inertia reactions. The actual dynamic motion of the whole structure is a cyclic motion of the center point coinciding with the rotation of the ring. The displacements measured by the inertia relief analysis, however, will be the static motion relative to the support point displacements.

The displacements are defined in a cylindrical coordinate system ($u_1 = u_r$, $u_2 = u_\phi$, $u_3 = u_z$). The elements used are BAR elements with a large cross-sectional area to minimize axial deformations. The BARs were offset a uniform radial distance from the grid points to demonstrate the offset option of the BAR element.

B. Input

1. Parameters:

   R = 10.0 (radius at end of BAR elements)

   R_1 = 11.0 (Radius at grid points)

   I = 10.0 (Bending inertia)

   ρ = 0.5 (Mass density)

   E = 1000. (Modulus of elasticity)

   A = 1000. (Cross-sectional area)

2. Loads:

   $P_{r,13}$ = 100

   f = 1.59 cps (Rotational velocity, ω = 1.0 radians per second)

3. Supports:

a) The $u_{r,1}$ direction is supported to restrict vertical translation.

b) The $u_{\phi,1}$ and $u_{\phi,13}$ directions are supported to restrict rotation and horizontal translation.

4. Grid Point Weight Generator Input:

Weight and moment of inertia are defined relative to point 19.

C. Theory

1. The Element Forces and Moments may be solved by the following analysis, as explained in Reference 7, Chapter 12.

a) Using symmetry the structure may be defined by the free-body diagram in figure 2. The static equilibrium equations at any angle are

$$A = A_o \cos\phi + \mu\phi \sin\phi \quad \text{(Axial Force)} \quad , \tag{1}$$

$$V = A_o \sin\phi + \mu\phi \cos\phi \quad \text{(Shear)} \quad , \tag{2}$$

and

$$M = M_o + r[\mu(1 - \cos\phi - \phi \sin\phi) + A_o(1 - \cos\phi)] \quad \text{(Bending Moment)} \tag{3}$$

b) Using energy and Castigliano's Theorem:

$$U = \frac{R}{2EI} \int_0^\pi M^2 \, d\phi \quad , \tag{4}$$

$$\frac{\delta U}{\delta M_o} = 0 \quad , \tag{5}$$

and

$$\frac{\delta U}{\delta A_o} = 0 \quad . \tag{6}$$

These are the deflections at the bottom which are fixed. The resulting two equations are used in step c.

c) Solving the equations in (b) gives the redundant forces:

$$A_o = -\frac{1}{2}\mu = -\frac{F}{4\pi} \quad , \tag{7}$$

and

$$M_o = \frac{R\mu}{2} = \frac{FR}{4} \quad . \tag{8}$$

d) Adding a dummy load and solving the problem with the above boundary conditions gives the displacement due to the point load:

$$\delta_f = \frac{FR^3}{\pi EI}\left(\frac{\pi^2}{8} - 1\right) \quad . \tag{9}$$

e) The axial stress and radial displacement due to the centrifugal load is

$$\sigma_\omega = \rho R^2 \omega^2 = 5.0 \times 10^2 \quad , \tag{10}$$

and

$$\delta_\omega = \frac{2\rho R^3 \omega^2}{E} = 1.0 \quad . \tag{11}$$

D. Results

1. The total result of summing the two loads is

	THEORY	NASTRAN
δ = Displacement $u_{r,13}$	1.75	1.734
M_o = Moment BAR #1, end A	-79.5	-80.48
M_1 = Moment BAR #12, end B	-238.5	-236.0

2. The structural mass characteristics as calculated by the grid point weight generator are

THEORETICAL	NASTRAN
X_{CG} = 11.0 from point 19	11.0
Mass = $\pi \times 10^4$ = 3.14159×10^4	3.1326×10^4
$I_{xx} = I_{yy} = \frac{\pi}{2} \times 10^6 = 1.5708 \times 10^6$	1.5663×10^6
$I_{zz} = \pi \times 10^6 = 3.14159 \times 10^6$	3.1326×10^6

(Inertias are about center of gravity)

NASTRAN gives slightly different answers due to the polygonal shape of the finite element model.

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E. Driver Decks and Sample Bulk Data

```
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  2   UMF       1977     20110
  3   TIME      5
  4   APP       DISPLACEMENT
  5   SØL       2,1
  6   CEND

  7   TITLE = INERTIA RELIEF ANALYSIS ØF A CIRCULAR RING
  8   LABEL = CØNCENTRATED AND CENTRIFUGAL LØADS
  9   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-1-1
 10   LØAD = 3
 11       ØUTPUT
 12            DISP = ALL
 13   ØLØAD = ALL
 14   SPCFØRCE = ALL
 15   STRESSES = ALL
 16   SET 1 = 1,6,7,12,13,18,19,24
 17   ELFØRCE = 1
 18   BEGIN BULK
 19   ENDDATA
```
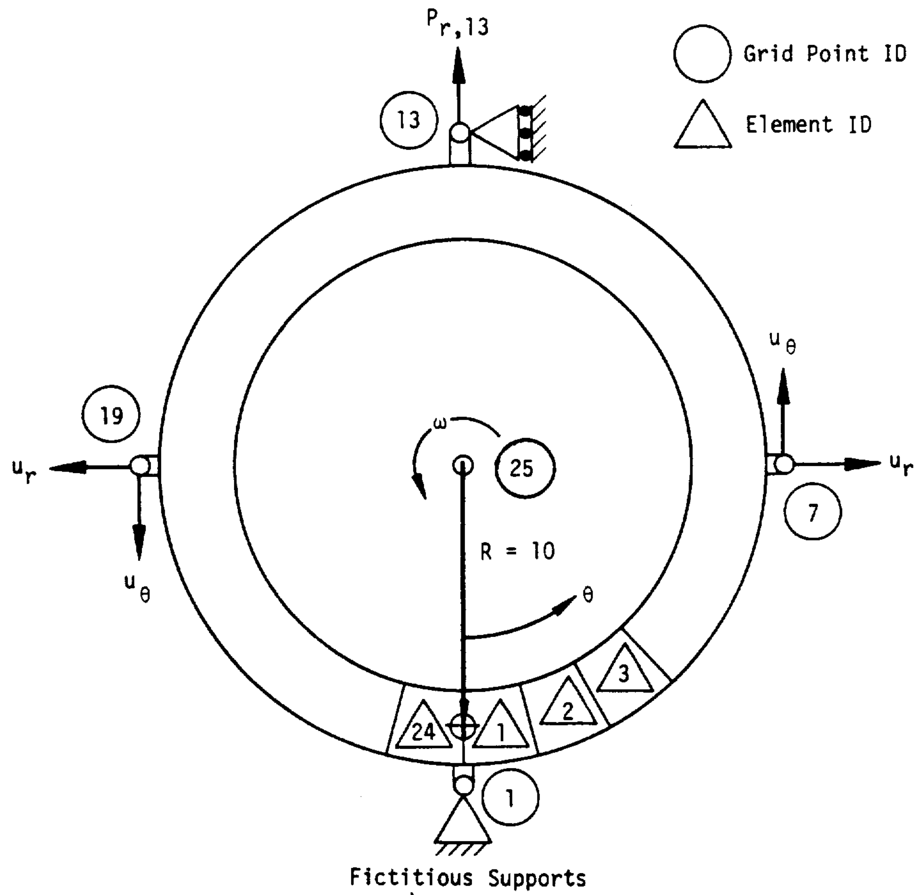

1	2	3	4	5	6	7	8	9	10
BARØR		5			1.0	.0	.0	1	
CBAR	1		1	2				1	+B1
+B1			-1.0	.0	.0	-1.0	.0	.0	
CØRD2C	2	0	.0	10.0	.0	.0	10.0	1.0	CCØRD
+CØRD	.0	9.0	.0						
FØRCE	1	13	2	100.0	1.0	.0	.0		
GRDSET		2				2	345		
GRID	1		11.0	.0	.0				
LØAD	3	1.0	1.0	1	1.0	2			
MAT1	1	1000.0	400.0		.5				+MAT1
+MAT1	100.	200.	300.						
PARAM	GRDPNT	19							
PBAR	5	1	1000.0	10.	10.				+P5
+P5	1.0	1.0	-1.0	-1.0					
RFØRCE	2	25	2	.159155	.0	.0	1.0		
SUPØRT	1	2	1	1	13	2			



Note: Grid points are offset from center line of ring.

Figure 1. Ring under concentrated and centrifugal loads.

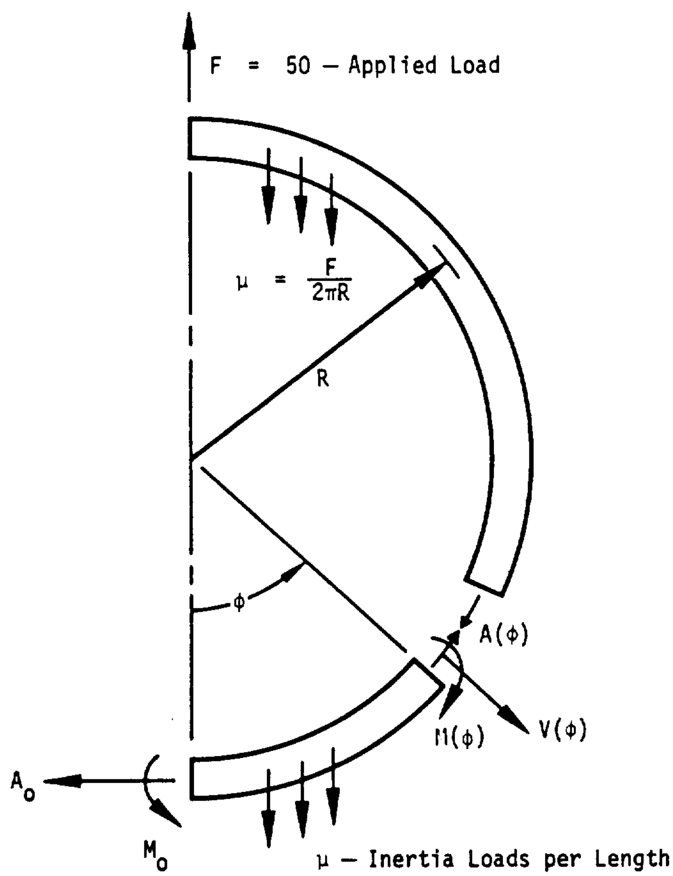


Figure 2. Free body diagram of loads in bending ring.

RIGID FORMAT NO. 2, Inertia Relief Analysis

Windmill Panel Sections for Automated Multi-stage Substructuring, Run 1, (2-2-1)

Windmill Panel Sections for Automated Multi-stage Substructuring, Run 2, (2-2-2)

Windmill Panel Sections for Automated Multi-stage Substructuring, Run 3, (2-2-3)

Windmill Panel Sections for Automated Multi-stage Substructuring, Run 4, (2-2-4)

Windmill Panel Sections for Automated Multi-stage Substructuring, Run 5, (2-2-5)

Windmill Panel Sections for Automated Multi-stage Substructuring, Run 6, (2-2-6)

Windmill Panel Sections for Automated Multi-stage Substructuring, Run 7, (2-2-7)

A. Description

This problem illustrates the fully automated multi-stage substructuring capability of NASTRAN. The single structure model for the Windmill panel problem is shown in Figure 1. Indicated in this figure are the three basic substructures used for the analysis. As can be seen, the entire structure can be composed of only these three components, thus taking advantage of symmetry. The detailed idealizations for the three basic substructures are shown in Figures 2 and 3. These figures show the three separate basic coordinate systems and the local coordinate systems for each of the three basic substructures created.

Of the total of seven runs involved, three Phase 1 runs are made, one for each basic substructure, using Rigid Format 2 in order to generate mass matrices. The combination and reduction to the final model is accomplished in seven distinct Phase 2 steps, plus eight equivalence operations. The sequence of combination steps taken is illustrated in Figures 4a and 4b. Figure 5 details the points retained in the "analysis set" following the Phase 2 Guyan reduction. A static solution, Rigid Format 1, is obtained for each of the three load cases specified. Run 4 produces actual plot output. Runs 5 and 6 demonstrate the Phase 3 data recovery for two of the basic substructures.

A seventh run is made to extract normal modes using Rigid Format 3 for the same reduced structure shown in Figure 5.

B. Input

1. Parameters:

r_o = 50.0 in (outer radius)

r_i = 10.0 in (inner radius)

t = 0.1 in (plate thickness)

E = 10×10^6 psi (modulus of elasticity)

ν = 0.25 (Poisson's ratio)

2. Boundary Conditions:

All points $u_z = \theta_x = \theta_y = \theta_z = 0$ (permanent constraint)

$u_x = 0$ at HUB grid points 13, 19, 37, 43

$u_y = 0$ at HUB grid points 1, 7, 25, 31

3. Loads:

First Subcase: centrifugal force due to unit angular velocity

Second Subcase: unsymmetric load - right panel in tension, bottom panel in compression, F = 100 uniformly distributed over each loaded edge

Third Subcase: F = 1.0 applied at HUB grid point 4 inward radially

4. Substructuring Parameters:

SØF(1) = SØF0,950 $ CDC

SØF(1) = FT18,950 $ IBM

SØF(1) = INPT,950 $ UNIVAC

PASSWØRD = DEMØ

ØPTIØNS = K, M, P

C. Theory

This problem is designed to illustrate the use of automated multi-stage substructuring. No closed form solution is available. Results are compared with non-substructured NASTRAN solutions.

D. Results

The solutions of the final reduced structure using both Rigid Format 1 and Rigid Format 3 are in excellent agreement with the non-substructured solutions. Displacements at selected points and eigenvalues are compared in Table 1. The values presented were obtained from executions on IBM equipment. Values obtained from CDC and UNIVAC are of the same order of magnitude with slight differences attributable to round-off of very small numbers.

E. <u>Driver Decks and Sample Bulk Data</u>

```
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 0b    NASTRAN FILES=(INPT,UMF) $ UNIVAC
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 2     UMF       1977    20210
 3     APP       DISPLACEMENT,SUBS
 4     SØL       2,0
 5     TIME      5
 6     DIAG      14,23
 7     CEND

 8     SUBSTRUCTURE PHASE1
 9     PASSWØRD = DEMØ
10a    SØF(1) = SØF0,950,NEW $ CDC
10b    SØF(1) = FT18,950,NEW $ IBM
10c    SØF(1) = INPT,950,NEW $ UNIVAC
11     RUN = STEP
12     ØPTION = K,M,P
13     NAME = HUB
14     SAVEPLØT = 1
15     SØFP TØC
16      ENDSUBS

17     TITLE = WINDMILL PANEL SECTIØNS FØR AUTØMATED MULTI-STAGE SUBSTRUCTURING
18     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-2-1
19     LABEL = SUBSTRUCTURE 1, RUN 1, PHASE 1
20     SPC = 30
21     SUBCASE 1
22     LABEL = RØTATIØNAL FØRCES DUE TØ UNIT ØMEGA ABØUT CENTER ØF STRUCTURE
23     LØAD = 1
24     SUBCASE 2
25     LABEL = CHECK ØN RELEASE FEATURE AT GRID PØINT 5
26     LØAD 3
27     ØUTPUT(PLØT)
28     SET 1 = ALL
29     PLØT
30     BEGIN BULK
31     ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØRD2C	1	0	.0	.0	.0	.0	.0	1.0	+CØR
+CØR	1.0	.0	.0						
CQDMEM	1	10	1	4	5	2			
FØRCE1	3	4	1.0	5	4				
GRDSET							3456		
GRID	1		-5.0	10.0					
MAT1	50	1.0+7		.25	2.5E-4	1.0E-6	70.0		
PQDMEM	10	50	.1						
RFØRCE	1	0	0	.1591579	.0	.0	1.0		
SPC1	30	1	13	19	37	43			

Card
No.

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0b  NASTRAN FILES=(INPT,UMF) $ UNIVAC
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2   UMF       1977    20220
3   APP       DISPLACEMENT,SUBS
4   SØL       2,0
5   TIME      5
6   DIAG      14,23
7   CEND

8   SUBSTRUCTURE PHASE1
9   PASSWØRD = DEMØ
10a SØF(1) = SØFO 950 $ CDC
10b SØF(1) = FT18,950 $ IBM
10c SØF(1) = INPT,950 $ UNIVAC
11  RUN = STEP
12  ØPTION = K,M,P
13  NAME = RØØT1
14  SAVEPLØT = 1
15  SØFP TØC
16  ENDSUBS

17  TITLE = WINDMILL PANEL SECTIØNS FØR AUTØMATED MULTI-STAGE SUBSTRUCTURING
18  SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-2-2
19  LABEL = SUBSTRUCTURE 2, RUN 2, PHASE 1
20  LØAD = 1
21  ØUTPUT(PLØT)
22  SET 1 = ALL
23  PLØT
24  BEGIN BULK
25  ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CQDMEM	1	10	3	4	2	1			
GRDSET							3456		
GRID	1		.0	27.5					
MAT1	50	1.0+7		.25	2.5E-4	1.0E-6	70.0		
PQDMEM	10	50	.1						
RFØRCE	1			.1591579	.0	.0	1.0		

```
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 0b   NASTRAN FILES=(INPT,UMF) $ UNIVAC
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 2    UMF        1977     20230
 3    APP        DISPLACEMENT,SUBS
 4    SØL        2,0
 5    TIME       5
 6    DIAG       14,23
 7    CEND

 8    SUBSTRUCTURE PHASE1
 9    PASSWØRD = DEMØ
10a   SØF(1) = SØF0,950 $ CDC
10b   SØF(1) = FT18,950 $ IBM
10c   SØF(1) = INPT,950 $ UNIVAC
11    RUN = STEP
12    ØPTIØN = K,M,P
13    NAME = VANE1
14    SAVEPLØT = 1
15    SØFP TØC
16    ENDSUBS

17    TITLE = WINDMILL PANEL SECTIØNS FØR AUTØMATED MULTI-STAGE SUBSTRUCTURING
18    LABEL = SUBSTRUCTURE 3, RUN 3, PHASE 1
19    SUBCASE 1
20    LABEL = RØTATIØAL FØRCES ABØUT CENTER ØF OVERALL STRUCTURE
21    LØAD = 1
22    SUBCASE 2
23    LABEL = EXTENSION ØF PANEL
24    LØAD = 2
25    ØUTPUT(PLØT)
26    SET 1 = ALL
27    PLØT
28    BEGIN BULK
29    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØRD2R	1		5.0	22.5	.0	5.0	22.5	1.0	+A
+A	.0	22.5	.0						
CQDMEM	1	10	3	4	2	1			
FØRCE1	2	1	25.0	4	2				
GRDSET						1	3456		
GRID	1		.0	22.5					
MAT1	50	1.0+7		.25	2.5E-4	1.0E-6	70.0		
PQDMEM	10	50	.1						
RFØRCE	1	9		.1591579	.0	.0	1.0		

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Card
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 0b   NASTRAN FILES=(UMF,INPT,PLT2) $ UNIVAC
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 2    UMF       1977    20240
 3    APP       DISPLACEMENT,SUBS
 4    SØL       1,0
 5    TIME      10
 6    DIAG      14,23
 7    CEND

 8    SUBSTRUCTURE PHASE2
 9    PASSWØRD = DEMØ
10a   SØF(1) = SØF0,950 $ CDC
10b   SØF(1) = FT18,950 $ IBM
10c   SØF(1) = INPT,950 $ UNIVAC
11    ØPTIØNS = K,M,P
12    PLØT VANE1
13    PLØT RØØT1
14    PLØT HUB
15    $
16    $   STEP I.   CØMBINE VANETØP
17    $
18        SØFPRINT TØC
19        EQUIV VANE1,VANE2
20             PREFIX = X
21        CØMBINE VANE1,VANE2
22             NAME = VANETØP
23             TØLERANCE = 0.02
24             ØUTPUT = 1,2,7,11,12,13,14,15,16,17
25             CØMPØNENT = VANE1
26                     TRANS = 100
27             CØMPØNENT = VANE2
28                     TRANS = 100
29                     SYMT = X
30        PLØT VANETØP
31        SØFPRINT TØC
32    $
33    $   STEP II.   CØMBINE RØØTTØP
34    $
35        EQUIV RØØT1,RØØT2
36             PREFIX = X
37        CØMBINE RØØT1,RØØT2
38             NAME = RØØTTØP
39             TØLERANCE = 0.02
40             ØUTPUT = 1,2,7,11,12,13,14,15,16,17
41             CØMPØNENT = RØØT2
42                     SYMT = X
43        PLØT RØØTTØP
44        SØFPRINT TØC
45    $
46    $   STEP III.   SEVEN STRUCTURE CØMBINE
47    $
48        EQUIV VANETØP,VANELFT
49             PREFIX = L
50        EQUIV VANETØP,VANERGT
51             PREFIX = R
52        EQUIV VANETØP,VANEBØT
```

```
     PREFIX = B
EQUIV RØØTTØP,RØØTLFT
     PREFIX = L
EQUIV RØØTTØP,RØØTRGT
     PREFIX = R
EQUIV RØØTTØP,RØØTBØT
     PREFIX = B
SØFPRINT TØC
$
CØMBINE VANETØP,RØØTTØP,VANELFT,RØØTLFT,VANEBØT,RØØTBØT,RØØTRGT
     NAME = RING
     TØLERANCE = 0.02
     ØUTPUT = 1,2,7,11,12,13,14,15,16,17
     CØMPØNENT = VANELFT
          TRANS = 400
     CØMPØNENT = RØØTLFT
          TRANS = 400
     CØMPØNENT = VANEBØT
          SYMT = Y
     CØMPØNENT = RØØTBØT
          SYMT = Y
     CØMPØNENT = RØØTRGT
          TRANS = 300
$
$    STEP IV.  CØMBINATIØN ØF BLADES
$
     CØMBINE RING,VANERGT
     NAME = BLADES
     TØLERANCE = 0.02
     ØUTPUT  =  1,2,7,11,12,13,14,15,16,17
     CØMPØNENT = VANERGT
          TRANS = 500
$
$    STEP V.  FINAL CØMBINE ØF WINDMILL WITH RELES ØPTIØN
$
     CØMBINE HUB,BLADES
     NAME = WINDMIL
     TØLERANCE = 0.02
     ØUTPUT = 1,2,9,11,12,13,14,15,16,17
     CØNNECT = 1000
SØFPRINT TØC
PLØT WINDMIL
$
$    STEP VI.   REDUCTION TØ BØUNDARY PØINTS
$
     REDUCE WINDMIL
     NAME = SMALLMIL
     BØUNDARY = 2000
RSAVE
ØUTPUT = 1,2,3,4,5,6,7,8,9
     SØFPRINT TØC
SØLVE SMALLMIL
RECØVER SMALLMIL
     PRINT WINDMIL
     SAVE HUB
     SAVE RVANE1
ENDSUBS

TITLE = WINDMILL PANEL SECTIØNS FØR AUTØMATED MULTI-STAGE SUBSTRUCTURING
SUBTITLE - NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-2-4
```

```
Card
 No.
112   LABEL = CØMBINE, REDUCE, SØLVE, AND RECØVER, RUN 4, PHASE 2
113   DISP = ALL
114   ØLØAD = ALL
115   MPC = 20
116   SUBCASE 1
117   LABEL = RØTATIØNAL FØRCES DUE TØ UNIT ØMEGA ABØUT CENTER ØF STRUCTURE
118   LØAD = 1
119   SUBCASE 2
120   LABEL = EXTENSIØN ØF RIGHT PANEL AND CØMPRESSIØN ØF BØTTØM PANEL
121   LØAD = 2
122   SUBCASE 3
123   LABEL = CHECK ØN RELEASE FEATURE AT GRID PØINT 5
124   LØAD = 3
125   PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-2-4
126   ØUTPUT(PLØT)
127   PLØTTER SC
128       SET 1 = ALL
129       AXES Z, X, Y
130       VIEW 0.0, 0.0, 0.0
131       FIND SCALE, ØRIGIN 1, SET 1, REGIØN 0.1, 0.1, 0.9, 0.9
132   PTITLE = SUBSTRUCTURES VANE1/RØØT1/HUB/VANETØP/RØØTTØP PLUS MILL
133       PLØT SET 1, ØRIGIN 1, LABEL BØTH
134   BEGIN BULK
135   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
BDYC	2000	VANE1	200	VANE2	200	LVANE1	200		+BC1
+BC1		LVANE2	200	BVANE1	200	BVANE2	200		+BC2
BDYS1	200	12	1	2	4	6	8		
GTRAN	100	VANE1	7	0					
LØADC	1	1.0	VANE1	1	1.0	VANE2	1	1.0	+LC1A
+LC1A			ROOT1	1	1.0	ROOT2	1	1.0	+LC1B
MPCS	20	HUB	108	1	-1.0				+MPC1
+MPC1		ROOT1	6	2	.94868336		1	.3162278	
RELES	1000	HUB	5	2	17	1	29	2	+REL
+REL	41	1	108	12					
TRANS	100		0.0	27.5	0.0	0.0	27.5	1.0	+A
+A	5.0	27.5	0.0						

```
Card
 No.

 0a   NASTRAN FILES=UMF $ CDC AND IBM
 0b   NASTRAN FILES=(INPT,UMF) $ UNIVAC
 1    ID        DEM2025,NASTRAN
 2    UMF       1977    20250
 3    APP       DISP,SUBS
 4    SØL       1,0
 5    TIME      5
 6    DIAG      14,23
 7    CEND

 8    SUBSTRUCTURE PHASE3
 9        PASSWØRD = DEMO
10a       SØF(1) = SØF0,950 $ CDC
10b       SØF(1) = FT18,950 $ IBM
10c       SØF(1) = INPT,950 $ UNIVAC
11    RECØVER RVANE1
12    ENDSUBS
13    TITLE = WINDMILL PANEL SECTIØNS FØR AUTØMATED MULTI-STAGE SUBSTRUCTURING
14    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-2-5
15    LABEL = RECØVER RVANE1, RUN 5, PHASE 3
16        DISP = ALL
17        STRESS = ALL
18    SUBCASE 1
19    LABEL = RØTATIØNAL FØRCES ABØUT CENTER ØF ØVERALL STRUCTURE
20    SUBCASE 2
21    LABEL = EXTENSIØN ØF RIGHT PANEL AND CØMPRESSIØN ØF BØTTØM PANEL
22    SUBCASE 3
23    LABEL = CHECK ØN RELEASE FEATURE AT GRID PØINT 5
24    BEGIN BULK
25    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØRD2R	1		5.0	22.5	.0	5.0	22.5	1.0	+A
+A	.0	22.5	.0						
CQDMEM	1	10	3	4	2	1			
FØRCE1	2	1	25.0	4	2				
GRDSET						1	3456		
GRID	1		.0	22.5					
MAT1	50	1.0+7		.25	2.5E-4	1.0E-6	70.0		
PQDMEM	10	50	.1						
RFØRCE	1	9		.1591579	.0	.0	1.0		

Card
No.

```
0a   NASTRAN FILES=UMF $ CDC AND IBM
0b   NASTRAN FILES=(INPT,UMF) $ UNIVAC
1    ID        DEM2026,NASTRAN
2    UMF       1977     20260
3    APP       DISPLACEMENT,SUBS
4    SØL       1,0
5    TIME      5
6    DIAG      14,23
7    CEND

8    SUBSTRUCTURE PHASE3
9        PASSWØRD = DEMØ
10a      SØF(1) = SØF0,950 $ CDC
10b      SØF(1) = FT18,950 $ IBM
10c      SØF(1) = INPT,950 $ UNIVAC
11       BRECØVER HUB
12   ENDSUBS

13   TITLE = WINDMILL PANEL SECTIØNS FØR AUTØMATED MULTI-STAGE SUBSTRUCTURING
14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-2-6
15   LABEL = RECØVER HUB, RUN 6, PHASE 3
16       DISP = ALL
17       STRESS = ALL
18   SPC = 30
19   SUBCASE 1
20   LABEL = RØTATIØNAL FØRCES DUE TØ UNIT ØMEGA ABØUT CENTER ØF STRUCTURE
21   SUBCASE 2
22   LABEL = EXTENSIØN ØF RIGHT PANEL AND CØMPRESSIØN ØF BØTTØM PANEL
23   SUBCASE 3
24   LABEL = CHECK ØN RELEASE FEATURE AT GRID PØINT 5
25   BEGIN BULK
26   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CØRD2C	1	0	.0	.0	.0	.0	.0	1.0	+CØR
+CØR	1.0	.0	.0						
CQDMEM	1	10	1	4	5	2			
FØRCE1	3	4	1.0	5	4				
GRDSET							3456		
GRID	1		-5.0	10.0					
MAT1	50	1.0+7		.25	2.5E-4	1.0E-6	70.0		
PQDMEM	10	50	.1						
RFØRCE	1	0	0	.1591579	.0	.0	1.0		
SPC1	30	1	13	19	37	43			

```
Card
 No.

 0a   NASTRAN FILES=UMF $ CDC AND IBM
 0b   NASTRAN FILES=(INPT,UMF) $ UNIVAC
 1    ID         DEM2027,NASTRAN
 2    UMF        1977     20270
 3    APP        DISP, SUBS
 4    SØL        3,0
 5    TIME       5
 6    DIAG       14,23
 7    CEND

 8    SUBSTRUCTURE PHASE2
 9        PASSWØRD = DEMØ
10a       SØF(1) = SØF0,950 $ CDC
10b       SØF(1) = FT18,950 $ IBM
10c       SØF(1) = INPT,950 $ UNIVAC
11        SØFPRINT TØC
12        EQUIV SMALLMIL,SMILLDYN
13             PREFIX = D
14        SØFPRINT TØC
15        SØLVE SMILLDYN
16        RECØVER SMILLDYN
17        PRINT DWINDMIL
18    ENDSUBS

19    TITLE = WINDMILL PANEL SECTIØNS FØR ATUØMATED MILI-STAGE SUBSTRUCTURING
20    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 2-2-7
21    LABEL = NØRMAL MØDES FØR SMALLMIL, RUN 7, PHASE 2
22    METHØD = 10
23    MPC = 21
24    VECTØR = ALL
25    BEGIN BULK
26    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
EIGR	10	INV	.0	.1	1	1			PEIG
+EIG	MAX								
MPCS	21	DHUB	108	1	-1.0				+MPC1
+MPC1		DROOT1	6	2	.9486833	6	1	.3162278	

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Table 1. Comparison of Displacements at Selected Points for Windmill Panel Problem

Name/Point/Comp	Subcase 1		Subcase 2		Eigenvector #1	
	Single Step	Substructure	Single Step	Substructure	Single Step	Substructure
VANE1/1/X	$-5.6x10^{-14}$	$-5.2x10^{-14}$	$-2.19155x10^{-5}$	$-2.19155x10^{-5}$	1.000000	-.999752
VANE1/1/Y	$-6.88493x10^{-7}$	$-6.88488x10^{-7}$	$8.6081x10^{-1}$	$8.6081x10^{-1}$	$-8.612x10^{-9}$	$3.297x10^{-7}$
RVANE1/1/X	$4.4x10^{-14}$	$2.1x10^{-13}$	$2.19155x10^{-5}$	$2.19155x10^{-5}$	1.000000	-.999748
RVANE1/1/Y	$-6.88493x10^{-7}$	$-6.88488x10^{-7}$	$3.85998x10^{-4}$	$-3.85997x10^{-4}$	$1.264x10^{-9}$	$-1.688x10^{-7}$
HUB/5/X	$-3.5x10^{-14}$	$-4.8x10^{-14}$	$1.04757z10^{-5}$	$1.04757x10^{-5}$	$-1.46899x10^{-1}$	$1.46636x10^{-1}$
HUB/5/Y	$6.70493x10^{-8}$	$6.70488x10^{-8}$	$-6.43969x10^{-7}$	$-6.4397x10^{-7}$	$-3.140x10^{-9}$	$-7.8304x10^{-6}$
Frequency, cps	-	-	-	-	288.3	288.3

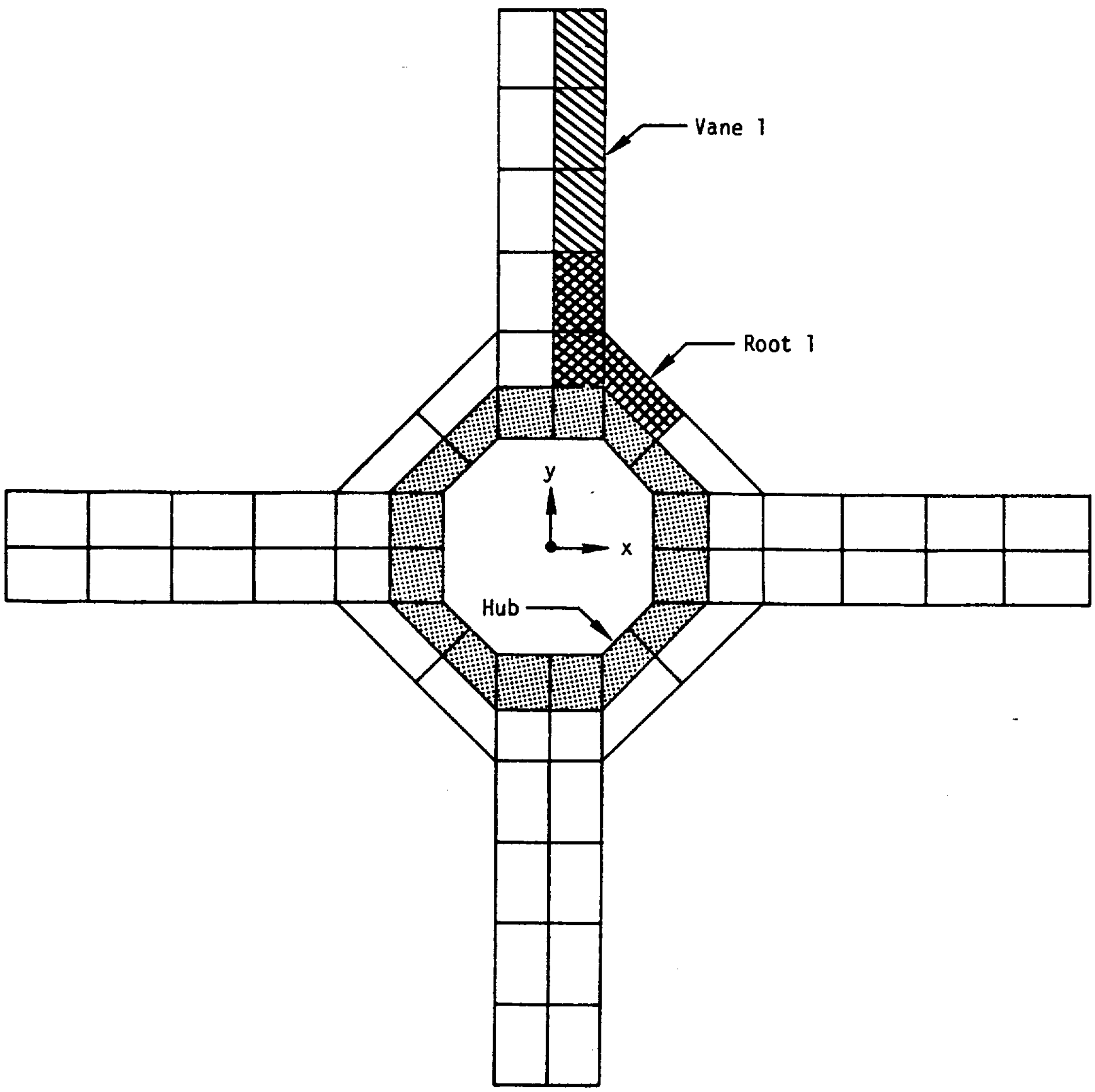


Figure 1. Windmill model, basic substructures.

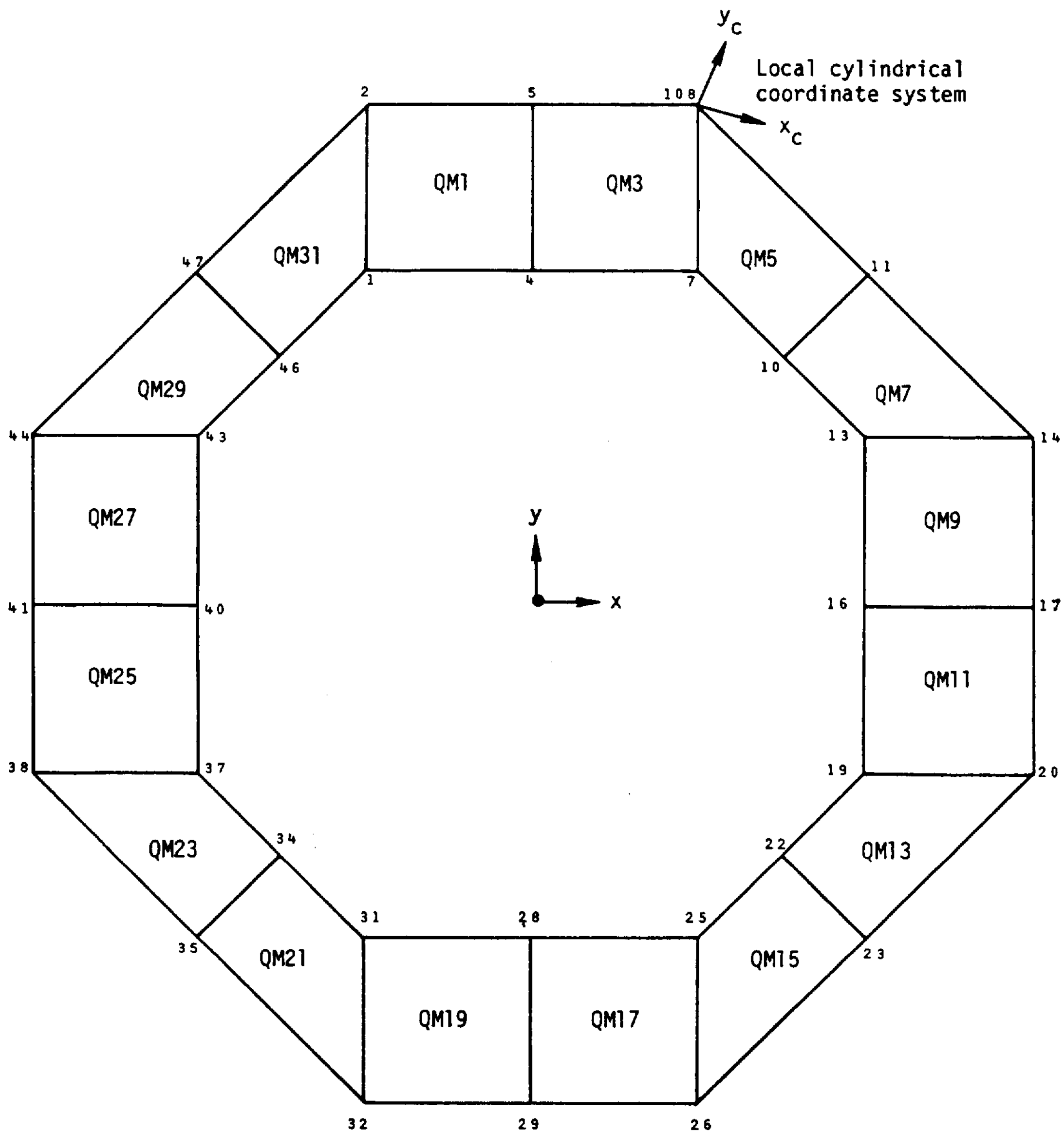


Figure 2. Hub substructure.

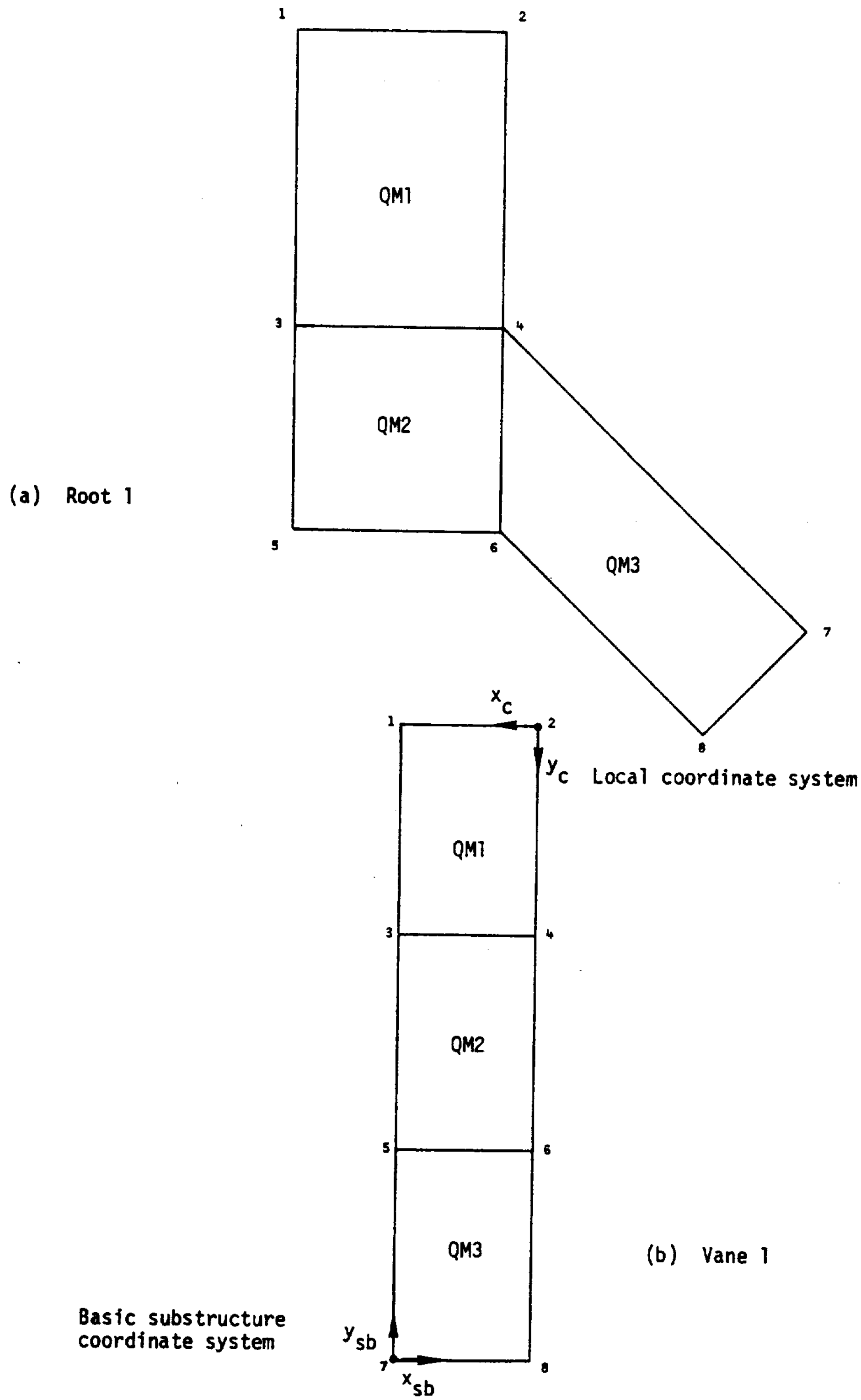


Figure 3. Windmill section substructures.

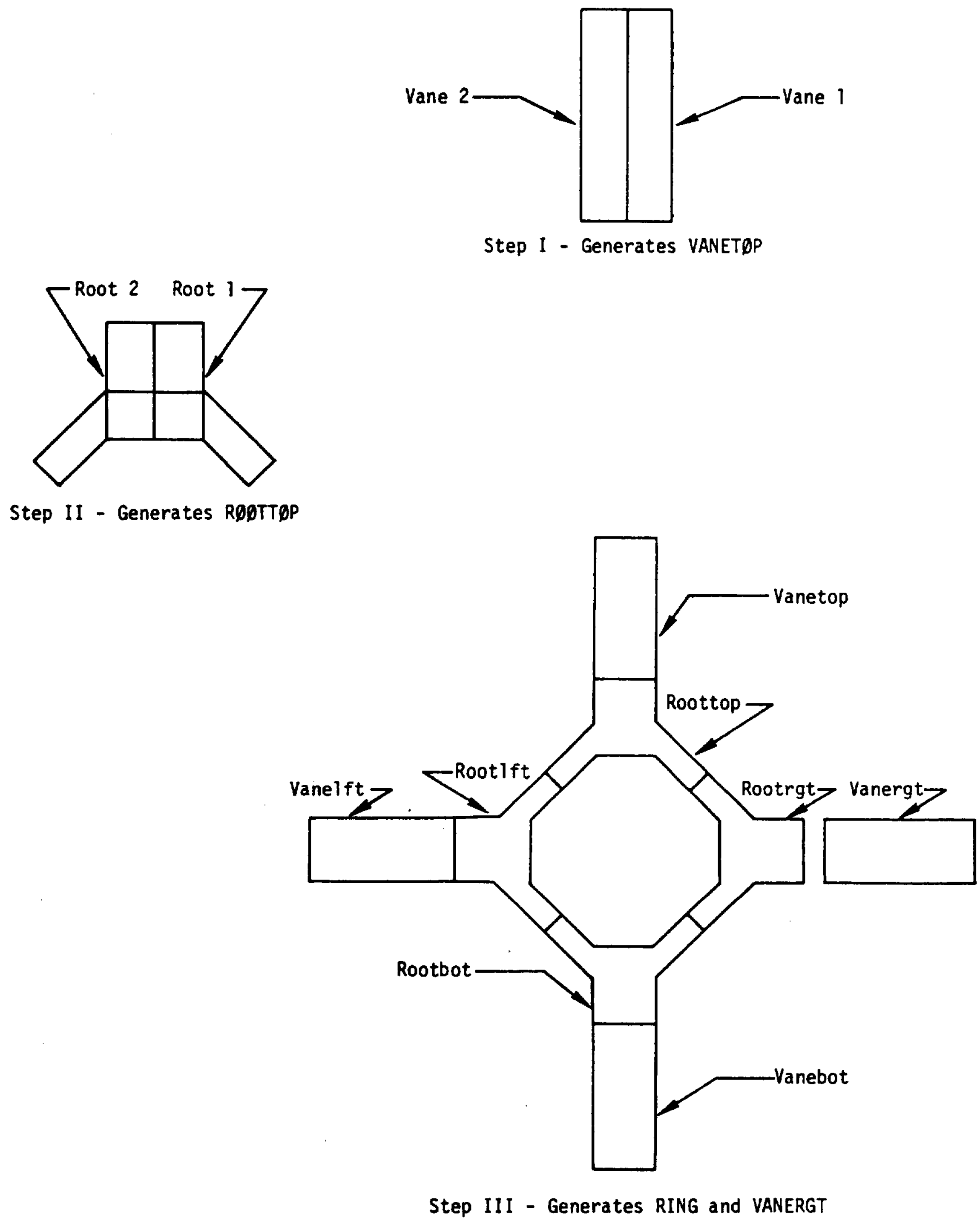
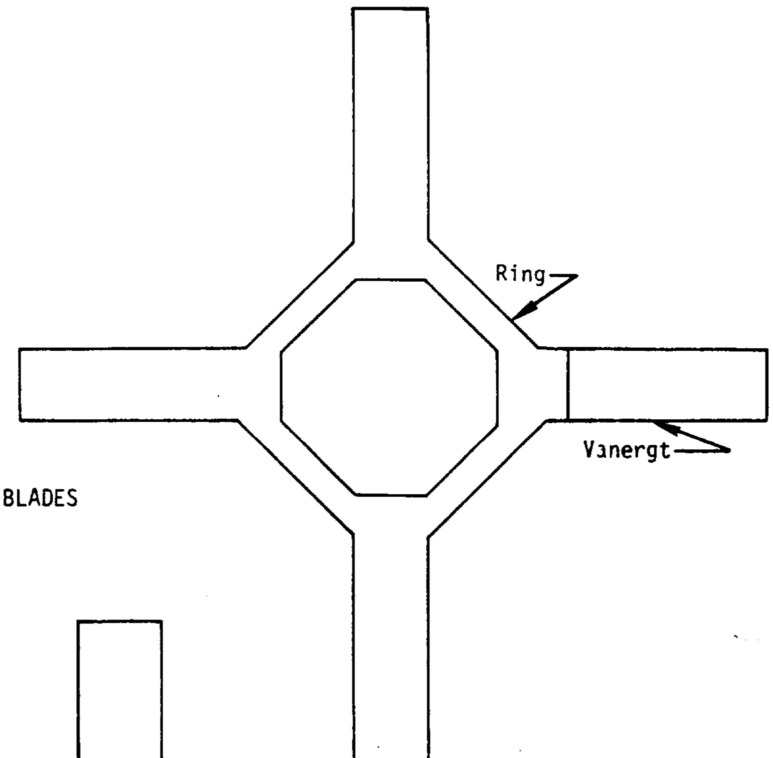
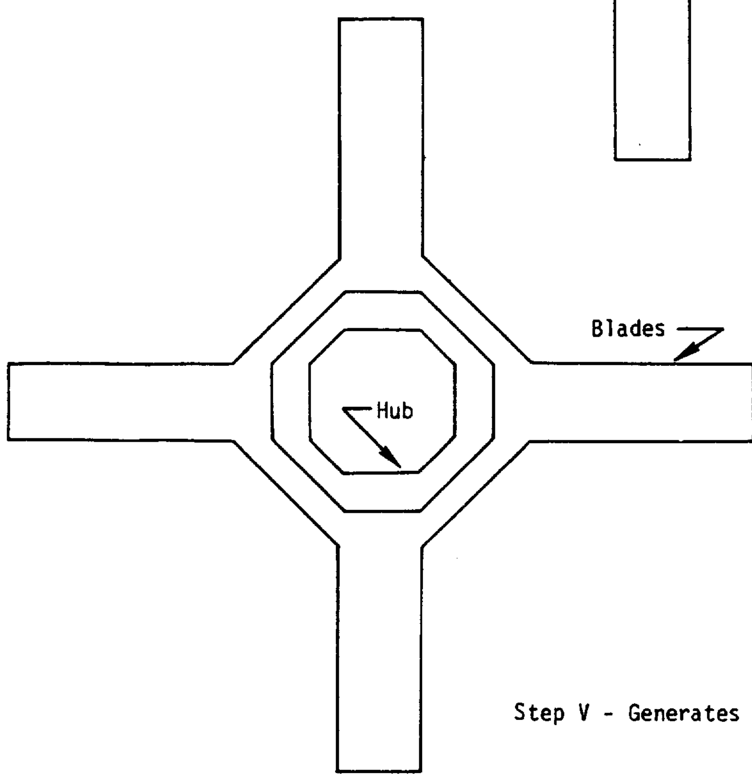


Figure 4. Sequence of combination steps.



Step IV - Generates BLADES



Step V - Generates WINDMILL

Figure 4. Sequence of combination steps (continued).

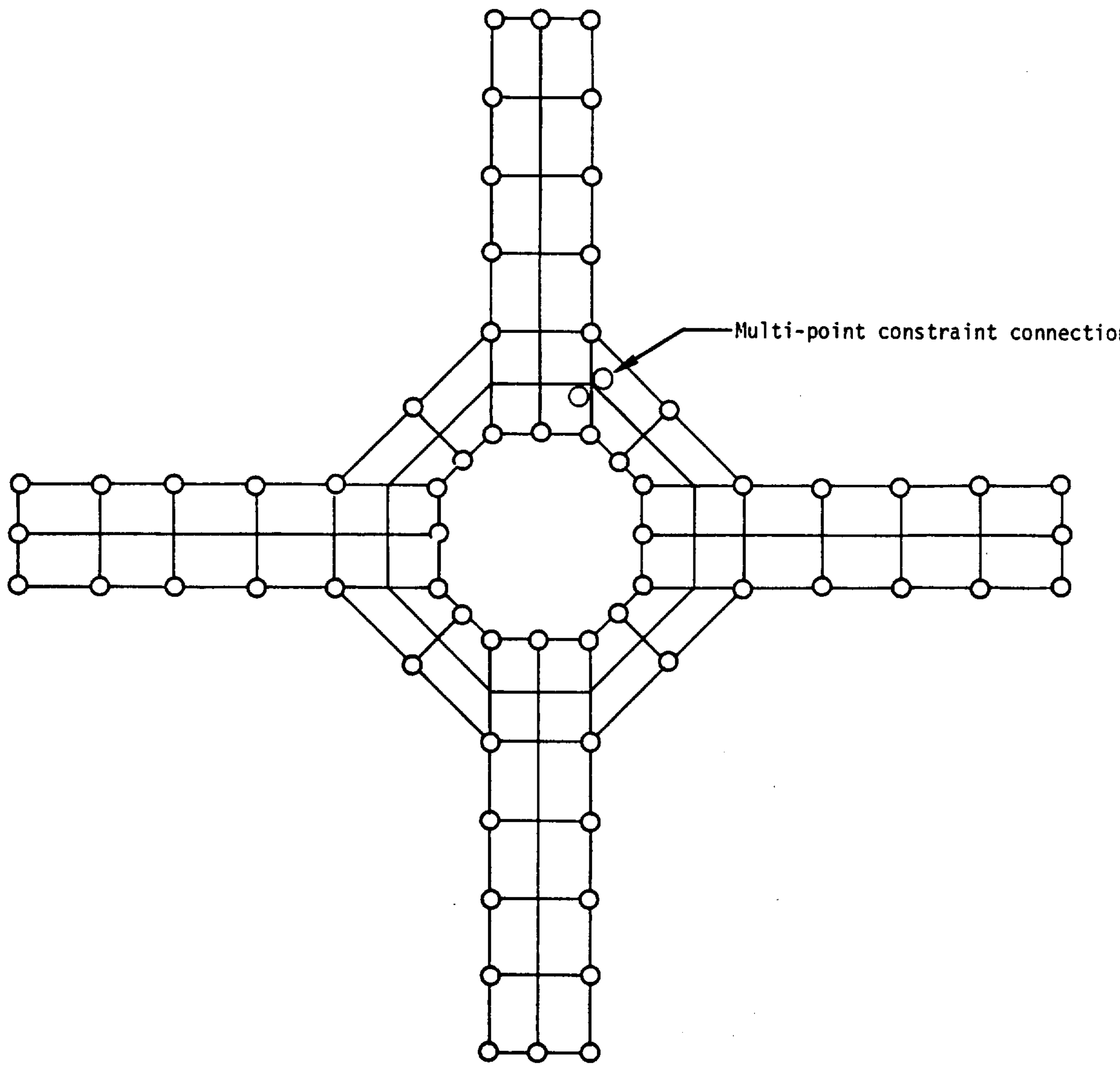


Figure 5. Solution grid points for windmill model.

RIGID FORMAT No. 3, Real Eigenvalue Analysis

Vibration of a 10x20 Plate (3-1-1)

Vibration of a 20x40 Plate (3-1-2)

Vibration of a 10x20 Plate (INPUT, 3-1-3)

Vibration of a 20x40 Plate (INPUT, 3-1-4)

A. Description

This problem demonstrates the solution for natural frequencies of a large-order problem. The structural model consists of a square plate with hinged supports on all boundaries. The 10x20 model (Problem 3-1-1), as shown in Figure 1, represents one half of the structure with symmetric boundary constraints on the mid-line to reduce the order of the problem and the bandwidth by one half. The 20x40 model (Problem 3-1-2) has the same dimensions, but with a finer mesh. Both configurations are developed via the INPUT module (Problems 3-1-3 and 3-1-4 for coarse mesh and fine mesh, respectively) to generate the QUAD1 elements.

Because only the bending modes are desired, the in-plane deflections and rotations normal to the plane are constrained. The inverse power method of eigenvalue extraction is selected for the smaller model and the FEER method (Reference 32) is selected for the larger model. Both structural mass density and non-structural mass-per-area are used to define the mass matrix.

An undeformed structure plot is executed without plot elements. This is expensive on most plotters since all four sides of each quadrilateral are drawn. For the deformed plots of each eigenvector, plot elements are used to draw an edge only once and to draw only selected coordinate lines (every second or fourth line depending on the model used).

B. Input

1. Parameters:

$\ell = w = 20.0$ (Length and width)

$I = \frac{1}{12}$ (Moment of inertia)

$t = 1.0$ (Thickness)

$E = 3.0 \times 10^7$ (Modulus of elasticity)

$\nu = 0.30$ (Poisson's ratio)

$\rho = 206.0439$ (Mass density, 200.0 structural and 6.0439 non-structural mass)

2. Boundary constraints:

along $x = 0$, $\theta_y = 0$ — Symmetric Boundary

$$\left.\begin{array}{l} \text{along } y = 0,\ u_z = \theta_y = 0 \\ \text{along } x = 10,\ u_z = \theta_x = 0 \\ \text{along } y = 20,\ u_z = \theta_y = 0 \end{array}\right\}\quad \text{Hinged Supports}$$

3. Eigenvalue extraction data:

Method: Inverse Power and FEER

Region of interest for Inverse Power: $.89 \le f \le 1.0$

Center point for FEER: .87

Number of desired roots: 3

Number of estimated roots: 1

C. Results

Table 1 lists the NASTRAN and theoretical natural frequencies as defined in Reference 8. Figures 2 and 3 present the first two mode shapes. The modal masses for these modes are equal to one-fourth the total mass or $m_i = 10302.2$.

Table 1. Natural Frequencies, cps.

Mode No.	Theoretical	NASTRAN 10x20 (INV)	NASTRAN 20x40 (FEER)
1	.9069	.9056	.9066
2	2.2672	2.2634	2.2663
3	4.5345	4.5329	4.5340

D. Driver Decks and Sample Bulk Data

Card No.

```
NASTRAN FILES=(UMF,PLT2)
ID        DEM3011,NASTRAN
UMF       1977    30110
APP       DISPLACEMENT
SØL       3,1
TIME      35
CEND

TITLE = VIBRATIØNS OF A 10 BY 20 PLATE
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-1
$
SPC = 37
          METHØD = 3       $ INV - ENCLØSES 1 MØDE - FINDS 3 RØØTS
$                       RØØTS ARE AT THE FØLLØWING FREQUENCIES (THEØRETICAL)
$         MØDE      M        N         FREQ
$         1         1        1         9.068997E-1
$         2         1        2         2.267249
$         5         1        3         4.534498
$         6         3        1         4.534498
$         7         3        2         5.894848
$         9         1        4         7.708647
$
ØUTPUT
    SET 1 = 1 THRU 11, 34 THRU 44, 56 THRU 66, 78 THRU 88, 111 THRU 121
    SET 2 = 1 THRU 12, 22,23,33,34,44,45,55,56,66,67,77,78,88,89,
          99,100, 110 THRU 121
          DISPLACEMENTS = 1
    SPCFØRCE = 2
$
$
$
PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-1
ØUTPUT(PLØT)
PLØTTER SC
    SET 1 INCLUDE PLØTEL
    SET 2 INCLUDE QUAD1
    MAXIMUM DEFØRMATIØN 1.0
    FIND SCALE, ØRIGIN 10
PTITLE = ALL QUADS IN THE PLATE
PLØT ØRIGIN 10, SET 2, LABELS
    FIND SCALE, ØRIGIN 11
PTITLE = MØDE SHAPES USING PLØTEL ELEMENTS
PLØT MØDAL DEFØRMATIØN 1, ØRIGIN 11, SHAPE
BEGIN BULK
ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	2	THRU	219					
CQUAD1	1	23	1	2	13	12	.00		
EIGR	2	INV	.85	.89	1	1	0		CSIMPL-I
+SIMPL-I	MAX								
GRDSET							126		
GRID	1		.00000	.00000	.00000				
MAT1	2	3.0+7		.300	200.00				+MAT1
+MAT1	30000.	28000.							
PARAM	GRDPNT	111							
PLØTEL	300	23	1						
PQUAD1	23	2	1.0	2	.0833333			6.04393	+PQUAD1
+PQUAD1	.5	.0							
SPC1	37	5	1	12	23	34	45	56	+31001H
+31001H	67	78	89	100	111	122	133	144	+31002H

```
Card
 No.

  0   NASTRAN FILES=(UMF,PLT2)
  1   ID        DEM3012,NASTRAN
  2   UMF       1977     30120
  3   APP       DISPLACEMENT
  4   SØL       3,1
  5   TIME      65
  6   CEND

  7   TITLE = VIBRATIØN ØF A 20 X 40 HALF PLATE
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-2
  9   $
 10   METHØD = 20 $ FEER - NØ MØDES
 11       SPC = 37
 12   $         RØØTS ARE AT THE FØLLØWING FREQUENCIES (THEØRETICAL)
 13   $         MØDE      M         N         FREQ
 14   $         1         1         1         9.068997E-1
 15   $         2         1         2         2.267249
 16   $         5         1         3         4.534498
 17   $         6         3         1         4.534498
 18   $         7         3         2         5.894848
 19   $         9         1         4         7.708647
 20   ØUTPUT
 21       SET 1 = 1 THRU 21, 64 THRU 84, 127 THRU 147, 190 THRU 210,
 22            253 THRU 273, 316 THRU 336, 379 THRU 399, 442 THRU 462,
 23            505 THRU 525, 568 THRU 588, 631 THRU 651, 694 THRU 714,
 24            757 THRU 777, 820 THRU 840, 841 THRU 861
 25       DISPLACEMENTS = 1
 26   $
 27   $
 28   $
 29   PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-2
 30   ØUTPUT(PLØT)
 31       PLØTTER SC
 32       SET 1 INCLUDE PLØTEL
 33       SET 2 INCLUDE QUAD1
 34       MAXIMUM DEFØRMATIØN 1.0
 35       FIND SCALE, ØRIGIN 10
 36   PTITLE = ALL QUADS IN THE PLATE
 37   PLØT ØRIGIN 10, SET 2, LABELS
 38       FIND SCALE, ØRIGIN 11
 39   PTITLE = MØDE SHAPES USING PLØTEL ELEMENTS
 40   PLØT MØDAL DEFØRMATIØN 1, ØRIGIN 11, SHAPE
 41   BEGIN BULK
 42   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	2	THRU	839					
CQUAD1	1	101	1	2	23	22	.0		
EIGR	20	FEER	.87			1			+FEER
+FEER	MAX								
GRDSET							126		
GRID	1	0	.0	.0	.0	0	126		
MAT1	2	3.0+7		.300	200.0				+MAT1
+MAT1	30000.	28000.							
PARAM	GRDPNT	421							
PLØTEL	1000	1	21		1001	21	861		
PQUAD1	101	2	1.0	2	.0833333			6.04393	+PQUAD1
+PQUAD1	.5	.0							
SPC1	37	5	1	22	43	64	85	106	+31001H
+31001H	127	148	169	190	211	232	253	274	+31002H

```
Card
 No.

  0   NASTRAN FILES=(UMF,PLT2)
  1    ID        DEM3013,NASTRAN
  2   UMF        1977    30130
  3   ALTER      1
  4   PARAM      //C,N,NØP/V,N,TRUE=-1 $
  5   INPUT,     ,GEOM2,,,/G1,G2,,G4,/C,N,3/C,N,1 $
  6   EQUIV      G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE $
  7   ENDALTER
  8   APP        DISPLACEMENT
  9   SØL        3,1
 10   DIAG 14
 11   TIME       35
 12   CEND

 13   TITLE = VIBRATIØNS ØF A 10 BY 20 PLATE
 14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-3
 15   $
 16            SPC = 10020
 17            METHØD = 3      $ INV - ENCLØSES 1 MØDE - FINDS 3 RØØTS
 18   $                 RØØTS ARE AT THE FØLLØWING FREQUENCIES (THEØRETICAL)
 19   $        MØDE      M         N         FREQ
 20   $        1         1         1         9.068997E-1
 21   $        2         1         2         2.267249
 22   $        5         1         3         4.534498
 23   $        6         3         1         4.534498
 24   $        7         3         2         5.894848
 25   $        9         1         4         7.708647
 26   $
 27   ØUTPUT
 28       SET 1 = 1 THRU 11, 34 THRU 44, 56 THRU 66, 78 THRU 88, 111 THRU 121
 29       SET 2 = 1 THRU 12, 22,23,33,34,44,45,55,56,66,67,77,78,88,89,
 30            99,100, 110 THRU 121
 31            DISPLACEMENTS = 1
 32       SPCFØRCE = 2
 33   $
 34   PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-3
 35   ØUTPUT(PLØT)
 36   PLØTTER SC
 37            SET 1 INCLUDE PLØTEL
 38            SET 2 INCLUDE QUAD1
 39            MAXIMUM DEFØRMATIØN 1.0
 40            FIND SCALE, ØRIGIN 10
 41   PTITLE = ALL QUADS IN THE PLATE
 42   PLØT ØRIGIN 10, SET 2, LABELS
 43            FIND SCALE, ØRIGIN 11
 44   PTITLE = MØDE SHAPES USING PLØTEL ELEMENTS
 45   PLØT MØDAL DEFØRMATIØN 1, ØRIGIN 11, SHAPE
 46   BEGIN BULK
 47   ENDDATA

 48         10        20       1.0       1.0       126       0.0       0.0
 49         35         5        35        34         0         0
```

1	2	3	4	5	6	7	8	9	10
EIGR	2	INV	.85	.89	1	1	0		CSIMPL-I
+SIMPL-I	MAX								
MAT1	2	3.0+7		.300	200.0				+MAT1
+MAT1	30000.	28000.							
PARAM	GRDPNT	111							
PLØTEL	300	23	1						
PQUAD1	101	2	1.0	2	.0833333			6.04393	+PQUAD1
+PQUAD1	.5	.0							

```
Card
 No.

  0   NASTRAN FILES=(UMF,PLT2)
  1   ID        DEM3014,NASTRAN
  2   UMF       1977     30140
  3   ALTER     1
  4   PARAM     //C,N,NØP/V,N,TRUE=-1  $
  5   INPUT,    ,GEØM2,,,/G1,G2,,G4,/C,N,3/C,N,1  $
  6   EQUIV     G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE  $
  7   ENDALTER
  8   APP       DISPLACEMENT
  9   SØL       3,1
 10   DIAG  14
 11   TIME      65
 12   CEND

 13   TITLE = VIBRATIØN ØF A 20 X 40 HALF PLATE
 14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-4
 15   $
 16   METHØD = 20 $ FEER - NØ MØDES
 17            SPC = 20040  $ INPUT VERSIØN
 18   $                RØØTS ARE AT THE FØLLØWING FREQUENCIES (THEØRETICAL)
 19   $        MØDE      M         N         FREQ
 20   $        1         1         1         9.068997E-1
 21   $        2         1         2         2.267249
 22   $        5         1         3         4.534498
 23   $        6         3         1         4.534498
 24   $        7         3         2         5.894848
 25   $        9         1         4         7.708647
 26   $
 27   ØUTPUT
 28       SET 1 = 1 THRU 21, 64 THRU 84, 127 THRU 147, 190 THRU 210, 253 THRU 273, 316 THRU 336,
 29            379 THRU 399, 442 THRU 462, 505 THRU 525, 568 THRU 588, 631 THRU 651, 694 THRU 714,
 30            757 THRU 777, 820 THRU 840, 841 THRU 861
 31            DISPLACEMENTS = 1
 32   $
 33   PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-1-4
 34   ØUTPUT(PLØT)
 35   PLØTTER SC
 36       SET 1 INCLUDE PLØTEL
 37       SET 2 INCLUDE QUAD1
 38       MAXIMUM DEFØRMATIØN 1.0
 39       FIND SCALE, ØRIGIN 10
 40   PTITLE = ALL QUADS IN THE PLATE
 41   PLØT ØRIGIN 10, SET 2, LABELS
 42       FIND SCALE, ØRIGIN 11
 43   PTITLE = MØDE SHAPES USING PLØTEL ELEMENTS
 44   PLØT MØDAL DEFØRMATIØN 1, ØRIGIN 11, SHAPE
 46   BEGIN BULK
 47   ENDDATA

 48        20        40      0.5      0.5       126       0.0       0.0
 49        35         5       35       34         0         0
```

1	2	3	4	5	6	7	8	9	10
EIGR	20	FEER	.87			1			+FEER
+FEER	MAX								
MAT1	2	3.0+7		.300	200.00				+MAT1
+MAT1	30000.	28000.							
PARAM	GRDPNT	421							
PLØTEL	1000	1	21		1001	21	861		
PQUAD1	101	2	1.0	2	.0833333			6.04393	+PQUAD1
+PQUAD1	.5	.0							

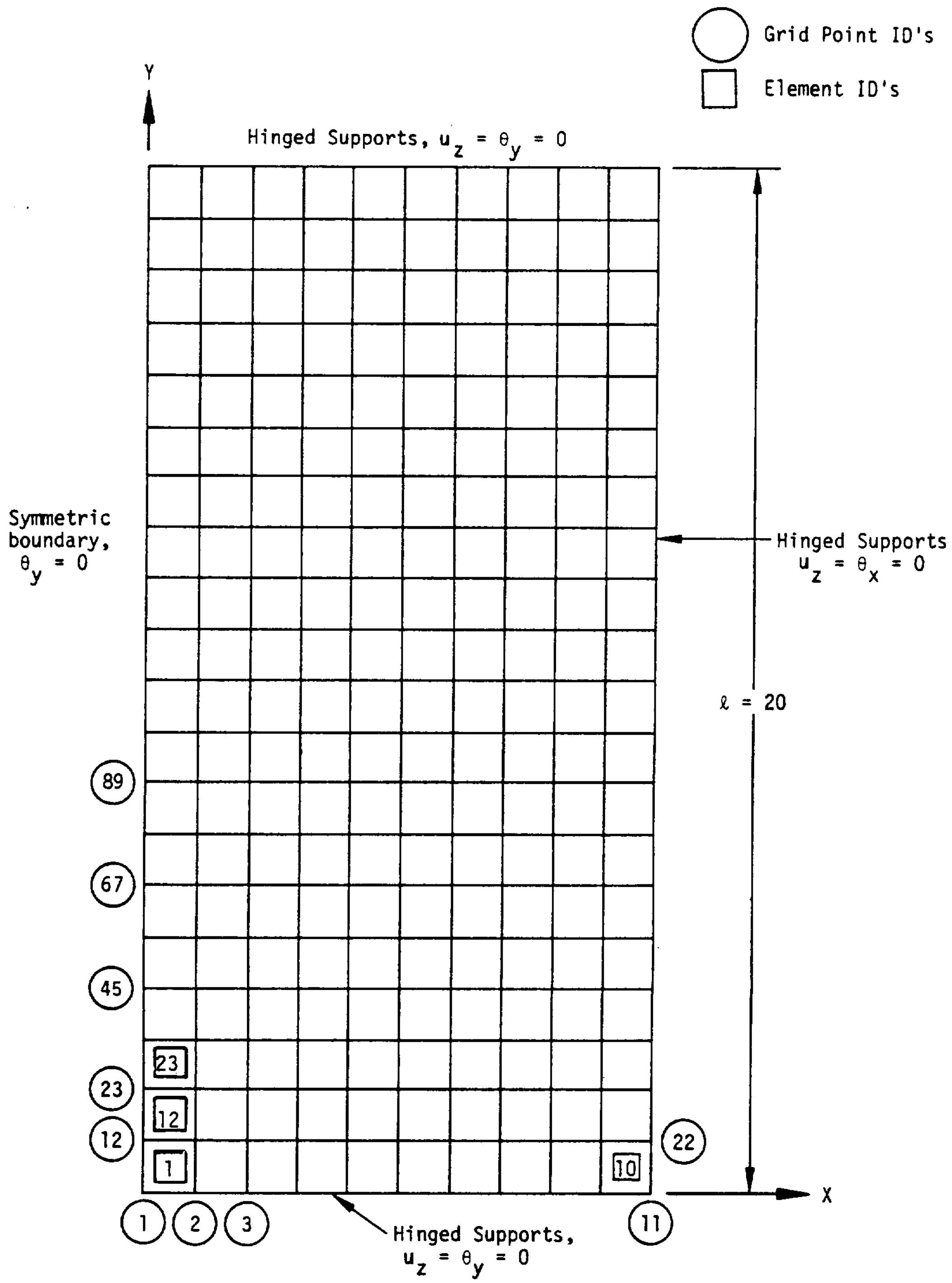


Figure 1. 10 x 20 Half plate model.

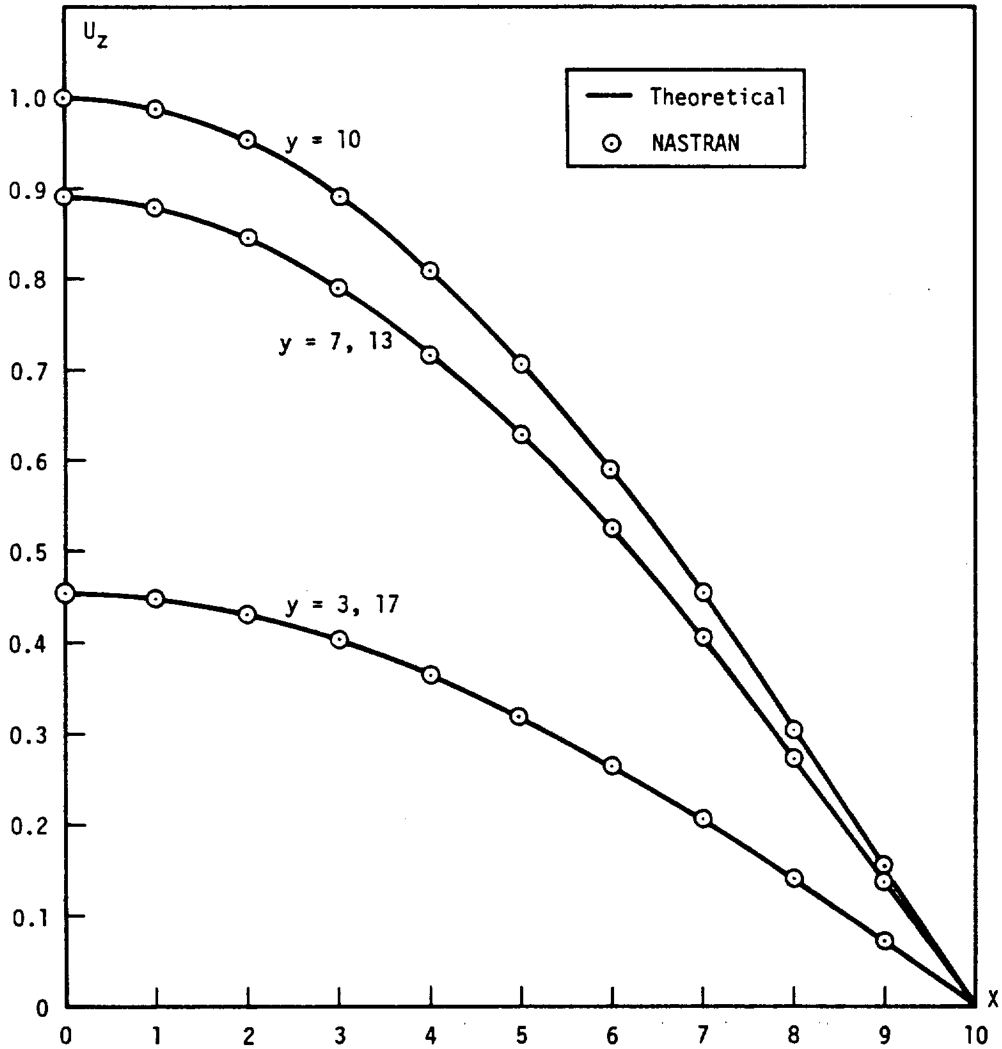


Figure 2. Comparison of displacements, first mode.

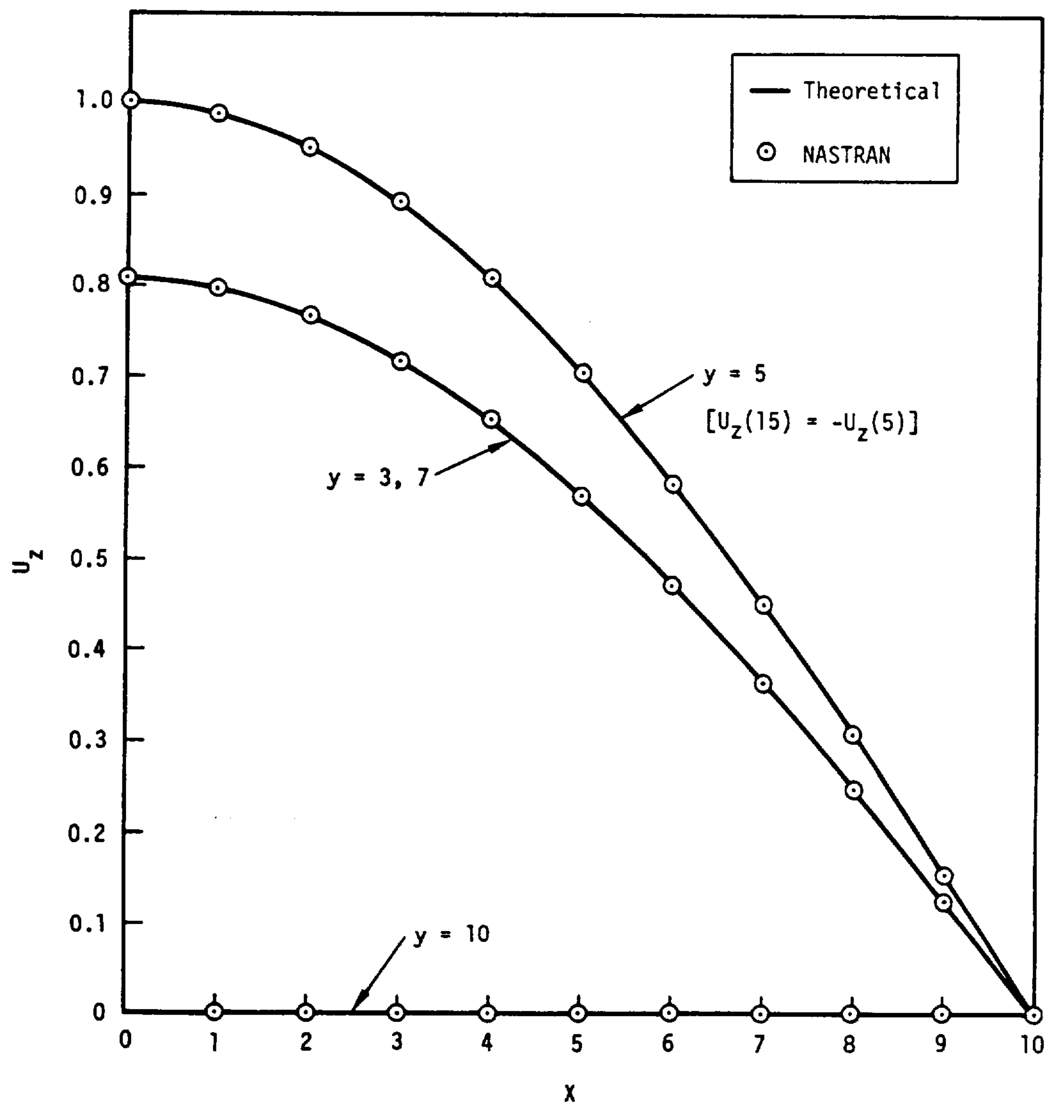


Figure 3. Second mode displacements.

Vibration of a Compressible Gas in a Rigid Spherical Tank (3-2-1)

A. Description

This problem demonstrates a compressible gas in a rigid spherical container. In NASTRAN a rigid boundary is the default for the fluid and, as such, no elements or boundary lists are necessary to model the container.

Aside from the NASTRAN bulk data cards currently implemented, this problem demonstrates the use of the hydroelastic data cards: AXIF, CFLUID2, CFLUID3, and RINGFL.

The lowest mode frequencies and their mode shapes for n = 0, 1 and 2 are analyzed where n is the Fourier harmonic number. Only the cosine series is analyzed.

B. Model

1. Parameters

   $R = 10.0$ m (Radius of sphere)

   $\rho = 1.0 \times 10^{-3}$ Kg/m^3 (Mass density of fluid)

   $B = 1.0 \times 10^{3}$ Newton/m^2 (Bulk modulus of fluid)

2. Figure 1 and 2 show the finite element model. The last 3 digits of the RINGFL identification number correspond approximately to the angle, θ, from the polar axis along a meridian.

C. Theory

From Reference 18, the pressure in the cylinder is proportional to a series of functions:

$$Q_{n,m} = \frac{J_{m+\frac{1}{2}}(X)}{\sqrt{X}} P_m^n (\cos\theta) \cos n\phi \, , \qquad \begin{array}{l} n \le m \\ m = 0,1,2 \end{array} \qquad (1)$$

where:

$Q_{n,m}$	Pressure coefficient for each mode
X	Nondimensional radius $= \frac{\omega_{mk}}{a} r$
ω_{mk}	Natural frequency for the k<u>th</u> mode number and m<u>th</u> radial number in radians per second
$J_{m+\frac{1}{2}}$	Bessel function of the first kind

r	radius
$a = \sqrt{\frac{B}{\rho}}$	speed of sound in the gas
P_m^n	associated Legendre functions
θ	meridinal angle
ϕ	circumferential angle
n	harmonic number
m	number of radial node lines

The solution for X and hence ω_{mk} is found by the use of the boundary condition that the flow through the container is zero.

$$\left\{ \frac{d}{dX} \left[\frac{J_{m+\frac{1}{2}}(X)}{\sqrt{X}} \right] \right\}_{r=R} = 0.0 \qquad (2)$$

where R is the outer radius .

This results in zero frequency for the first root. Multiple roots for other modes can be seen in Table 1. The finite element model assumes different pressure distributions in the two angular directions which causes the difference in frequencies.

D. Results

Table 1 and Figure 3 summarize the NASTRAN and analytic results for the lowest nonzero root in each harmonic. Table 1 lists the theoretical natural frequencies, the NASTRAN frequencies, the percent error in frequency, and the maximum percent error in pressure at the wall as compared to the maximum value. Figure 3 shows the distribution of the harmonic pressure at the wall versus the meridinal angle. The theoretical pressure distributions correspond to the Legendre functions P_o^0 (cos θ), P_o^1 (cos θ), and P_o^2 (cos θ) which are proportional to cos θ, sin θ, and $\sin^2\theta$ respectively.

E. Driver Decks and Sample Bulk Data

Card No.	
0	NASTRAN FILES=UMF
1	ID DEM3021,NASTRAN
2	UMF 1977 30210
3	APP DISPLACEMENT
4	SØL 3,3
5	TIME 20
6	CEND
7	TITLE = VIBRATIØN ØF A CØMPRESSIBLE GAS IN A RIGID SPHERICAL TANK.
8	SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-2-1
9	METHØD = 1
10	AXISYMMETRIC = FLUID
11	ØUTPUT
12	HARMØNICS = ALL
13	SET 1 = 1000 THRU 2030, 2090,2150,3022,3090,3157,4018,4090,
14	4162,5015,5090,5165,6012,6089,6167,7011,7090,7168,8010,8090,
15	8170,9009,9090,9171,10000 THRU 10180
16	PRESSURE = 1
17	BEGIN BULK
18	ENDDATA

1	2	3	4	5	6	7	8	9	10
AXIF	100		.001	1.0+3	NO				+AXIF
+AXIF	0	THRU	2						
CFLUID2	1	1090	1045						
CFLUID3	4	2060	2030	1045					
CØRD2S	100	0	.0	.0	10.0	.0	.0	20.0	+CØRD2S
+CØRD2S	.0	1.0	.0						
EIGR	1	INV	14.0	60.0	2	7		1.0-6	+EIGR-1
+EIGR-1	MAX								
RINGFL	1045	1.00000	45.0000		1090	1.00000	90.0000		

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Table 1. Comparison of NASTRAN and analytical results.

Harmonic	Natural Frequency (Hertz)			Pressure
	Analytical	NASTRAN	% Error	Max. % Error at Wall
0	33.1279	33.2383	0.33	1.19
1	33.1279	33.2060	0.24	0.47
2	53.1915	53.3352	0.27	0.91

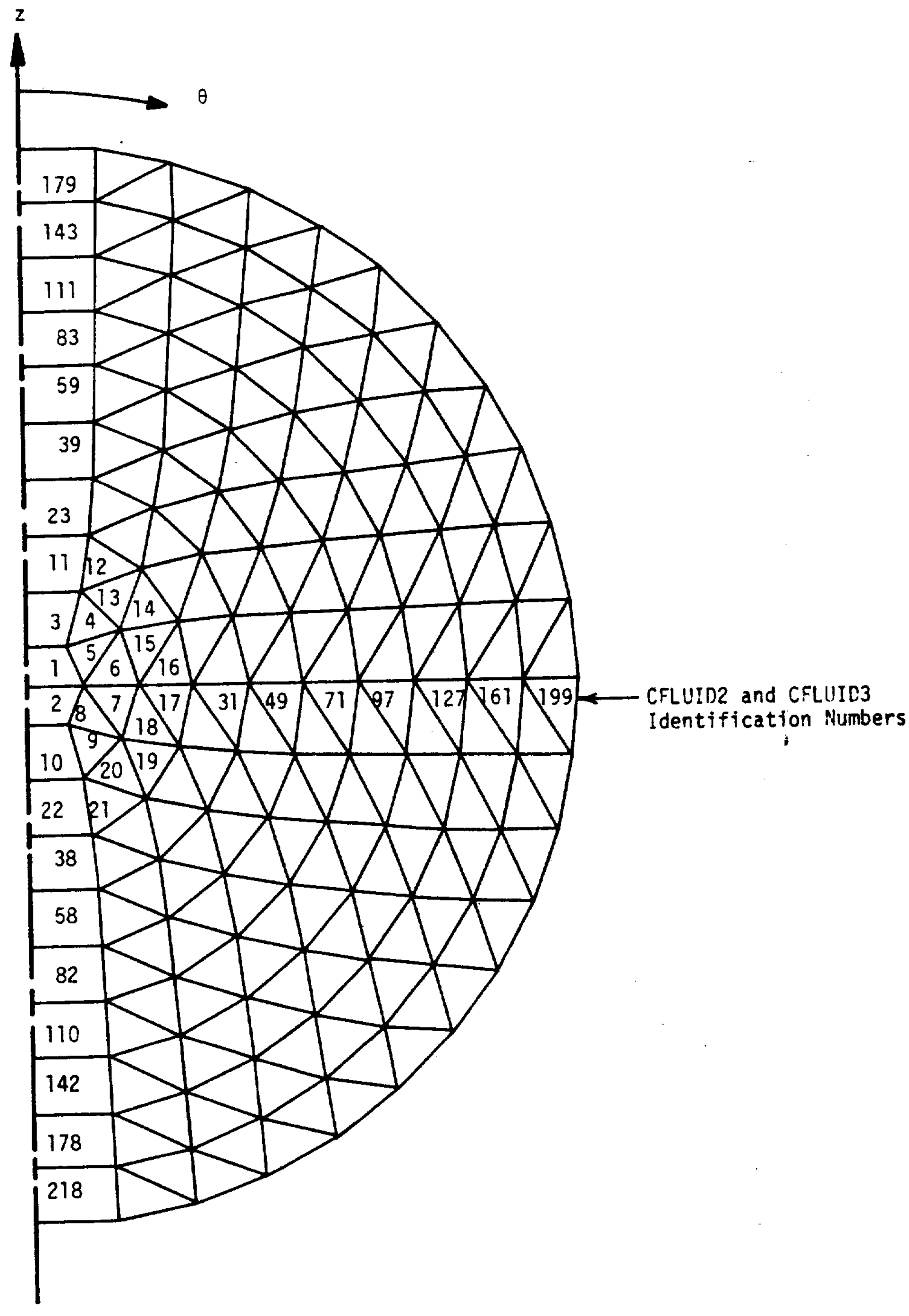

Figure 1. Gas filled rigid spherical tank model.

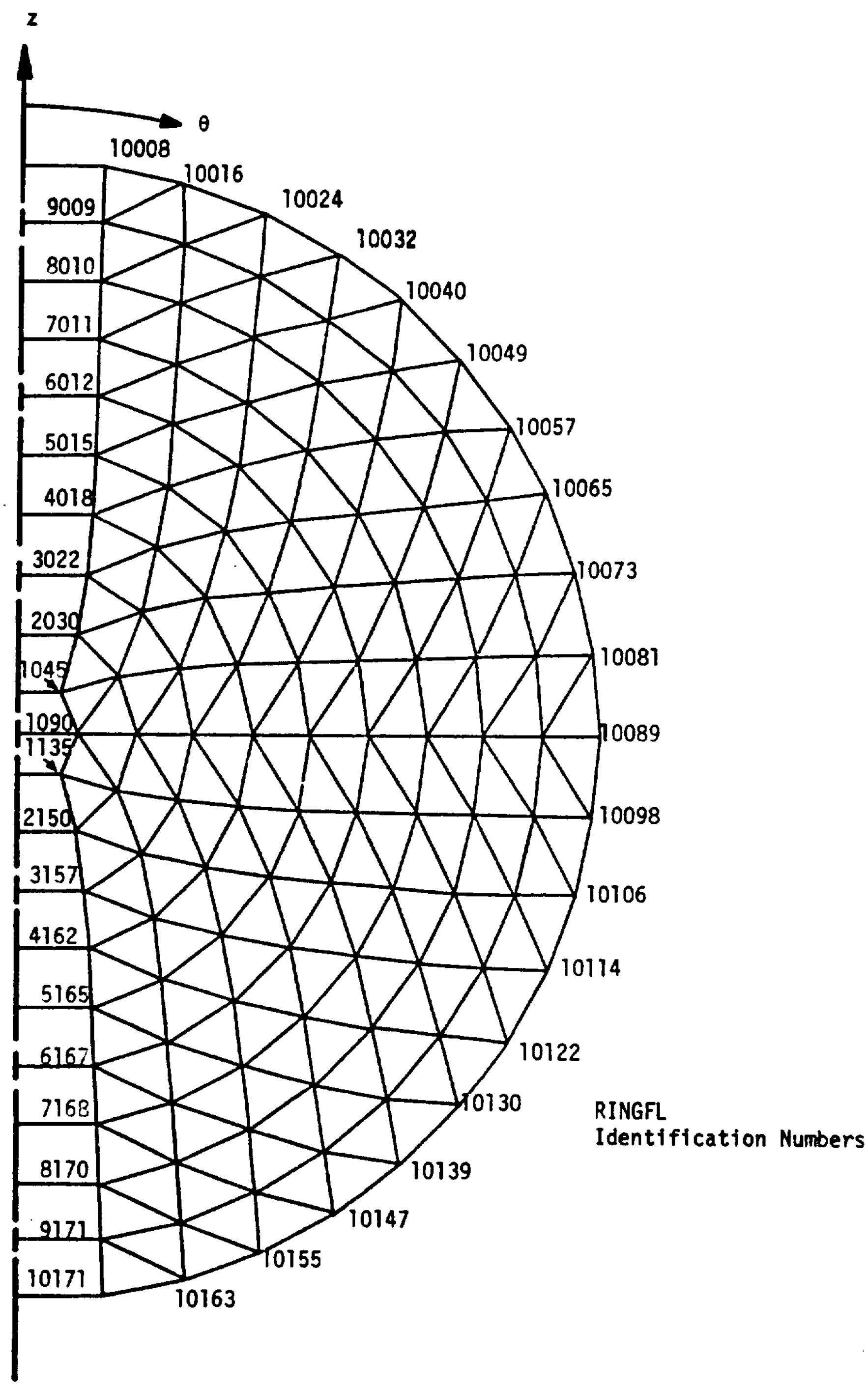

Figure 2. Gas filled rigid spherical tank model.

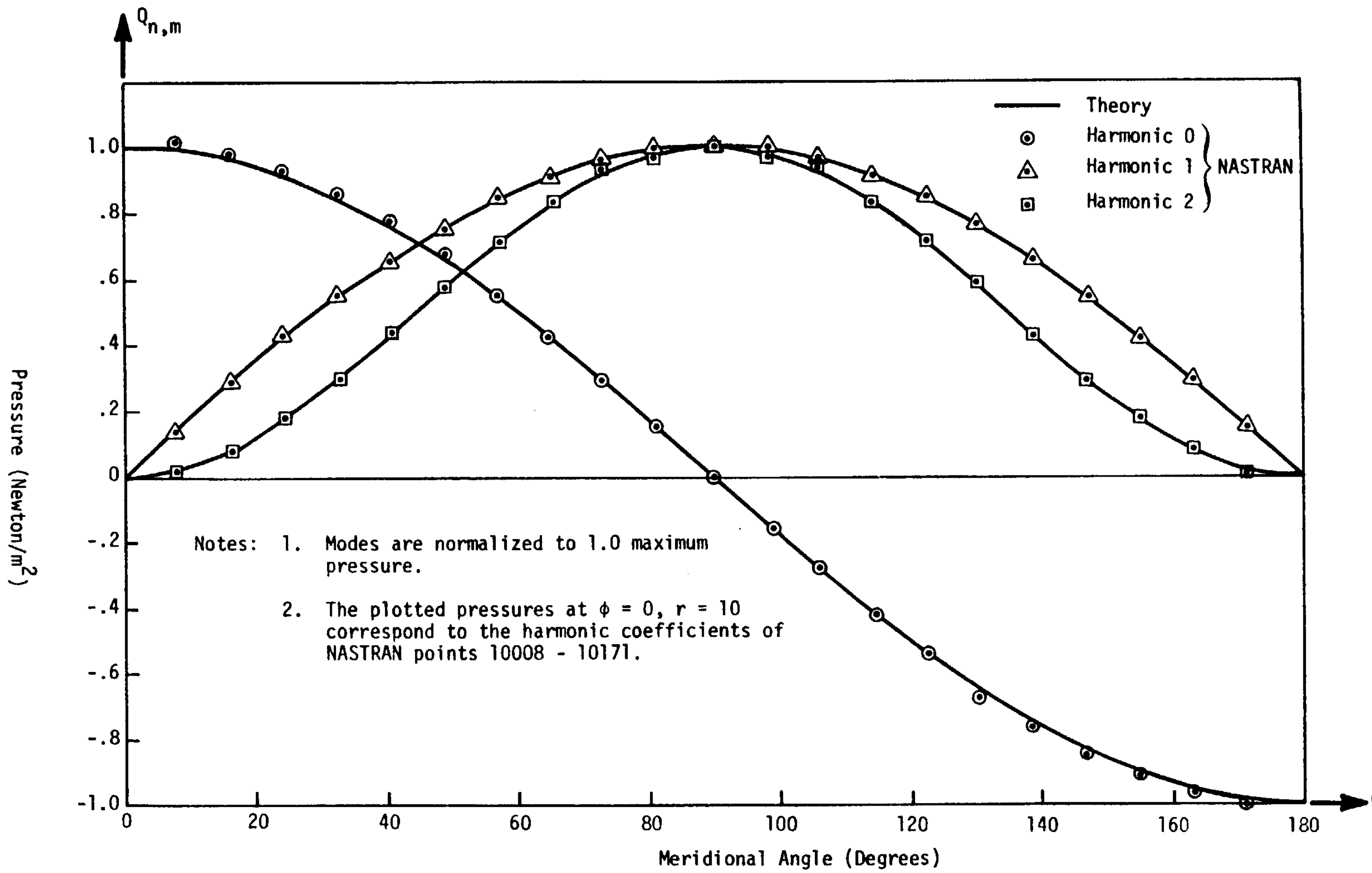

Figure 3. Pressure at tank wall - first finite modes.

RIGID FORMAT No. 3, Real Eigenvalue Analysis

Vibration of a Liquid in a Half-Filled Rigid Sphere (3-3-1)

A. Description

The model is similar to Demonstration Problem No. 3-2-1 except that a hemispherical fluid model with a free surface is analyzed. Additional cards demonstrated are the free surface list (FSLIST) and free surface points (FREEPT). The effective gravity for the fluid is found on the AXIF card. The fluid is considered incompressible.

The lowest three eigenvalues and eigenvectors for the cosine and sine series of $n = 1$ are analyzed, where n is the harmonic order.

B. Input

1. Parameters

g	$= 10.0 \text{ ft/sec}^2$	(Gravity)
R	$= 10.0 \text{ ft}$	(Radius of hemisphere)
ρ	$= 1.255014 \text{ lb-sec}^2/\text{ft}^4$	(Fluid mass density)
B	$= \infty$	(Bulk modulus of fluid, incompressible)

2. Figure 1 shows the finite element model.

C. Results

Reference 17 gives the derivations and analytical results. In particular, the parameters used in the reference are:

$$\left.\begin{aligned} e &= 0 \quad \text{(half-filled sphere)} , \\ \lambda &= \frac{\omega^2 R}{g} \text{ (dimensionless eigenvalue)} \quad . \end{aligned}\right\} \quad (1)$$

Table 2 of Reference 17 lists the eigenvalues, λ_1, λ_2, and λ_3 for the first three modes. Figure 13 of Reference 17 shows the mode shapes.

The analytic and NASTRAN results are compared in Table 1. The frequencies are listed and the resulting percentage errors are given. The maximum percent error of the surface displacement, relative to the largest displacement, is tabulated for each mode.

The free surface displacements may be obtained by the equation:

$$u = \frac{p}{\rho g} \quad , \tag{2}$$

where p is the pressure at the free surface recorded in the NASTRAN output. Note that, since an Eulerian reference frame is used, the pressure at the original (undisturbed) surface is equal to the gravity head produced by motions of the surface. Special FREEPT data cards could also have been used for output. Since the results are scaled for normalization anyway, the harmonic pressures may be used directly as displacements.

Figure 2 is a graph of the shape of the free surface for each distinct root. Both analytic and NASTRAN results are scaled to unit maximum displacements. Because the cosine series and the sine series produce identical eigenvalues, the resulting eigenvectors may be linear combinations of both series. In other words the points of maximum displacement will not necessarily occur at $\phi = 0°$ or $\phi = 90°$. Since the results are scaled, however, and the results at $\phi = 0$ are proportional to the results at any other angle, the results at $\phi = 0$ were used.

Table 1. Comparison of natural frequencies and free surface mode shapes from the reference and NASTRAN.

Mode Number	Natural Frequency (Hertz)			Mode Shape
	Reference	NASTRAN	NASTRAN % Error	Maximum % Error, ε
1	0.1991	0.1988	-0.1	$\varepsilon < 1$ %
2	0.3678	0.3691	0.3	$\varepsilon < 2.6$%
3	0.4684	0.4766	1.8	$\varepsilon < 4$ %

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0     NASTRAN FILES=UMF
  1     ID        DEM3031,NASTRAN
  2     UMF       1977     30310
  3     APP       DISPLACEMENT
  4     SØL       3,3
  5     TIME      20
  6     CEND

  7     TITLE = VIBRATIØN ØF A LIQUID IN A HALF FILLED RIGID SPHERE.
  8     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-3-1
  9         METHØD = 1
 10         AXISYMMETRIC = FLUID
 11     ØUTPUT
 12         HARMONICS = ALL
 13     SET 1 = 1 THRU 1000,1090,2090,3090,4090,5090,6089,7090,8090,9090,
 14     10089,11090,12089,13089,14090,15090,16089,17090,18089,19090,20089
 15         PRESSURE = 1
 16     BEGIN BULK
 17     ENDDATA
```

1	2	3	4	5	6	7	8	9	10
AXIF	100	10.0	1.255014	.0	YES				+AXIF
+AXIF	1								
CFLUID2	1	1135	1090						
CFLUID3	2	2120	2090	1090					
CØRD2S	100	0	.0	.0	10.0	.0	.0	20.0	+CØRD2S
+CØRD2S	.0	1.0	.0						
EIGR	1	INV	.1	.5	6	7		1.0-5	+EIGR-1
+EIGR-1	MAX								
FREEPT	4090		109	90.0	118	180.0	127	270.0	
FSLIST		AXIS	1090	2090	3090	4090	5090	6089	+1-FSL
+1-FSL	7090	8090	9090	10089	11090	12089	13089	14090	+2-FSL
RINGFL	1090	.500000	90.0000		1135	.50000	135.000		

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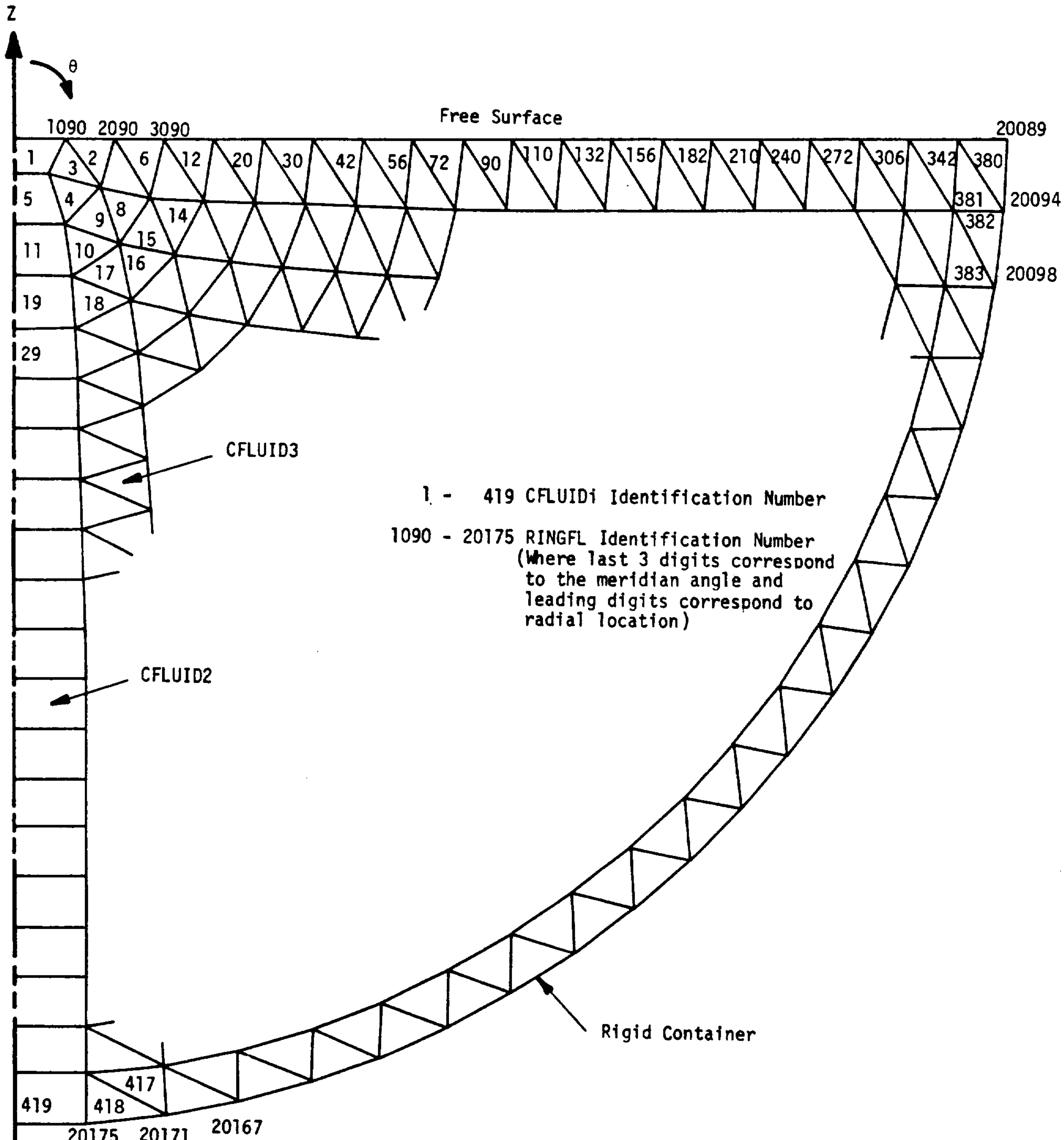

Figure 1. Rigid sphere half filled with a liquid.

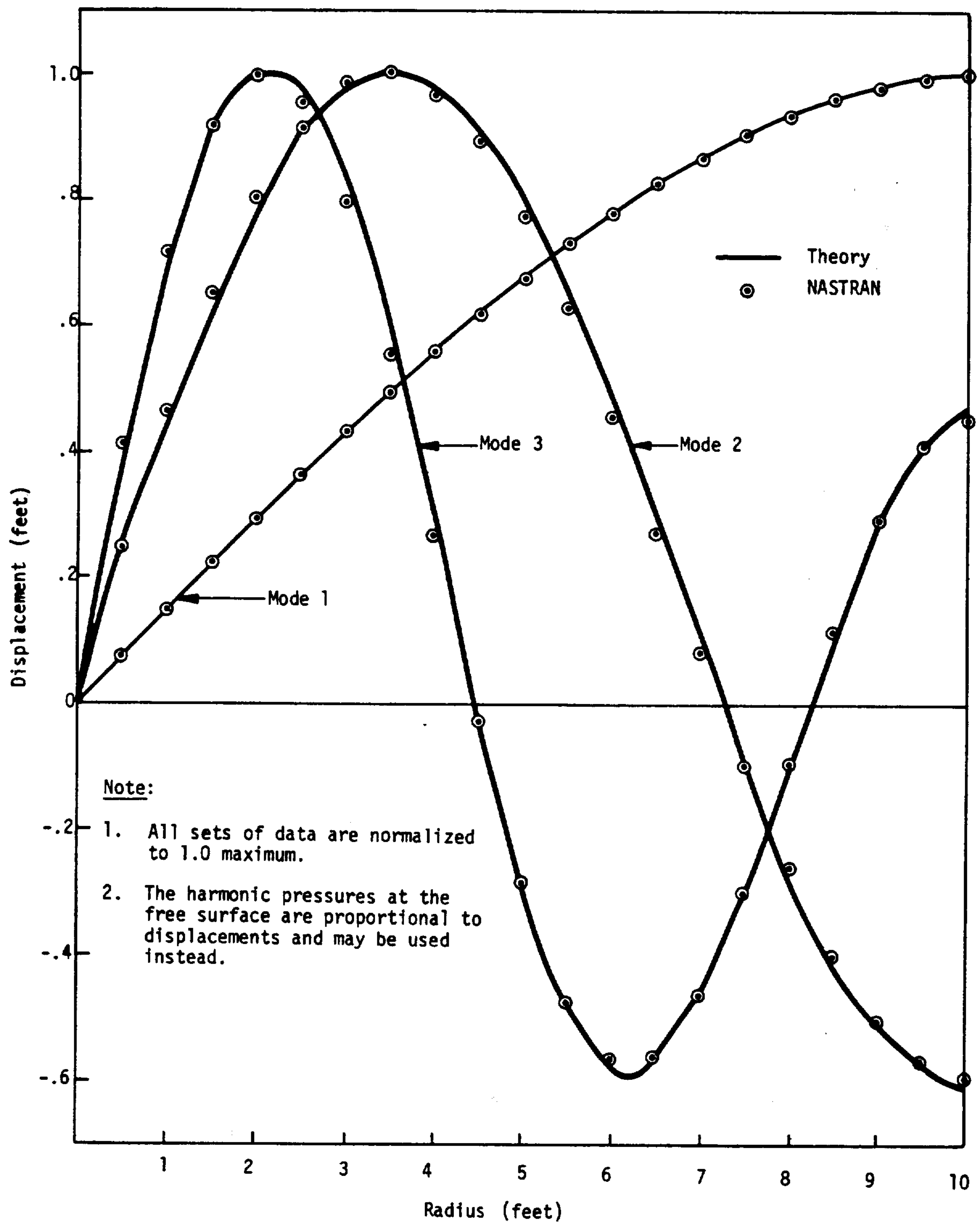

Figure 2. Free surface mode shapes.

RIGID FORMAT No. 3, Real Eigenvalue Analysis

Acoustic Cavity Analysis (3-4-1)

A. Description

This problem illustrates the use of NASTRAN to determine the acoustic modes in a cavity containing both axisymmetric regions and evenly spaced radial slots. The motor cavity of Stage III of the Minuteman III missile is selected for analysis. The finite element model, shown in Figure 1, consists of six slots and a long, slender central cavity of irregular shape. The model consists of AXIF2, AXIF3, and AXIF4 finite elements in the central cavity, and SLØT3 and SLØT4 finite elements in the slotted region.

The axisymmetric radial and longitudinal acoustic modes are desired (N = 0) for this problem. The harmonic index N specifies the Fourier Series terms to be analyzed. For example, N = 1 defines the lateral motion where the velocity is normal to the center axis. Repeated runs with N = 0, 1, ... M/2 may be necessary to extract all possible modes where M is the number of radial slots specified.

B. Input

Parameters:

$\rho = 1.143 \times 10^{-7}$	(Fluid density)
$\beta = 20.58$	(Fluid bulk modulus)
N = 0	(Harmonic index)
WD = 4.0	(Slot width)
MD = 6	(Number of slots)

C. Results

The vibration mode frequencies for harmonic n = 0 as determined with NASTRAN are shown in Table 1. Also shown are the vibration mode frequencies as determined with an acoustic model and reported in Reference 19.

D. Driver Decks and Sample Bulk Data

```
Card
 No.
  0    NASTRAN FILES=(UMF,PLT2)
  1    ID       DEM3041,NASTRAN
  2    UMF      1977    30410
  3    APP      DISPLACEMENT
  4    SØL      3,0
  5    TIME     3
  6    CEND

  7    TITLE = ACØUSTIC CAVITY ANALYSIS
  8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-4-1
  9    SET 1 =   1 THRU  210
 10    SET 2 = 101 THRU  131,  200 THRU  230,  300 THRU  321,  401 THRU  430,
 11            523 THRU  530,  624 THRU  630,  725 THRU  730,  825 THRU  830,
 12            926 THRU  930, 1026 THRU 1030
 13        METHØD = 1
 14        PRESSURE = 1
 15        STRESS = 2
 16    PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-4-1
 17    ØUTPUT(PLØT)
 18    PLØTTER SC
 19        SET 1 INCLUDE PLØTEL
 20        MAXIMUM DEFØRMATIØN 5.0
 21        AXES MZ,Y,X
 22        VIEW -20.0, 45.0, 0.0
 23        FIND SCALE, ØRIGIN 1, SET 1
 24    PTITLE = RØCKET MØTØR CAVITY USING PLØTEL ELEMENTS
 25    PLØT SET 1, ØRIGIN 1, LABEL GRID PØINTS
 26    PTITLE = MØDE SHAPES ØF MØTØR CAVITY USING PLØTEL ELEMENTS
 27    PLØT MØDAL DEFØRMATIØN, SET 1, ØRIGIN 1, VECTØR R
 28    BEGIN BULK
 29    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
AXSLØT	.1143-6	20.58	0	4.	6				
CAXIF2	101	11	12						
CAXIF3	200	12	19	13					
CAXIF3	410	34	39	35					
CSLØT3	422	89	94	95					
CSLØT4	423	94	100	101	95				
EIGR	1	INV	100.0	500.0	6	7			+EIG1
+EIG1	MAX								
GRID	500		.0	65.25			123456		
GRIDF	1	10.							
GRIDS	89	4.6	43.85		87				
PLØTEL	1	201	500		2	500	501		
SLBDY			89	94	100	107	115	125	+BDY
+BDY	145	165	185	205					
SUPØRT	1	1							

Table 1. Natural frequencies for the third stage, Minuteman III, motor cavity.

Mode	Frequency, Hz	
	NASTRAN	Experimental
1	0.0	0.0
2	90.1	93.0
3	199.5	200.0
4	310.4	312.0
5	388.0	388.0
6	449.1	466.0
7	512.8	518.0

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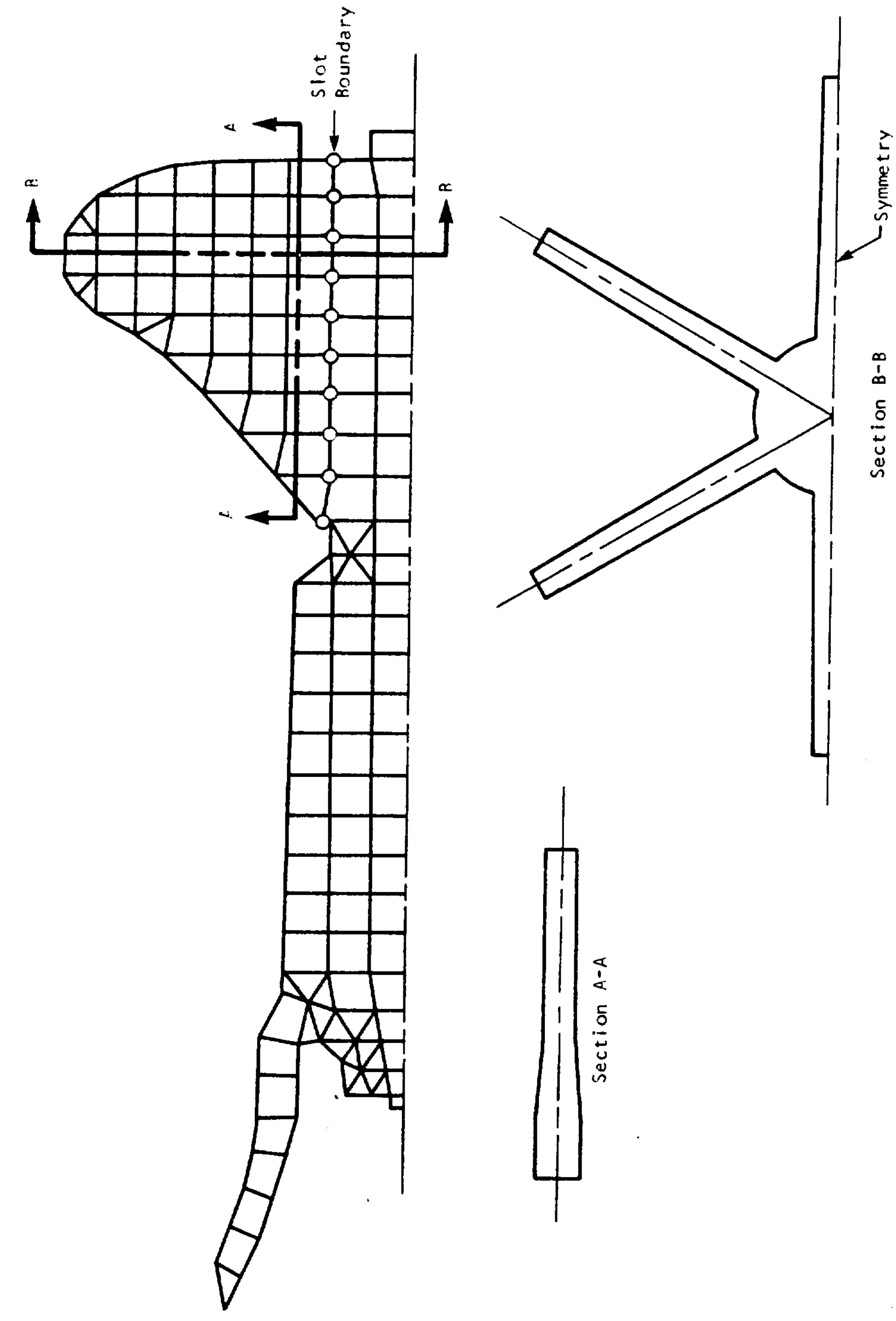

Figure 1. Minuteman III, Stage III, Rocket Motor Cavity

RIGID FORMAT No. 3 (APP HEAT), Nonlinear Heat Conduction
Nonlinear Heat Transfer in an Infinite Slab (3-5-1)

A. Description

This problem demonstrates NASTRAN's capability to solve nonlinear steady state heat conduction problems. The infinite slab is subjected to uniform heat addition per unit volume. There is no heat flux on one face and the other face is kept at zero degrees. The conductivity is temperature dependent. This is a one dimensional problem, since there is no temperature gradient parallel to the surfaces of the slab.

B. Input

The NASTRAN model is shown in Figure 1. Linear elements BAR, CØNRØD, RØD and TUBE with areas of π square units and boundary condition element HBDY (PØINT) are used. The heat addition is specified on a QVØL card and is referenced in Case Control by a LØAD card. The area factor for the HBDY is given on the PHBDY card and heat flux is zero. The initial temperatures are given on a TEMPD card and referenced in Case Control by a TEMP (MATERIAL) card. The conductivity is specified on a MAT4 card and is made temperature dependent by the MATT4 card referencing table TABLEM3. The convergence parameter, the maximum number of iterations and an option to have the residual vector output are specified on PARAM cards. The temperature at the outer surface is specified by an SPC card. Temperature output is punched on TEMP bulk data cards for future use in static analysis.

C. Theory

The conductivity, k, is defined by

$$k(T) = 1 + T/100 \quad , \tag{1}$$

where T is the temperature.

The heat flow per area, q, is

$$q(x) = -k \frac{dT}{dx} = -(1 + T/100) \frac{dT}{dx} \quad . \tag{2}$$

The heat input per volume, q_v, affects the heat flow by the equation

$$\frac{dq(x)}{dx} = q_v \quad . \tag{3}$$

A convenient substitution of variables in Equations (2) and (3) is

$$u = -\int q(x)dx = (T + T^2/200) \quad . \tag{4}$$

Differentiation and substitution for q in Equation (3) results in the second-order equation in u:

$$\frac{d^2u}{dx^2} = -q_v \quad . \tag{5}$$

From the following boundary conditions

$$u = 0 \quad \text{at} \quad x = \ell \quad ,$$

and

$$\frac{du}{dx} = 0 \quad \text{at} \quad x = 0 \quad ,$$

the solution to Equation (5) is

$$u = \frac{q_v}{2} (\ell^2 - x^2) \quad . \tag{6}$$

Therefore the solution for the temperature is

$$T = 100 \,[-1 \pm (1+q_v(\ell^2 - x^2)/100)^{\frac{1}{2}}] \quad . \tag{7}$$

Since heat is flowing into the system, the positive temperature solution will occur.

D. Results

A comparison with NASTRAN results is shown in Table 1.

Table 1. Comparison of theoretical and NASTRAN temperatures for nonlinear heat conduction in an infinite slab.

Grid Point	Theoretical Temperature	NASTRAN Solution
1	73.20	73.13
2	69.56	69.53
3	58.11	58.11
4	36.93	36.93
5	0.00	0.00

E. Driver Decks and Sample Bulk Data

Card No.	
0	NASTRAN FILES=UMF
1	ID DEM3051,NASTRAN
2	UMF 1977 30510
3	APP HEAT
4	SØL 3,1
5	TIME 10
6	CEND
7	TITLE = NØNLINEAR HEAT TRANSFER IN AN INFINITE SLAB
8	SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-5-1
9	ØLØAD = ALL
10	SPCFØRCE = ALL
11	THERMAL(PRINT,PUNCH) = ALL
12	ELFØRCE = ALL
13	TEMPERATURE(MATERIAL) = 201
14	SPC = 350
15	LØAD = 252
16	BEGIN BULK
17	ENDDATA

1	2	3	4	5	6	7	8	9	10
CBAR	1	101	1	2	.0	1.0	.0	1	
CHBDY	5	105	POINT	1					+HBDY5
+HBDY5					-1.0	.0	.0		
CØNRØD	3	2	3	200	3.14159				
CRØD	2	102	3	4					
CTUBE	4	103	4	5					
GRID	1		.0	.0	.0				
MAT4	200	1.0							
MATT4	200	200							
PARAM	EPSHT	.001							HEAT
PARAM	MAXIT	30							HEAT
PBAR	101	200	3.14159						
PHBDY	105		3.14159						
PRØD	102	200	3.14159						
PTUBE	103	200	2.0	.0					
QVØL	252	12.5	1	THRU	4				
SPC	350	5	.0						
TABLEM3	200	.0	1.0						+T200
+T200	.0	1.0	100.0	2.0	ENDT				
TEMPD	201	.0							

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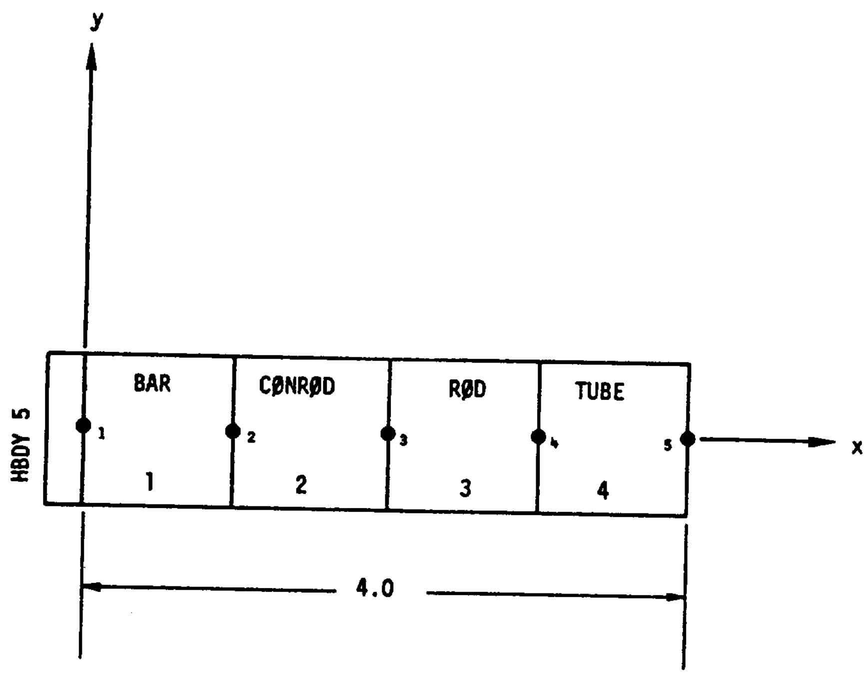

Grid 1 Flux = 0.0
Grid 5 Temperature = 0.0

Figure 1. Slab modeled with linear elements

RIGID FORMAT No. 3, Approach Heat,
Nonlinear Radiation and Conduction of a Cylinder (3-6-1)

A. Description

This problem illustrates the solution of a combined conduction and radiation heat transfer analysis. The model is a two-dimensional representation of a long cylinder subject to radiant heat from a distant source. The shell has internal radiation exchange, external radiation loss, and conduction around the circumference.

B. Input

The NASTRAN Model, shown in Figure 1, uses RØD elements to represent the circumferential heat flow and HBDY elements to represent the inside and outside surfaces. The radiation exchange factors for the inside of the cylinder are defined on the RADMTX data cards. The incoming vector flux is defined on the QVECT data card. The model parameters are:

$R = 2.0$ ft	(Radius of shell)
$t = .001$ ft	(Thickness)
$\ell = 20.306$ ft	(Axial length)
$\varepsilon = \alpha = 0.1$	(Emissivity and absorptivity)
$q_v = 425$ BTU/(ft^2-hr)	(Source flux density)
$k = 94.5$ BTU/(hr-ft-°F)	(Conductivity of shell)
$\sigma = .174 \times 10^{-8}$ BTU/(ft^2-hr-°R^4)	(Stefan-Boltzmann radiation constant)

C. Theory

A closed-form solution to this problem is not available. However, the solution may be validated by checking the global net heat flow, the local net heat exchange, and the estimated average temperature.

An estimate of the average temperature may be obtained from the equations:

$$Q_{in} = \alpha q_v \ell R \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta = 2\alpha\ell R q_v \quad , \qquad (1)$$

and

$$Q_{out} = \varepsilon\sigma\bar{T}^4 \, (2\pi R\ell) \quad , \qquad (2)$$

where Q_{in} is the total input from the source, Q_{out} is the net flux radiated outward and $\bar{T}$ is the average absolute temperature.

Since the net heat flow must be zero in a steady-state analysis, Equations (1) and (2) are equated to obtain:

$$\bar{T}^4 = \frac{q_v}{\pi\sigma} \tag{3}$$

D. Results

The resulting temperature distribution around the circumference of the shell is shown in Figure 2. The average value of temperature from the NASTRAN results shows 57.87° F. The estimated average temperature from Equation (3) above is 68°. The difference is due to the non-uniform radiation effects.

A second check is provided by computing the global net heat flow error in the system. Summing the net flow into each element gives a net heat flow error several orders of magnitude less than the total heat from the source. As a further check, the local net heat flow error at grid point 2 was calculated by summing the contributions from the connected elements. The heat flow terms shown in Figure 3, as calculated by NASTRAN, were:

Q_2 = 59.420 (Flow through RØD #2 (flux · area))

Q_3 = 97.862 (Flow through RØD #3 (flux · area))

Q_{r42} = -133.564 (Inside radiation flow into HBDY #42)

Q_{r43} = -85.352 (Inside radiation flow into HBDY #43)

Q_{r22} = -305.418 (Outside radiation into HBDY #22)

Q_{r23} = -257.930 (Outside radiation into HBDY #23)

Q_{v22} = 481.157 (Vector flux input to HBDY #22)

Q_{v23} = 381.848 (Vector flux input to HBDY #23)

The net flow error into grid point 2 is:

$$\bar{Q}_2 = \frac{1}{2}(Q_{r22} + Q_{r23} + Q_{r42} + Q_{r43} + Q_{v22} + Q_{v23}) + Q_2 - Q_3 = 1.9 \text{ BTU} \tag{4}$$

This error is less than 1% of the total heat flow input at the point.

E. Driver Decks and Sample Bulk Data

Card No.

```
 0   NASTRAN FILES=UMF
 1   ID        DEM3061,NASTRAN
 2   UMF       1977     30610
 3   TIME      5
 4   APP       HEAT
 5   SØL       3,1
 6   CEND

 7   TITLE = NØNLINEAR RADIATIØN AND CØNDUCTIØN ØF A CYLINDER
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-6-1
 9   LØAD = 102
10   TEMP(MATERIAL) = 201
11   ØUTPUT
12   THERMAL = ALL
13   ØLØAD = ALL
14   ELFØRCE = ALL
15   BEGIN BULK
16   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CHBDY	2.	101	LINE	20	1				+B1
+B1					1.0				
CØRD2C	1							1.0	+CØRD1
+CØRD1	1.0								
CRØD	1	100	20	1	2	100	1	2	
GRDSET		1							
GRID	1		2.0	18.					
MAT4	100	94.5	36.7						
PARAM	EPSHT	.001							HEAT
PARAM	MAXIT	20							HEAT
PARAM	SIGMA	.174-8							HEAT
PARAM	TABS	460.							HEAT
PHBDY	101		20.306	.1					
PRØD	100	100	.020306						
QVECT	102	425.	-1.	.0	.0	21	22	23	+Q102
+Q102	24	25	26	27	28	29	30	31	+Q102A
RADLST	21	THRU	40	41	THRU	60			
RADMTX	21	.0	.15643	.30902	.45399	.58779	.70711	.80902	+R21
+R21	.89101	.95106	.98769	1.0	.98769	.95106	.89101	.80902	+R21A
TEMPD	201	200.0							
TLØAD2	105	106			.0	1.+6			

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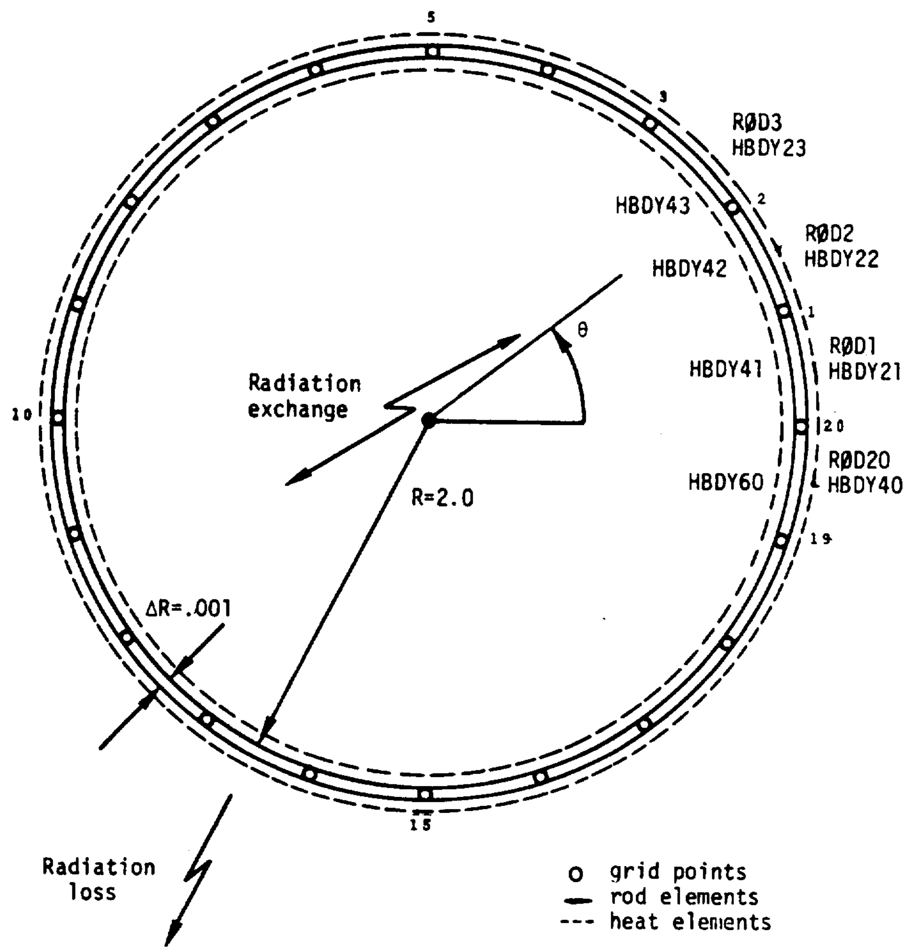


Figure 1. Cross section of thin wall shell.

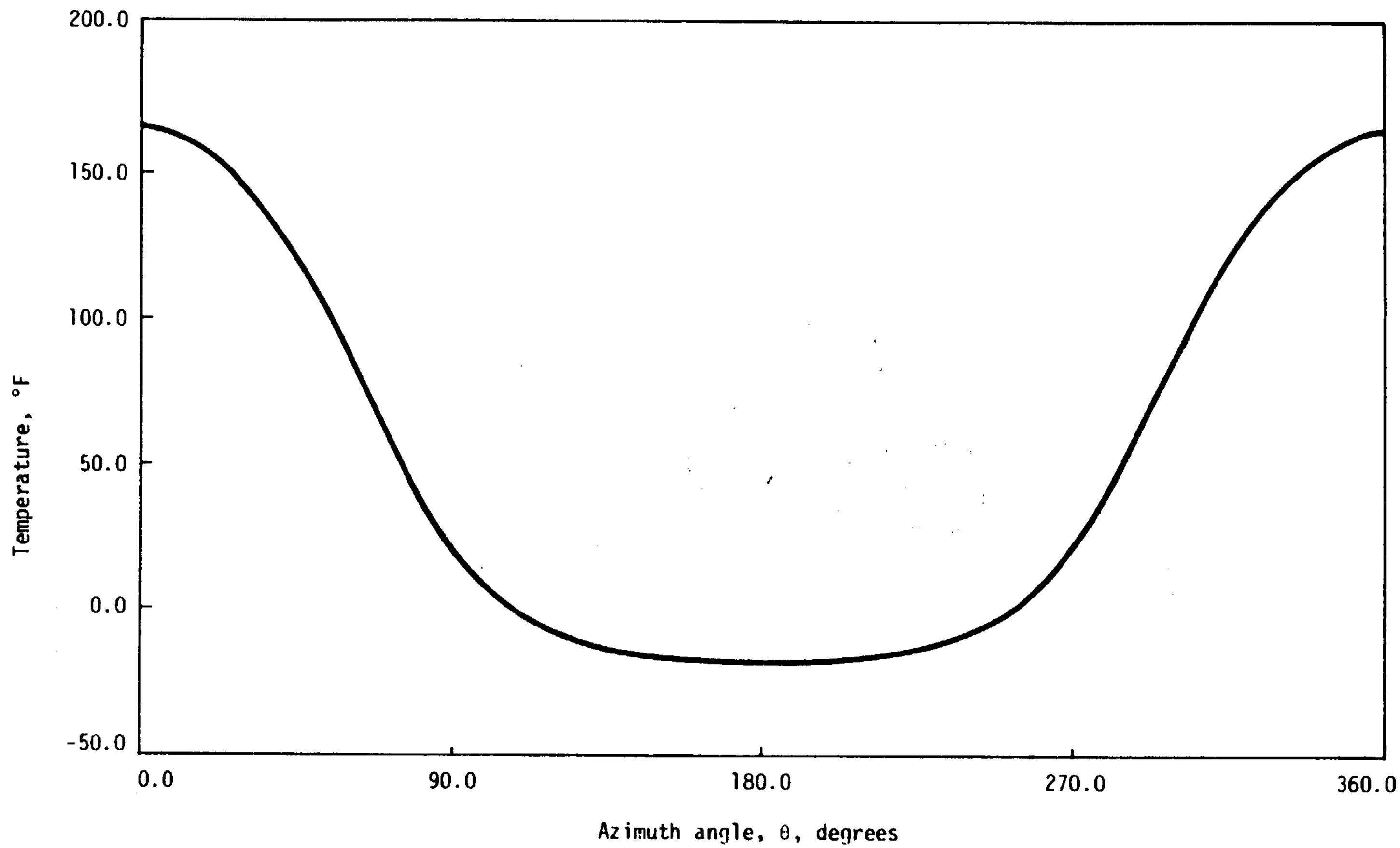


Figure 2. Temperature in stationary cylinder, with conduction and radiation heat transfer.

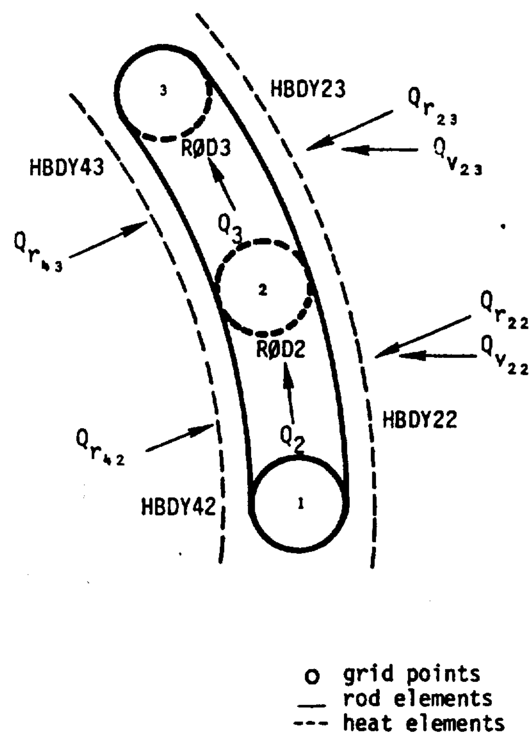


Figure 3. Illustration for heat exchange computation at a grid point.

RIGID FORMAT No. 3, Real Eigenvalue Analysis

Vibrations of a Linear Tapered Cantilever Plate (3-7-1)

A. Description

This problem demonstrates the use of the higher order triangular bending element TRPLT1 to solve a normal modes analysis. The structural model is that of a thin, isotropic plate with tapered cross section and cantilevered at one end. Figure 1 presents the plate geometry and finite element idealization.

B. Input

E	$= 3.0 \times 10^7$ lb/in^2	(Modulus of elasticity)
I_o	$= 4.3877 \times 10^{-5}$ in^4	(Maximum bending inertia)
t_o	$= 0.0807$ in	(Maximum thickness)
a	$= 5.0$ in	(Length)
ν	$= .3$	(Poisson's ratio)
ρ	$= 7.3698$ lb sec^2/in^4	(Mass density)

C. Theory

The theory for the tapered plate elements is developed in Reference 33. In this reference, a frequency parameter Ω is defined as

$$\Omega = \omega\, a^2 \sqrt{\frac{\rho t_o}{D_o}} \quad , \tag{1}$$

where a = length,

ρ = mass density,

ω = circular frequency,

and t_o = thickness.

The bending rigidity, D_o, is defined as

$$D_o = \frac{E t_o^3}{12(1-\nu^2)} \quad . \tag{2}$$

D. Results

The results of the NASTRAN analysis using the TRPLT1 element are presented in Table 1. For purposes of comparison, results are presented from an experiment described by Plunkett in Reference 34. In this table the modes are identified by m and n where m represents the number of nodal lines perpendicular to the support and n represents the number of nodal lines parallel to the support.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM3071,NASTRAN
  2   UMF       1977     30710
  3   APP       DISPLACEMENT
  4   SØL       3,0
  5   TIME      10
  6   CEND

  7   TITLE = VIBRATIØNS ØF A LINEARLY TAPERED CANTILEVERED PLATE
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-7-1
  9   METHØD = 3
 10   SPC = 2
 11   ØUTPUT
 12   VECTØR = ALL
 13   BEGIN BULK
 14   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CTRPLT1	1	6	13	8	3	2	1	9	+TR1
+TR1									
EIGR	3	INV	.0001	1.0	4	4	0		+ABC
+ABC	MAX								
GRDSET							126		
GRID	1		0.0	0.0	0.0				
MAT1	4	3.0+7		.3	7.3698-4				
PARAM	CØUPMASS	1							
PTRPLT1	6	4	4.3977-5		1.0E-10				+TP2
+TP2									
SPC1	2	345	1	2	3	4	5		

Table 1. Frequency Parameters for a Linearly Tapered Rectangular Cantilever Plate; $\nu = 0.3$

Mode		Frequency Parameter $\Omega_{mn} = \omega_{mn} a^2 \left(\frac{\rho t_o}{D_o}\right)^{1/2}$	
m	n	TRPLT1	Experiment
0	0	2.25	2.47
1	0	10.0	10.6
0	1	13.6	14.5
1	1	27.0	28.7
0	2	32.8	34.4
0	3	47.3	47.4
2	0	53.3	52.5
1	2	57.7	54.0

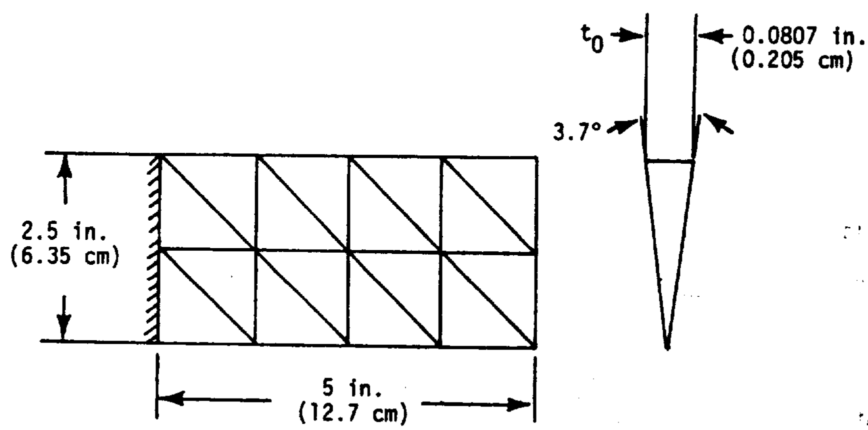


Figure 1. Geometry and element idealization for a cantilevered plate with tapered cross section.

RIGID FORMAT No. 3, Real Eigenvalue Analysis

Vibration of a Helicopter Main Rotor Pylon on a Rigid Body Fuselage (3-8-1)

A. Description

The use of rigid elements in modeling a helicopter main rotor pylon on a rigid body fuselage is illustrated with this problem. The structure to be modeled is shown in Figure 1. The finite element model schematic is presented in Figure 2.

The forces of multipoint constraint created by the rigid elements are recovered using a rigid format alter and the EQMCK module (Reference 35).

B. Input

The details of this model are discussed in Reference 36. In addition to rigid elements, the finite element model utilizes bars, scalar springs, and concentrated masses.

C. Results

The computed normal mode frequencies and generalized masses are presented in Table 1.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0    NASTRAN FILES=UMF
  1    ID         DEM3081,NASTRAN
  2    UMF        1977    30810
  3    APP        DISPLACEMENT
  4    SØL        3,0
  5    DIAG       14
  6    $ ALTER TØ SUPPØRT REQUEST FØR FØRCES ØF MULTI-PØINT CØNSTRAINT
  7    ALTER      109 $
  8    EQMCK      CASECC,EQEXIN,GPL,BGPDT,SIL,USET,KGG,GM,UGV,PGG,QG,CSTM/ØQM1/
  9               C,N,0/C,N,0/C,N,-1 $
 10    ØFP        ØGM1,,,,,//V,N,CARDNØ $
 11    SAVE       CARDNØ $
 12    ENDALTER   $
 13    TIME       14
 14    DIAG       21, 22
 15    CEND

 16    TITLE = HELICØPTER MAIN RØTØR PYLØN ØN A RIGID BØDY FUSELAGE
 17    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 3-8-1
 18    LABEL = NØRMAL MØDES ANALYSIS USING RIGID ELEMENTS
 19    METHØD = 1000
 20    ØUTPUT
 21    ECHØ=BØTH
 22         VECTØR = ALL
 23         MPCFØRCE = ALL
 24    BEGIN BULK
 25    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CBAR	3530251	353025	200070	200078	1.0	.0	.0	1	MR G/B
CELAS2	189831	28125.	189073	1	18983	1			FWD R X
CØNM2	209	209	0	7297.399					BASICWT
+BASICWT	4.7561+6		5.3412+7			5.3697+7			
CRIGD1	353252	200078	200079						
CRIGD2	2091	209	19765	1236					
CRIGD3	200078	200078	123456						+CRG31
+CRG31	MSET	189073	123456	189077	123456	211073	123456		+CRG32
CRIGDR	357000	19765	200078	3					
EIGR	1000	GIV				15			+EIGR
+EIGR	MAX								
GRID	209	0	191.7117	.001757	56.03001	0			
MAT1	1	1.0+6	1.0+6						
ØMIT	200070	456							
PARAM	GRDPNT	0							
PBAR	353025	1	100.	1950.	1950.	1480.			
SUPØRT	209	123456							

Table 1. Results for Helicopter Main Rotor Pylon on Rigid Body Fuselage.

Mode No.	Natural frequencies (Hz)	Generalized masses ($lb\text{-}sec^2/in$)
1	0.0	23.088
2	0.0	23.088
3	0.0	23.088
4	0.0	4.7452
5	0.0	21.991
6	0.0	3051.5
7	2.987	3.058
8	3.372	6.502
9	24.47	.8486
10	26.82	.8414
11	61.54	.5886
12	70.34	.4855
13	113.3	.3867
14	117.4	.3940
15	165.6	1.257

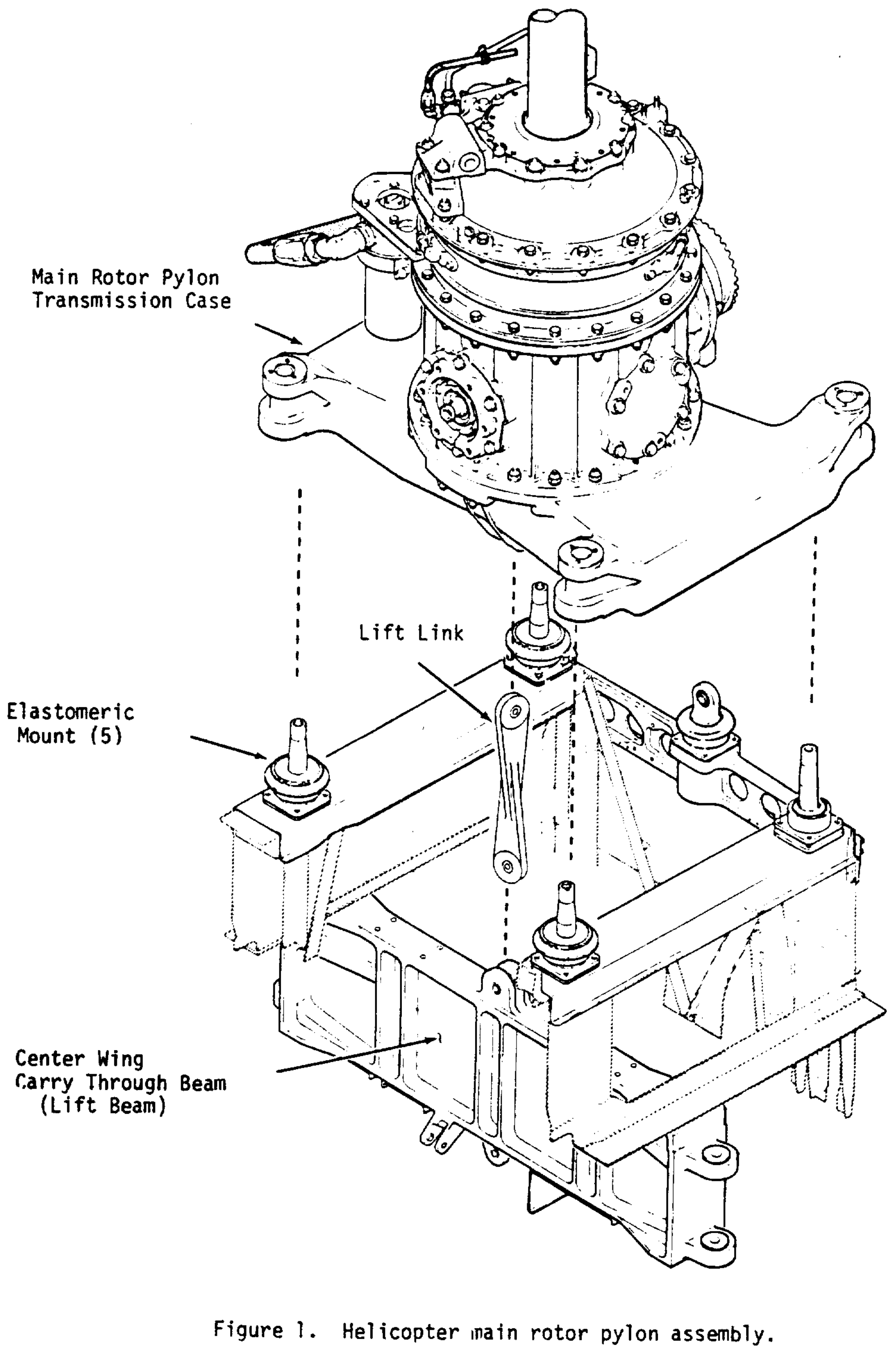


Figure 1. Helicopter main rotor pylon assembly.

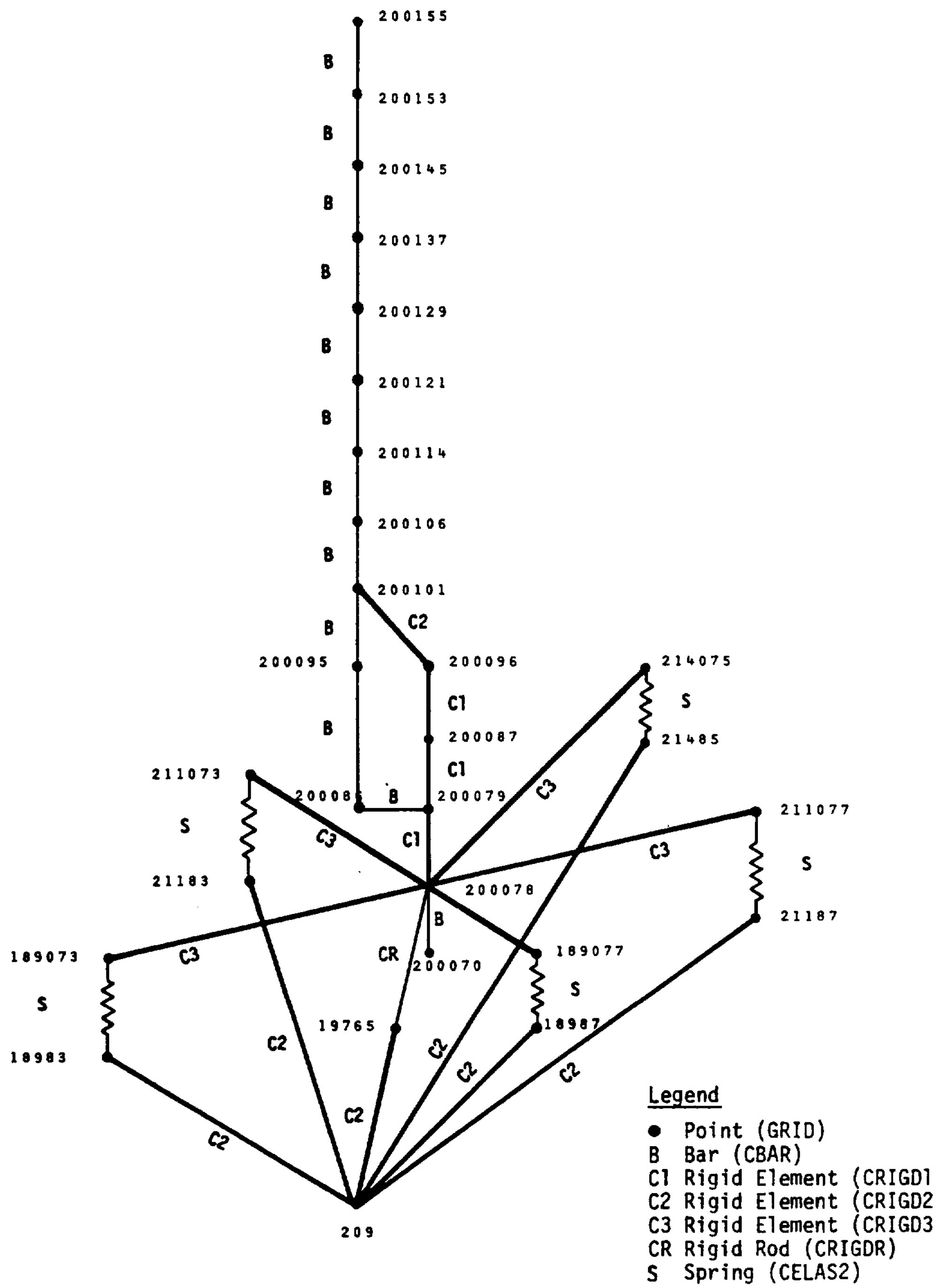


Figure 2. Finite element model schematic.

RIGID FORMAT No. 4, Differential Stiffness Analysis
Differential Stiffness Analysis for a Hanging Cable (4-1-1)

A. Description

NASTRAN provides an iteration procedure for nonlinear differential stiffness (or geometric stiffness) solutions. As described in Section 7 of the NASTRAN Theoretical Manual, the internal loads are recalculated for each iteration. The changes in direction of these internal loads are used to correct the previous solution. External loads retain their original orientation; however, they do travel with the grid point.

A classical nonlinear goemetric problem is that of a hanging cable which assumes the shape of a catenary when a uniform gravity load is applied. As shown in Figure 1, the model is given a circular shape initially. The resulting displacements of the grid points, when added to their original locations, provide a close approximation to the catenary.

B. Input

The NASTRAN model consists of nine BAR elements connected to ten GRID points evenly spaced on a quarter circle. The bending stiffness of the elements is a nominally small value necessary to provide a non-singular, linear solution.

The axial stiffness of the elements is a sufficiently large value to limit extensional displacements. The basic parameters are

$$R = 10.0 \text{ ft} \quad \text{(initial radius)},$$

$$w = 1.288 \text{ lb/ft} \quad \text{(Height per length)},$$

and

$$L = 5\pi.$$

C. Theory

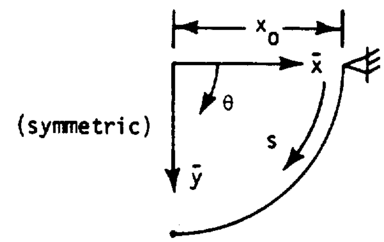
Using the coordinate system illustrated on the next page, the coordinate positions of the initial circular shape are defined by the equations

$$x = R \cos \theta, \tag{1}$$

$$y = R \sin \theta, \tag{2}$$

and

$$s = R\,\theta, \tag{3}$$



where s is the arc length and θ is measured in radians. Solving Equation (3) for θ and substituting into Equations (1) and (2), the expressions for the circular shape are

$$\bar{x} = R \cos\left(\frac{s}{R}\right) \quad , \tag{4}$$

and

$$\bar{y} = R \sin\left(\frac{s}{R}\right) \quad . \tag{5}$$

With reference to the coordinate system illustrated below, the differential equation for the deformed shape (see Reference 25) is

$$\frac{dy'}{dx} = \frac{w}{H}\left(1 + (y')^2\right)^{1/2} \quad , \tag{6}$$

where

w is the weight per unit length,

H is the tension at x = 0,

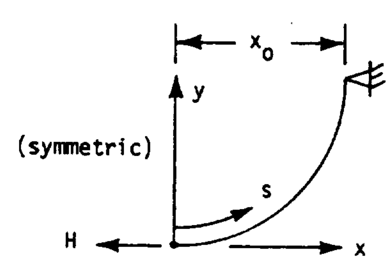
and $y' = dy/dx$ is the slope of the resulting curve.

Dividing both sides of Equation (6) by the radical term and integrating, results in the equation

$$\sinh^{-1} y' = \frac{wx}{H} + C_1 . \tag{7}$$

Since $y' = 0$ at $x = 0$ and $C_1 = 0$, then

$$y' = \sinh\left(\frac{wx}{H}\right) \quad . \tag{8}$$



Integrating again and applying the known boundary condition $y = 0$ at $x = 0$, the equation for the shape is

$$y = \frac{H}{W}\left[\cosh \frac{wx}{H} - 1\right] . \tag{9}$$

Since the length of the cable is known but the horizontal force H is unknown, the two may be related by integrating for the arc length L which is

$$L = \frac{H}{w} \sinh \frac{wx_o}{H} \quad , \tag{10}$$

where x_o is one-half the distance between supports. If w, x_o, and L are given, Equation (10) is solved for H (for $x_o = 10.0$, $w/H = .1719266$) and Equation (9) is evaluated to obtain the actual shape. However, for a given position s along the cable, the coordinates x and y would be

$$x = \frac{H}{w} \sinh^{-1} \left(\frac{ws}{H}\right) , \tag{11}$$

and

$$y = \frac{H}{w}\left[\left(1 + \left(\frac{ws}{H}\right)^2\right)^{1/2} - 1\right] . \tag{12}$$

The location of points on the initial circular shape are defined in the coordinate system used for the deflected shape using

$$x_o = \overline{x} \tag{13}$$

and

$$y_o = R - \overline{y} . \tag{14}$$

The deflections of points on the cable are computed with the equations

$$u_x = x - x_o \tag{15}$$

and

$$u_y = y - y_o . \tag{16}$$

D. Results

NASTRAN and theoretical results are presented in Table 1 below. Deflections are measured from the initial shape at selected locations.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0     NASTRAN FILES=UMF
  1     ID        DEM4011,NASTRAN
  2     UMF       1977     40110
  3     APP       DISP
  4     SØL       4,0
  5     TIME      10
  6     CEND
  7     TITLE = DIFFERENTIAL STIFFNESS ANALYSIS FØR A HANGING CABLE
  8     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 4-1-1
  9     LABEL = INITIAL SHAPE IS A CIRCLE, FINAL SHAPE IS A CATENARY
 10     DISP = ALL
 11     SPCF = ALL
 12     LØAD = 32
 13     SPC = 2
 14     STRESS = ALL
 15     FØRCE = ALL
 16     ØLØAD = ALL
 17     SUBCASE 1
 18     LABEL = LINEAR SØLUTIØN
 19     SUBCASE 2
 20     LABEL = NØNLINEAR SØLUTIØN
 21     BEGIN BULK
 22     ENDDATA
```

1	2	3	4	5	6	7	8	9	10
BARØR					-1.0	1.0	0.0	1	
CBAR	10	10	10	11					
CØRD2C	10	0	.0	.0	.0	.0	.0	1.0	+CS1
+CS1	1.0	.0	.0						
GRAV	32	0	32.2	0.0	1.0	.0			
GRDSET		10				0	345		
GRID	10		10.0	.0					
MAT1	1	5.5+5		.3	.4				
PARAM	BETAD	8							DIFFSTIF
PARAM	NT	18							DIFFSTIF
PARAM	FPSIØ	1.0-5							DIFFSTIF
PBAR	10	1	.1	1.0-6	1.0-6				
SPC	2	10	12	.0	19	1	.0		

Table 1. Comparison of NASTRAN Results to Theoretical Predictions.

Grid Point	s	θ	u_x - Horizontal		u_y - Vertical	
			Theory	NASTRAN	Theory	NASTRAN
11	13.962	10	-.4856	-.4739	-.1119	-.0408
13	10.472	30	-.8043	-.7666	-.2286	-.1269
15	6.981	50	-.5175	-.4612	.0030	.1470
17	3.491	70	-.1110	-.0877	.5698	.7973
19	.0	90	.0	.0	.9338	1.2167

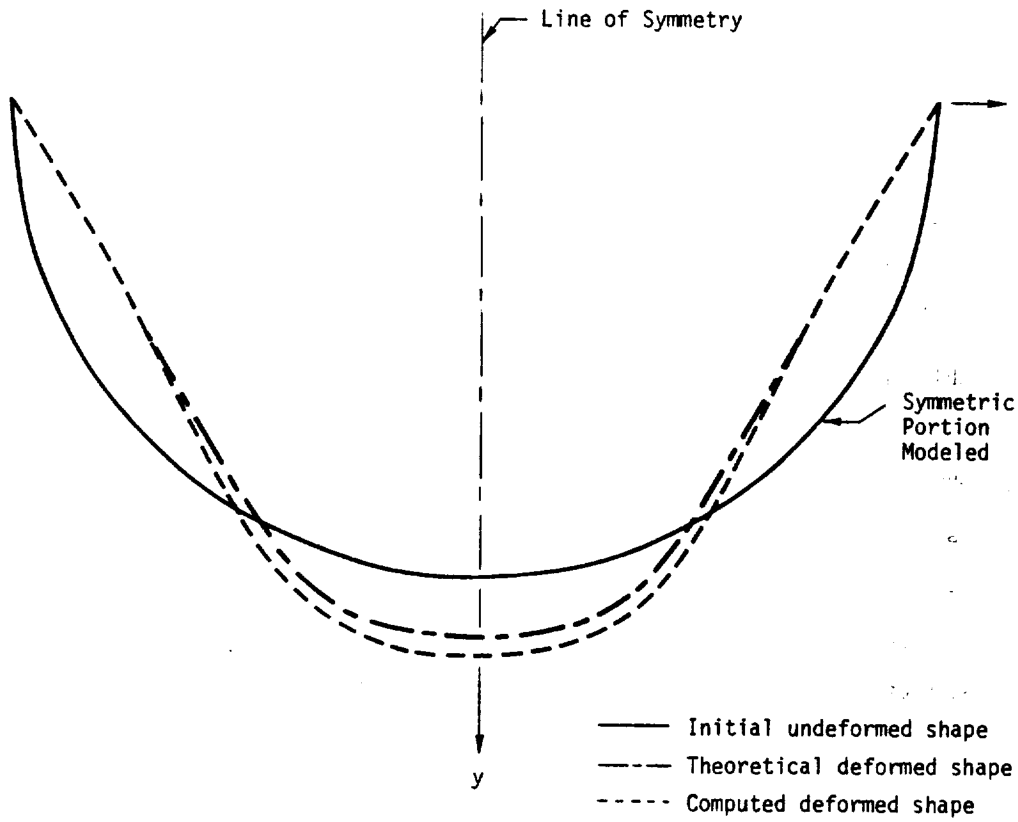


Figure 1. Hanging cable.

RIGID FORMAT No. 5, Buckling Analysis

Symmetric Buckling of a Cylinder (5-1-1)

A. Description

This problem demonstrates the use of buckling analysis to extract the critical loads and the resulting displacements of a cylinder under axial loads. The Buckling Analysis rigid format solves the statics problem to obtain the internal loads in the elements. The internal loads define the differential stiffness matrix $[K^d]$ which is proportional to the applied load. The load factors, λ_i, which causes buckling are defined by the equation:

$$[\lambda_i[K^d] + [K]]\{u_i\} \quad = \quad 0 \quad , \tag{1}$$

where [K] is the linear stiffness matrix. This equation is solved by the Real Eigenvalue Analysis methods for positive values of λ_i. The vectors $\{u_i\}$ are treated in the same manner as in real eigenvalue analysis.

The problem is illustrated in Figure 1; it consists of a short, large radius cylinder under a purely axial compression load. A section of arc of 6 degrees is used to model the axisymmetric motions of the whole cylinder as shown in Figure 2.

All three types of structure plots are requested: undeformed, static and modal deformed. The undeformed perspective plot is fully labeled for checkout of the problem. The modal orthographic plots specify a range of vectors $\{u_i\}$ which includes all roots. A longitudinal edge view of the model is also plotted for easy identification of mode shapes.

B. Input

1. Parameters:

R	=	80	(Radius)
h	=	50	(Height)
E	=	1.0×10^4	(Modulus of elasticity)
ν	=	0.0	(Poissons ratio)
t	=	2.5	(Thickness)
I_b	=	1.30208	(Bending inertia)

2. Loads:

   $p = 1.89745 \times 10^3/3°$ ARC

3. Constraints:

   a) The center point (17) is constrained in u_z.

   c) All points are constrained in u_θ, θ_r, and θ_z.

   d) The top and bottom edges are constrained in u_r.

4. Eigenvalue Extraction Data:

   a) Method: Unsymmetrical Determinant

   b) Region of Interest: $.10 < \lambda < 2.5$

   c) Number of estimated roots = 4

   d) Number of desired roots = 4

   e) Normalization: Maximum deflection

C. Results

The solution to this problem is derived in Reference 9, p. 439. For axisymmetric buckling, the number of half-waves which occur when the shell buckles at minimum load are:

$$m \cong \frac{h}{\pi}\sqrt[4]{\frac{12(1-\nu^2)}{R^2t^2}} \quad , \tag{2}$$

where m is the closest integer to the right-hand values.

The corresponding critical stress is:

$$\sigma_{cr} = \frac{Et^2m^2\pi^2}{12h^2(1-\nu^2)} + \frac{Eh^2}{R^2m^2\pi^2} \quad . \tag{3}$$

Using the values given, the lowest bulkling mode consists of a full sine wave. The NASTRAN results and the theoretical solutions for the critical load for each buckling mode are listed below:

Number of Half Waves m	NASTRAN	ANALYTICAL
1	2.2889	2.2978
2	.99424	1.0
3	1.2744	1.26402
4	2.0070	1.86420

D. Driver Decks and Sample Bulk Data

Card
No.

```
NASTRAN FILES=(UMF,PLT2)
ID       DEM5011,NASTRAN
UMF      1977    50110
APP      DISPLACEMENT
SØL      5,1
TIME     16
CEND

TITLE = SYMMETRIC BUCKLING ØF A CYLINDER
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 5-1-1
SPC = 1
    ØUTPUT
         SET 1 = 1 THRU 33
         SET 2 = 2,6,10,14,18,22,26,30,34,38,42,46,50,54,58,62,66,70,
                 74,78
    DISPLACEMENTS = 1
         SPCFØRCE = ALL
         ELFØRCE = 2
         ELSTRESS = 2
$
SUBCASE 1
LABEL =          STATICS SØLUTIØN
LØAD = 100
    ØUTPUT
         ØLØAD = ALL
$
SUBCASE 2
LABEL =          BUCKLING SØLUTIØN
    METHØD = 300
$
$
PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 5-1-1
ØUTPUT(PLØT)
PLØTTER SC
         SET 1 INCLUDE TRIA1
$
         PERSPECTIVE PRØJECTIØN
         AXES Y, X, MZ
    FIND SCALE,ØRIGIN 1, VANTAGE PØINT
PTITLE = PERSPECTIVE VIEW ØF MØDEL
PLØT LABELS,SYMBØLS 6,5
$
         ØRTHØGRAPHIC PRØJECTIØN
         MAXIMUM DEFØRMATIØN 3.0
    FIND SCALE, ØRIGIN 2
PTITLE = STATIC LØAD UNDERLAY ØF CYLINDRICAL SURFACE
PLØT STATIC DEFØRMATIØN 0,1, ØRIGIN 2, LABELS, SHAPE
PTITLE = MØDE SHAPES ØF CYLINDRICAL SURFACE WITH VECTØRS
PLØT MØDAL DEFØRMATIØN  2, RANGE 0.5, 3.0,
    ØRIGIN 2, VECTØR R, SYMBØLS 5,6
         VIEW 0.0, 0.0, 0.0
    FIND SCALE, ØRIGIN 1
PTITLE = LØNGITUDINAL EDGE VIEW SHØWING BUCKLING MØDES
PLØT MØDAL DEFØRMATIØN 0,2, RANGE 0.0, 200.0, ØRIGIN 1, SHAPE
BEGIN BULK
ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	5	9	13	17	21	25	29	+CNG11
+CNG11	33	37	41	45	49	53	57	61	+CNG12
CØRD2C	100	0	25.0	.0	80.0	50.0	.0	80.0	+CØRD100
+CØRD100	25.0	.0	.0						
CTRIA1	1	200	1	2	51	.0			
EIGB	300	UDET	.10	2.5	4	4	0	1.5E-05	+EIGB300
+EIGB300	MAX								
FØRCE	1	1	100	1.0+3	.0	.0	.5		
GRDSET							462		
GRID	1	100	80.0	-3.0	-25.0	100			
LØAD	100	1.0	1.89745	1					
MAT1	400	10000.00		.0					
PARAM	IRES	1							
PTRIA1	200	400	2.5	400	1.30208				+PTRIA1*
+PTRIA1*	1.51022	0.00							
SEQGP	51	2.5	52	3.5	54	5.5	55	6.5	
SPC	50038	17	3	.0					
SPC1	50037	1	1	2	3	31	32	33	
SPCADD	1	50037	50038						

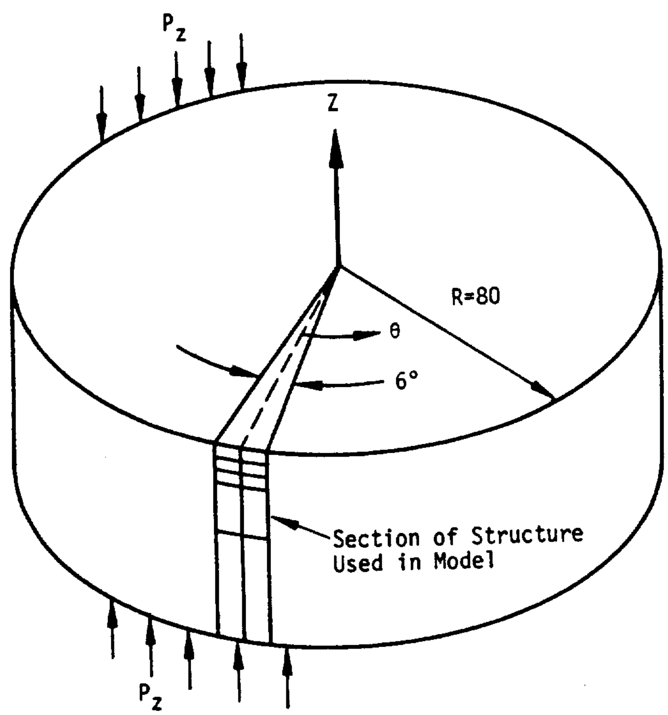


Figure 1. Cylinder under axial load.

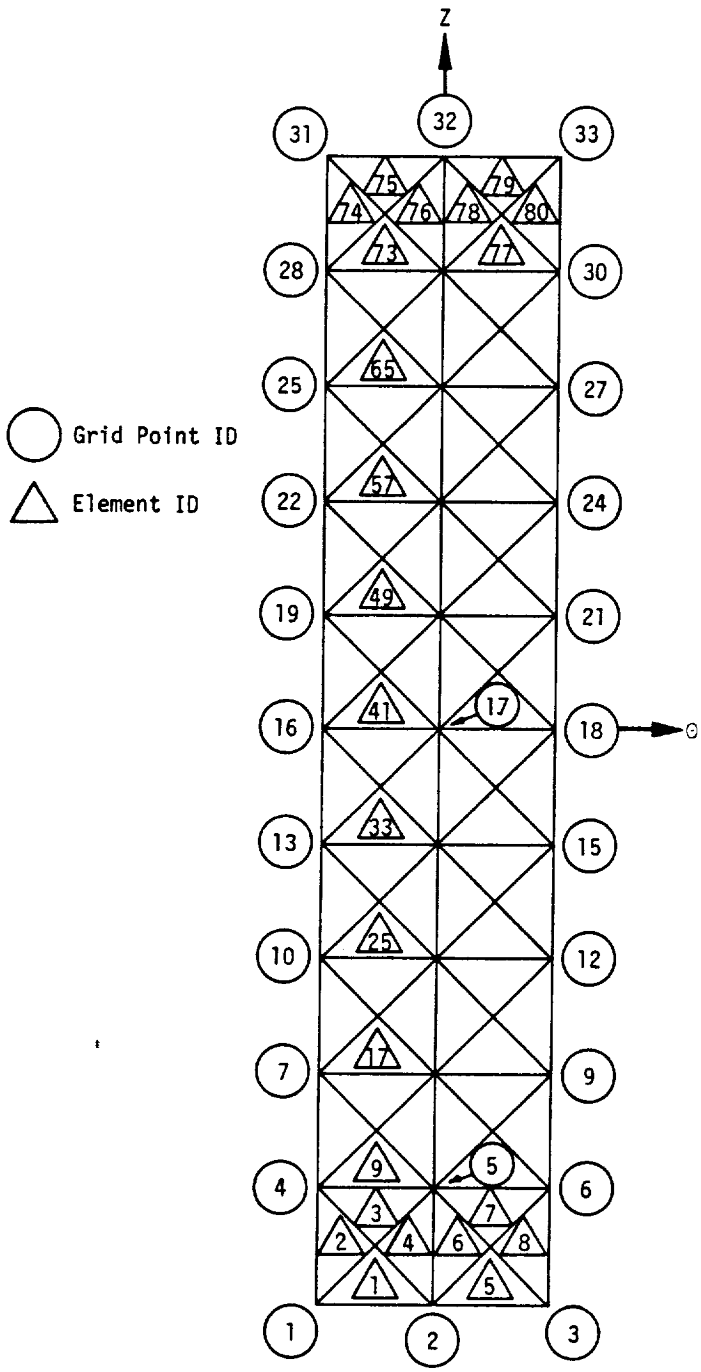


Figure 2. Finite element model of cylinder.

Buckling of a Tapered Column Fixed at the Base (5-2-1)

A. Description

A buckling analysis of a tapered column fixed at the base is presented. The shallow shell element TRSHL, with membrane and bending stiffness combined, is utilized for modeling the column shown in Figure 1(a). The finite element model representation is shown in Figure 1(b) (See Reference 31, pp. 190-194). Note that a vertical plane of symmetry is utilized allowing the model to represent only half the structure.

B. Input

1. Parameters:

$E = 3.0 \times 10^7$	pounds/inch^2	(Young's modulus)
$G = 1.5 \times 10^7$	pounds/inch^2	(Shear modulus)
$L = 3.0$	inches	(Height)
$a = 6.056$	inches	(Length)

The area moment of inertia at any cross section is espressed as

$$I_x = I_1 \left(\frac{x}{a}\right)^4 . \tag{1}$$

Refering to Figure 1, I_1 and I_2 are the moments of inertia at the top (x=a) and bottom (x=0) of the column respectively and $I_1/I_2 = 0.2$. For this problem $3I_1 = 2$ and $3I_2 = 10$. The thickness varies linearly from the top (t = 2.0) to the bottom (t = 3.0) of the column.

2. Constraints:

$\theta_y, \theta_z = 0$	(All grid points)
$x, y, z, \theta_x = 0$	(Grids 1, 2 and 3)
$x = 0$	(Grids 4, 7 10, 13)

3. Loads:

$F_y = -166.66$	(Grids 13 and 15)
$F_y = -666.66$	(Grid 14)

C. Theory

The theoretical solution to this problem is developed on pages 125 - 130 of Reference 23. The reference defines the buckling factor as

$$\lambda = \frac{P_{cr}L^2}{EI_2}\,, \tag{2}$$

where, for this problem, $\lambda = 1.505$.

D. Results

NASTRAN results for this problem, as modeled with the TRSHL element, are presented below.

Buckling Factor $\lambda = \frac{P_{cr}L^2}{EI_2}$	
TRSHL	Theory
1.543	1.505

Table 1. Comparison of NASTRAN and analytical results, clamped-free ends (subcase 1).

CATEGORY	MAXIMUM ANALYTICAL VALUE	MAXIMUM NASTRAN DIFFERENCE	PER CENT ERROR
Displacement	-1.1054×10^{-2}	2.9424×10^{-4}	2.66
Constraint Force	0	*	*
Element Force	0	*	*
Element Stress	$5.1965 \times 10^{+3}$	0.671	0.01

*These results vary with the computer. The very small numbers are essentially zero when compared to subcase 2 results.

Table 2. Comparison of NASTRAN and analytical results, clamped-pinned ends (subcase 2).

CATEGORY	MAXIMUM ANALYTICAL VALUE	MAXIMUM NASTRAN DIFFERENCE	PER CENT ERROR
Displacement	4.3936×10^{-3}	8.024×10^{-6}	0.18
Constraint Force	$-2.2859 \times 10^{+2}$	6.0841	2.66
Element Force	$2.2859 \times 10^{+2}$	6.0846	2.66
Element Stress	$5.1965 \times 10^{+3}$	4.4136×10	0.85

Simply-Supported Rectangular Plate with a Thermal Gradient (1-11-1)
Simply-Supported Rectangular Plate with a Thermal Gradient (INPUT, 1-11-2)

A. Description

This problem illustrates the solution of a general thermal load on a plate with the use of an equivalent linear thermal gradient. The thermal field is a function of three dimensions, demonstrated by the TEMPP1 card. The plate is modeled with the general quadrilateral, QUAD1, elements as shown in Figure 1. Two planes of symmetry are used. This problem is repeated via the INPUT module to generate the QUAD1 elements.

B. Input

E	=	3.0×10^5 pounds/inch^2	(Youngs modulus)
ν	=	0.3	(Poisson's ratio)
ρ	=	1.0 pound-sec.^{2}/inch^4	(Mass density)
α	=	0.01 inch/°F/inch	(Thermal expansion coefficient)
T_R	=	0.0 °F	(Reference temperature)
T_o	=	2.5 °F	(Temperature difference)
a	=	10.0 inch	(Width)
b	=	20.0 inch	(Length)
t	=	0.5 inch	(Thickness)

The thermal field is

$$T = T_o\left(\cos \frac{\pi x}{a}\right)\left(\cos \frac{\pi y}{b}\right)\left(\frac{2z}{t}\right)^3 ,$$

anc

$$= 160.0\left(\cos \frac{\pi x}{10}\right)\left(\cos \frac{\pi y}{20}\right) z^3 \text{ °F} .$$

C. Theory

The plate was solved using a minimum energy solution. The net moments, $\{M_N\}$, in the plate are equal to the sum of the elastic moments, $\{M_e\}$, and the thermal moments, $\{M_t\}$.

$$\{M_N\} = \{M_t\} + \{M_e\} , \tag{1}$$

where the thermal moment is

$$\{M_t\} = \alpha T_o' \, D(1+\nu) \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \quad , \tag{2}$$

and

$$D = \frac{Et^3}{12(1-\nu^2)} \quad .$$

and $T_o' = 6T_o/5t$ is the effective thermal gradient.

The elastic moment is defined by the curvatures, χ, with the eauation:

$$\{M_e\} = D \begin{Bmatrix} \chi_x + \nu\chi_y \\ \chi_y + \nu\chi_x \\ \frac{(1-\nu)}{2} \chi_{xy} \end{Bmatrix} \quad . \tag{3}$$

Assuming a normal displacement function, W, of

$$W = \sum_n \sum_m W_{nm} \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} \quad , \tag{4}$$

then

$$\left.\begin{aligned} \chi_x &= \frac{\partial^2 W}{\partial x^2} = -\sum_n \sum_m \pi^2 W_{nm} \left(\frac{n}{a}\right)^2 \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} \quad , \\ \chi_y &= \frac{\partial^2 W}{\partial y^2} = -\sum_n \sum_m \pi^2 W_{nm} \left(\frac{m}{a}\right)^2 \cos\frac{n\pi x}{a} \cos\frac{m\pi y}{b} \quad , \\ \chi_{xy} &= 2\frac{\partial^2 W}{\partial x \partial y} = 2\sum_n \sum_m \pi^2 W_{nm} \left(\frac{nm}{ab}\right) \sin\frac{n\pi x}{a} \sin\frac{m\pi y}{b} \quad . \end{aligned}\right\} \tag{5}$$

The work done by the thermal load is:

$$U = \int_A \{\chi\}^T \{M_t\} \, dA + \frac{1}{2} \int_A \{\chi\}^T \{M_e\} \, dA \quad , \tag{6}$$

where A is the surface area. Performing the substitution and integrating results in the energy expression:

$$U = -\frac{\alpha T'_o D(1+\nu)\pi^2 (a^2+b^2)}{4ab} W_{11} + \frac{D}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\pi^4 ab}{4} \left(\frac{n^2}{a^2} + \frac{m^2}{b^2}\right)^2 W_{nm}^2 \ . \qquad (7)$$

The static solution exists at a minimum energy:

$$\frac{\partial U}{\partial W_{nm}} = 0 \ . \qquad (8)$$

This results in all but W_{11} equal to zero. The displacement function is therefore:

$$W(x,y) = \frac{\alpha T'_o (1+\nu) a^2 b^2}{\pi^2 (a^2 + b^2)} \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \ . \qquad (9)$$

Solving for moments by differentiating W and using equation (3) results in the equations for element moments:

$$M_x = \alpha T'_o D(1+\nu) \left[1 - \frac{b^2+\nu a^2}{a^2+b^2}\right] \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \ , \qquad (10)$$

$$M_y = \alpha T'_o D(1+\nu) \left[1 - \frac{a^2+\nu b^2}{a^2+b^2}\right] \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \ , \qquad (11)$$

$$M_{xy} = \frac{\alpha T'_o D(1-\nu^2) ab}{a^2 + b^2} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \ . \qquad (12)$$

D. Results

Figure 2 compares the element forces given by the above equation and the NASTRAN results. Figure 3 compares the normal displacements. The maximum errors for displacements, constraint forces, element forces and element stresses are listed in Table 1.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1111,NASTRAN
  2   UMF       1977    11110
  3   APP       DISPLACEMENT
  4   SØL       1,3
  5   TIME      9
  6   CEND

  7   TITLE = SIMPLY SUPPØRTED RECTANGULAR PLATE WITH A THERMAL GRADIENT
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-11-1
  9             SPC = 1
 10             TEMP(LØAD) = 20
 11       ØUTPUT
 12             DISPLACEMENT = ALL
 13             SPCFØRCE = ALL
 14             ELFØRCE = ALL
 15             STRESSES = ALL
 16   BEGIN BULK
 17   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	2	THRU	59					
CQUAD1	1	101	1	2	8	7			
GRDSET							6		
GRID	1		.00	.00	.00				
MAT1	1	3.0+5		.3	1.0	.01	.0		
PARAM	IRES	1							
PQUAD1	101	1	.5	1	.0104167				+PQUAD1
+PQUAD1	.25	-0.25							
SPC1	1	34	6	12	18	24	30	36	+SPC-34
+SPC-34	42	48	54	60	66				
TEMPP1	20	1	.0	5.90786	2.46161	-2.46161			

```
Card
 No.

  0   NASTRAN FILES=UMF
  1   ID        DEM1112,NASTRAN
  2   UMF       1977    11120
  3   APP       DISPLACEMENT
  4   TIME      9
  5   SØL       1,3
  6   DIAG      14
  7   ALTER     1
  8   PARAM     //C,N,NØP/N,N,TRUE=-1  $
  9   INPUT,    ,,,GEØM4,/G1,G2,,G4,/C,N,3/C,N,1  $   QUAD1 ELEMENT
 10   EQUIV     G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE  $
 11   ENDALTER
 12   CEND

 13   TITLE = SIMPLY-SUPPØRTED RECTANGULAR PLATE WITH THERMAL GRADIENT
 14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-11-2
 15             SPC = 5010
 16             TEMP(LØAD) = 20
 17       ØUTPUT
 18             DISPLACEMENT = ALL
 19             SPCFØRCE = ALL
 20             ELFØRCE = ALL
 21             STRESSES = ALL
 22   BEGIN BULK
 23   ENDDATA

 24        5       10      1.0      1.0      6      0.0      0.0
 25      421      125       53       34      0        0
```

1	2	3	4	5	6	7	8	9	10
MAT1	1	3.0+5		.3	1.0	.01	.0		
PQUAD1	101	1	.5	1	.0104167				+PQUAD1
+PQUAD1	.25	-0.25							
TEMPP1	20	1	.0	5.90786	2.46161	-2.46161			
ENDDATA									

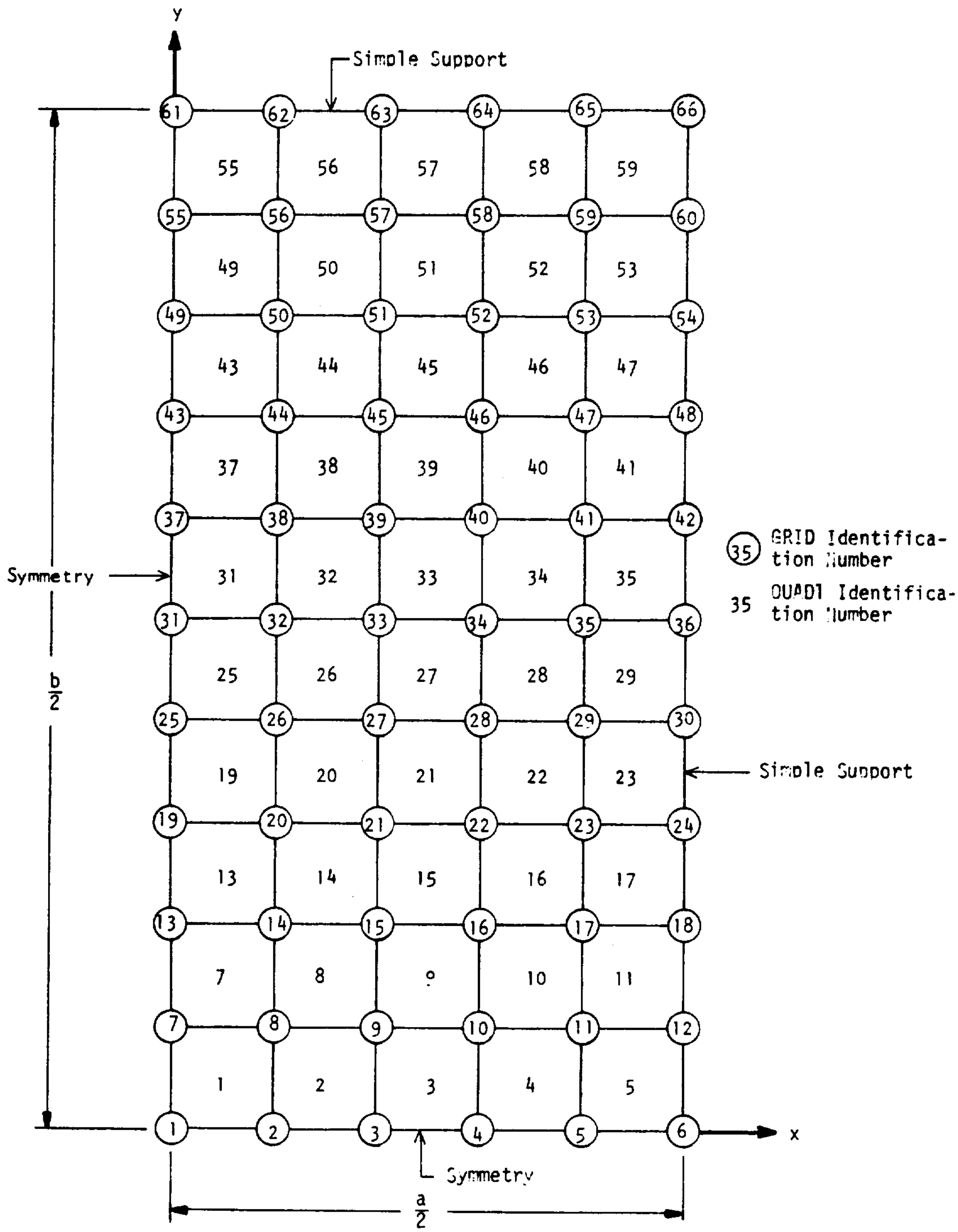


Figure 1. Simply-supported rectangular plate with a thermal gradient.

Table 1. Comparison of analytical and NASTRAN results.

CATEGORY	MAXIMUM ANALYTICAL	MAXIMUM DIFFERENCE	PER CENT ERROR
Displacement	6.2898×10^{-1}	-1.5464×10^{-3}	-0.25
Constraint Force	150.0	-.9594	-0.65
Element Mom., M_x	1.4770×10^{2}	-1.1767	-0.80
Element Stress	7.764618×10^{3}	-90.33275	-1.16

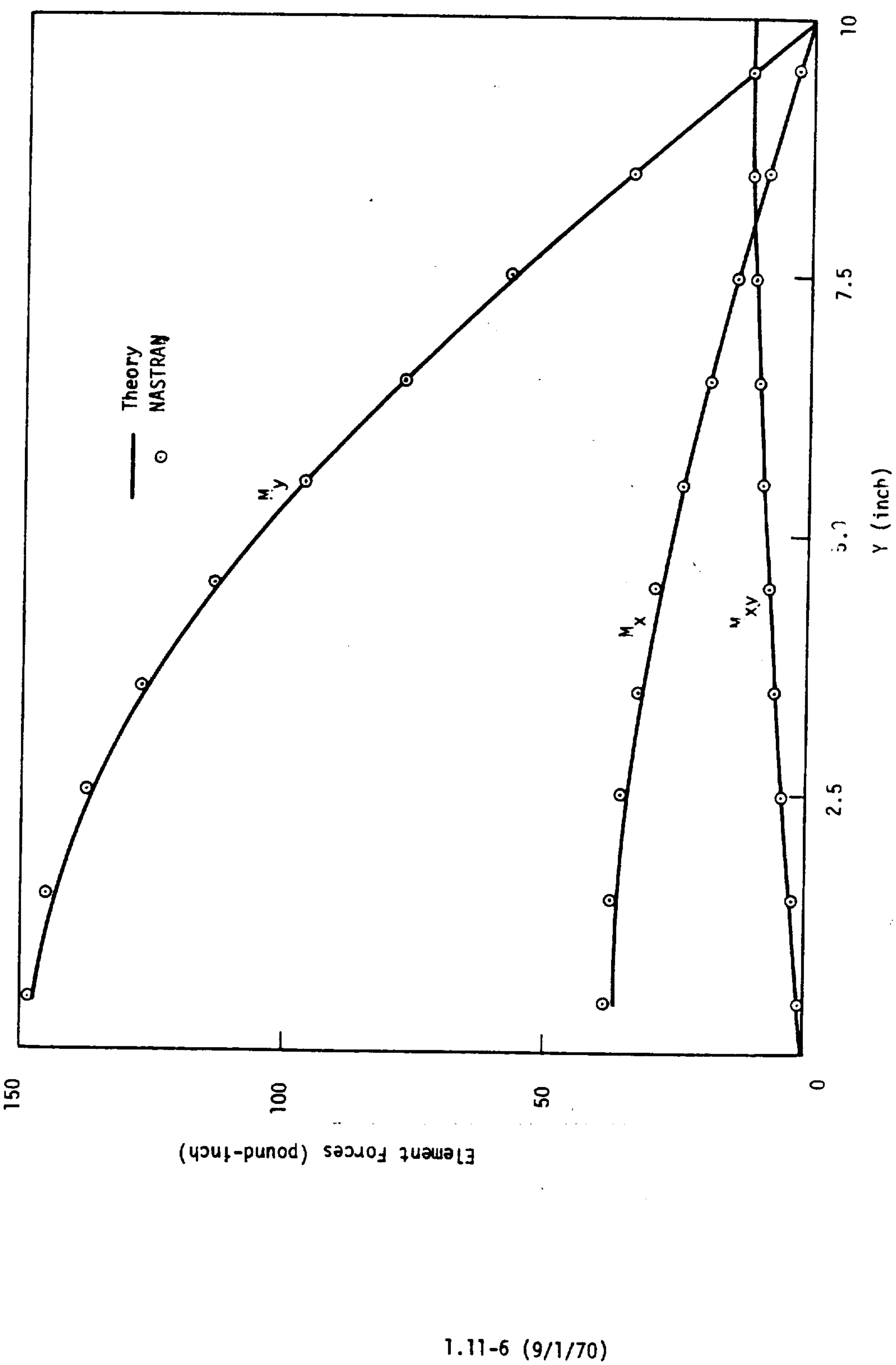


Figure 2. Element forces at x = 0.5.

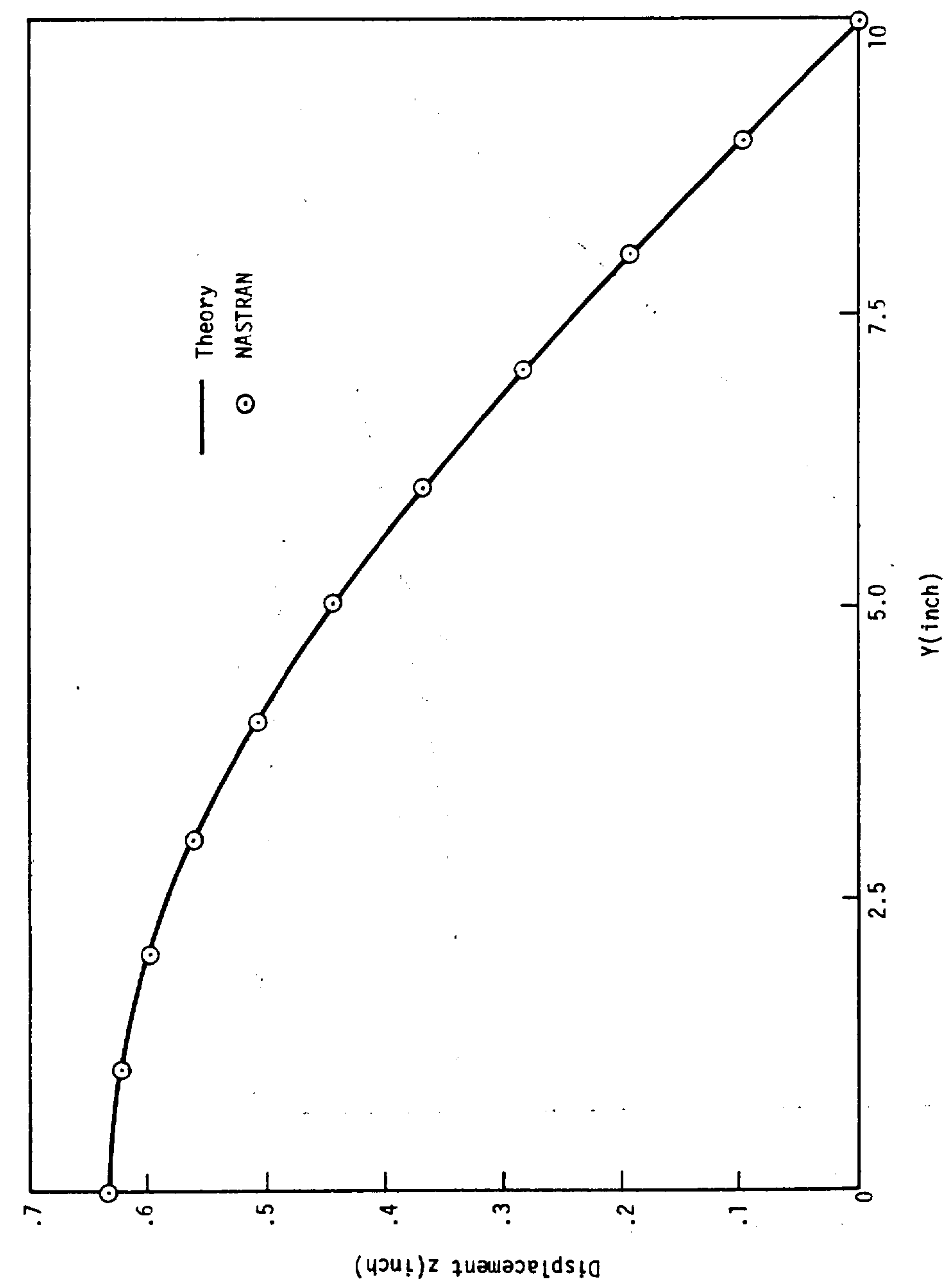


Figure 3. Displacement at x = 5.0.

RIGID FORMAT No. 1 (APP HEAT), Heat Conduction Analysis
Linear Steady State Heat Conduction Through a Washer
Using Solid Elements (1-12-1)
Linear Steady State Heat Conduction Through a Washer
Using Ring Elements (1-12-2)

A. Description

This problem illustrates the capability of NASTRAN to solve heat conduction problems. The washer, shown in Figure 1, has a radial heat conduction with the temperature specified at the outside and a film heat transfer condition at the inner edge. Due to symmetry about the axis and the assumption of negligible axial gradients, the temperature depends only upon the radius.

B. Input

The first NASTRAN model is shown in Figure 2. The solid elements (HEXA1, HEXA2, WEDGE and TETRA) and boundary condition element (HBDY, type AREA4) are used. The conductivity of the material is specified on a MAT4 card. Temperatures are specified at the outer boundary with SPC cards. Punched temperature output is placed on TEMP bulk data cards suitable for static analysis.

Another variation of the problem is shown in Figure 3. Solid of revolution elements (TRIARG and TRAPRG) and boundary condition element (HBDY, type REV) are used. The conductivity of the material and the convective film coefficient are specified on a MAT4 card. The CHBDY card references a scalar point at which the ambient temperature is specified using an SPC card. An SPC1 card is used to constrain the temperature to zero degrees at gridpoints on the outer surface.

C. Theory

The mathematical theory for the continuum is simple, and can be solved in closed form. The differential equation is

$$\frac{1}{r} \frac{\partial}{\partial r} \left(rk\frac{\partial U}{\partial r}\right) = 0 \quad . \tag{1}$$

The boundary conditions are

$$-k\frac{\partial U}{\partial r} = H(U_a - U) \text{ at } r = r_1 \quad , \tag{2}$$

and

$$U = 0 \qquad \text{at } r = r_2 \quad . \tag{3}$$

The solution is

$$U(r) = \frac{HU_a}{(k/r_1) + H \ln(r_2/r_1)} \ln(r_2/r)$$

$$= 288.516 \ln(2/r)$$

D. Results

A comparison with the NASTRAN results is shown in Table 1.

Table 1. Comparison of Theoretical and NASTRAN Temperatures for Heat Conduction in a Washer.

r(radius)	Theoretical Temperatures	NASTRAN Temperatures (Solids)*	NASTRAN Temperatures (Rings)*
1.0	199.984	202.396	199.932
1.1	172.486	173.904	172.448
1.2	147.381	148.833	147.355
1.3	124.288	124.783	124.269
1.4	102.906	102.852	102.894
1.5	83.001	82.913	82.992
1.6	64.380	64.306	64.375
1.7	46.889	46.832	46.886
1.8	30.398	30.356	30.397
1.9	14.799	14.773	14.798
2.0	0.000	0.000	0.000

*These are the average temperatures at a radius.

E. Driver Decks and Sample Bulk Data

Card No.	
0	NASTRAN FILES=UMF
1	ID DEM1121,NASTRAN
2	UMF 1977 11210
3	TIME 1
4	APP HEAT
5	SØL 1,1
6	CEND
7	TITLE = LINEAR STEADY STATE HEAT CØNDUCTIØN THRØUGH A WASHER
8	SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-12-1
9	LABEL = SØLID ELEMENTS,SURFACE FILM HEAT TRANSFER
10	ØLØAD = ALL
11	SPCFØRCES = ALL
12	THERMAL(PRINT,PUNCH) = ALL
13	ELFØRCE = ALL
14	SUBCASE 123
15	LABEL = TEMPERATURE SPECIFIED AT ØUTER BØUNDARY
16	SPC = 351
17	LØAD = 251
18	BEGIN BULK
19	ENDDATA

1	2	3	4	5	6	7	8	9	10
CHBDY	701	702	AREA4	1	12	112	101		
CHEXA1	1	200	1	2	13	12	101	102	+SØL1
+SØL1	113	112							
CHEXA2	2	200	2	3	14	13	102	103	+SØL2
+SØL2	114	113							
CØRD2C	111	0	.0	.0	.0	.0	.0	100.0	+CØRD111
+CØRD111	100.0	.0	.0						
CTETRA	3	200	104	114	3	103			
CWEDGE	8	200	4	5	15	104	105	115	
GRDSET						111			
GRID	1	111	1.0	.0	.0				
MAT4	200	1.0							
PARAM	IRES	1							
PHBDY	702								
QBDY1	251	288.5	701						
SEQGP	12	1.1	13	2.1	14	3.1	15	4.1	
SPC	351	11	1	.0	22	1	.0		

Card No.

```
0   NASTRAN FILES=UMF
1   ID        DEM1122,NASTRAN
2   UMF       1977     11220
3   APP       HEAT
4   SØL       1,0
5   TIME      10
6   CEND

7   TITLE = LINEAR STEADY STATE CØNDITIØN THRØUGH A WASHER
8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-12-2
9   LABEL = RING ELEMENTS, FILM HEAT TRANSFER
10  ØUTPUT
11  ØLØAD = ALL
12  SPCFØRCE = ALL
13  THERMAL (PRINT,PUNCH) = ALL
14  ELFØRCE = ALL
15  SPC = 350
16  BEGIN BULK
17  ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CHBDY	14	100	REV	1	12				+HBDY14
+HBDY14	23	23							
CTRAPRG	7	4	5	16	15	.0	200		
GRID	1		1.0	.0	.0				
MAT4	200	1.0							
PHBDY	100	300							
SEQGP	12	1.1	13	2.1	14	3.1	15	4.1	
SPC	352	23		488.5					
SPC1	351	1	11	22					
SPCADD	350	351	352						
SPØINT	23								
TEMPD	201	.0							

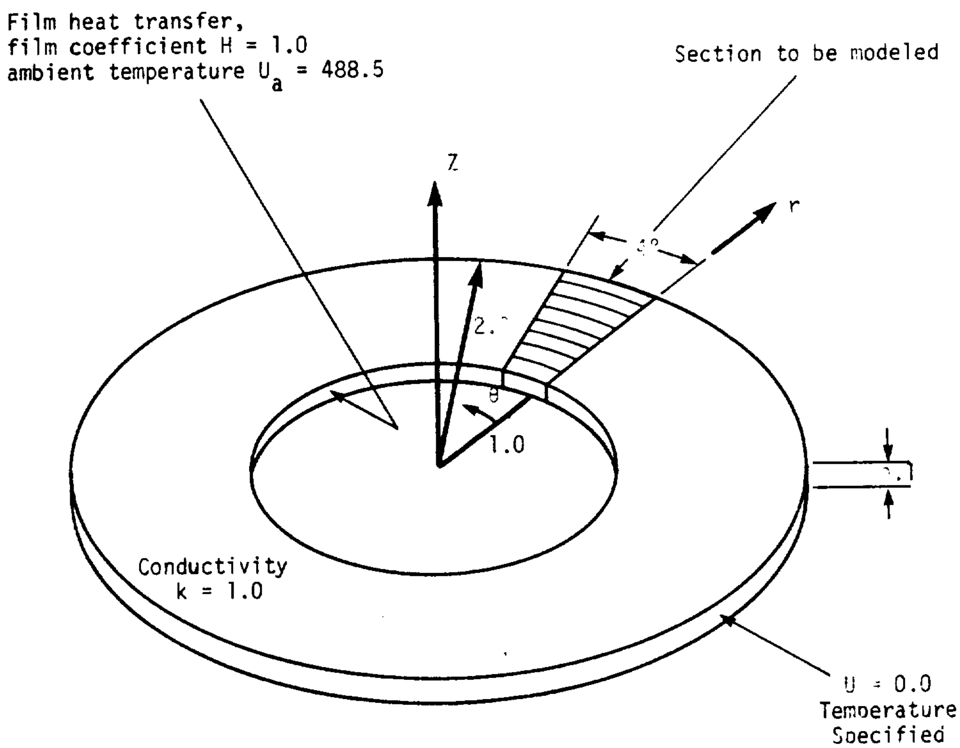
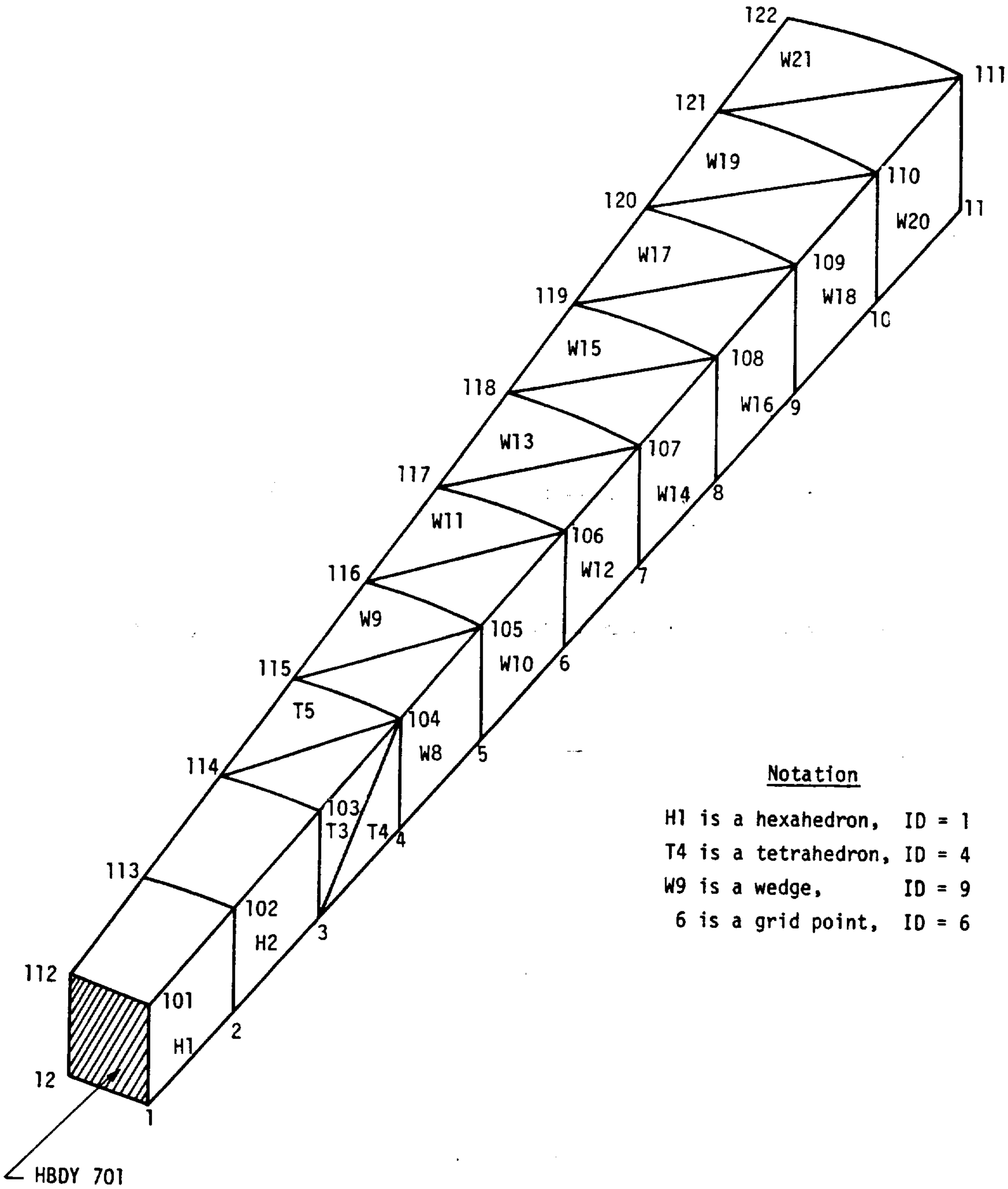


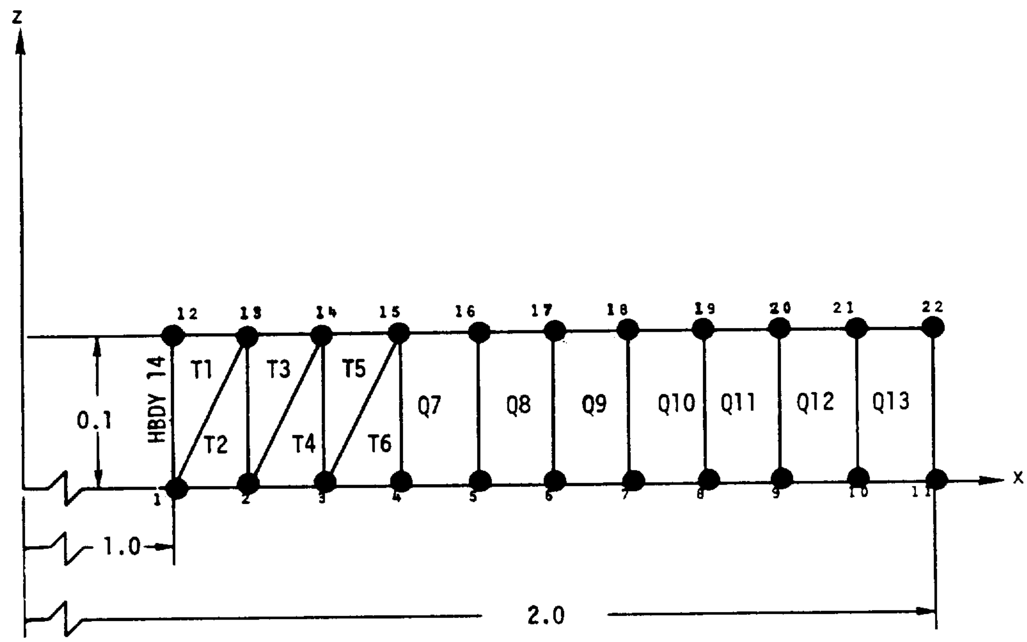
Figure 1. Washer Analyzed in Heat Conduction Demonstration Problem



<u>Notation</u>

H1 is a hexahedron, ID = 1
T4 is a tetrahedron, ID = 4
W9 is a wedge, ID = 9
6 is a grid point, ID = 6

Figure 2. Elements and Grid Points



T TRIARG elements

Q TRAPRG elements

U_a = 488.5 at left end

U_a = 0.0 at right end

Figure 3. Section of a pipe, modeled with ring elements

RIGID FORMAT No. 1, Static Analysis

Thermal and Pressure Loads on a Long Pipe Using Linear Isoparametric Elements (1-13-1)
Thermal and Pressure Loads on a Long Pipe Using Quadratic Isoparametric Elements (1-13-2)
Thermal and Pressure Loads on a Long Pipe Using Cubic Isoparametric Elements (1-13-3)

A. Description

These problems demonstrate the use of the linear, quadratic and cubic isoparametric solid elements, IHEX1, IHEX2 and IHEX3, respectively. A long pipe, assumed to be in a state of plane strain, is subjected to an internal pressure and a thermal gradient in the radial direction. The structure modeled is shown in Figure 1. The finite element NASTRAN models for each of the elements are shown in Figures 2, 3 and 4.

B. Input

1. Parameters:

$r_{inner} = a = 4$ in.	(radius to the inner surface)
$r_{outer} = b = 5$ in.	(radius to the outer surface)
$E = 30.\text{x}10^6$ psi	(Young's Modulus)
$\nu = 0.3$	(Poisson's Ratio)
$\alpha = 1.428 \times 10^{-5}$	(thermal expansion coefficient)
$\rho = 7.535 \times 10^{-4} \dfrac{\text{lb-sec}^2}{\text{in}^4}$	(mass density)
$p = 10$ psi	(inner surface pressure)
$T_i = 100.0°F$	(inner surface temperature)
$T_o = 0.0°F$	(outer surface temperature)

2. Boundary Conditions:

$u_\theta = 0$ at all points on the right side

$u_\theta = 0$ at all points on the left side

$u_z = 0$ at all points on the bottom surface

$u_z = 0$ at all points on the top surface

3. Loads:

Subcase 1,

$p = 10$ psi (internal pressure)

Subcase 2,

$$T_r = \frac{(T_i - T_o)}{\ln(\frac{b}{a})} \ln(\frac{b}{r}) = \frac{100}{\ln(1.25)} \ln(\frac{5}{r})$$, where r is any radius.

C. Theory

1. Subcase 1

The normal stresses due to the pressure load (Reference 24) are obtained by

$$\sigma_r = -\frac{a^2 b^2}{(b^2 - a^2)} \frac{p}{r^2} + \frac{pa^2}{(b^2 - a^2)} , \tag{1}$$

$$\sigma_\theta = \frac{a^2 b^2}{(b^2 - a^2)} \frac{p}{r^2} + \frac{pa^2}{(b^2 - a^2)} , \tag{2}$$

and

$$\sigma_z = 2\nu \frac{pa^2}{(b^2 - a^2)} . \tag{3}$$

where r is the radius and all shearing stresses are zero.

The displacement in the radial direction is

$$u_r = \frac{(1-2\nu)(1+\nu)}{E} r \frac{pa^2}{(b^2 - a^2)} + \frac{(1+\nu)}{E} \frac{1}{r} \frac{pa^2 b^2}{(b^2 - a^2)} , \tag{4}$$

and all other displacements are zero.

2. Subcase 2

The stresses in the radial and tangential directions due to the thermal load (Reference 24) are given by

$$\sigma_r = \frac{\alpha E T_i}{2(1-\nu)\ln(\frac{b}{a})} \left[-\ln(\frac{b}{r}) - \frac{a^2}{(b^2 - a^2)} (1 - \frac{b^2}{r^2}) \ln(\frac{b}{a}) \right] , \tag{5}$$

and

$$\sigma_\theta = \frac{\alpha E T_i}{2(1-\nu)\ln(\frac{b}{a})} \left[1 - \ln(\frac{b}{r}) - \frac{a^2}{(b^2 - a^2)} (1 + \frac{b^2}{r^2}) \ln(\frac{b}{a}) \right] . \tag{6}$$

The stress in the axial direction is obtained via the procedure contained in the reference as

$$\sigma_z = \frac{\alpha E T_i}{2(1-\nu)\ln(\frac{b}{a})} \left[\nu - \frac{2a^2\nu}{(b^2-a^2)} \ln(\frac{b}{a}) - 2 \ln(\frac{b}{r}) \right] . \tag{7}$$

All shearing stresses are zero.

The displacement in the radial direction is

$$u_r = \frac{(1 + \nu)}{(1 + \nu)} \alpha \frac{T_i}{\ln(\frac{b}{a})} \left\{ - \frac{1}{r} \left[\frac{a^2b^2}{2(b^2-a^2)} \ln(\frac{b}{a}) \right] \right.$$

$$\left. + \frac{r}{4} \left[2 \ln(\frac{b}{r}) + 1 + (1-2\nu) \left(1 - \frac{2a^2}{(b^2-a^2)} \ln(\frac{b}{a}) \right) \right] \right\} . \tag{8}$$

D. Results

Representative displacements and stresses for the finite element results compared to theoretical predictions are plotted in Figures 5 and 6. Note that five IHEX1 elements were used along the radial thickness whereas one element was used for each of the IHEX2 and IHEX3 cases. Two values for the stress occur at the boundary of two adjacent IHEX1 elements resulting in a sawtooth pattern.

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E. Driver Decks and Sample Bulk Data

```
Card
 No.

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 1   ID        DEM1131,NASTRAN
 2   UMF       1977     11310
 3   APP       DISPLACEMENT
 4   SØL       1,0
 5   TIME      5
 6   CEND

 7   TITLE = LØADS ØN A LØNG PIPE USING LINEAR ISØPARAMETRIC ELEMENTS
 8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-13-1
 9   DISP = ALL
10   STRESS = ALL
11   SPC = 100
12   SUBCASE 1
13   LABEL = PRESSURE LØAD
14   LØAD = 400
15   SUBCASE 2
16   LABEL = THERMAL LØAD
17   TEMP(LØAD) = 500
18   BEGIN BULK
19   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CIHEX1	1	200	1	2	20	19	7	8	+HEX1-1
+HEX1-1	26	25							
CNGRNT	1	6	11	16	21	26	31	36	
CØRD2C	1	0	.0	.0	.0	.0	.0	100.0	+CØRD2-1
+CØRD2-1	100.0	.0	.0						
GRDSET		1				1	456		
GRID	1		4.0	-14.0					
MAT1	300	3.+7		.3	7.535-4	1.428-5	.0		
PIHEX	200	300		4	4.5	10.0			
PLØAD3	400	-10.0	1	1	25	21	7	31	
SPC1	100	2	1	THRU	18				
TEMP	500	1	100.0	7	100.0	13	100.0		
TEMPD	500	.0							

Card No.

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2 UMF 1977 11320
3 APP DISPLACEMENT
4 SØL 1,0
5 TIME 5
6 CEND

7 TITLE = LØADS ØN A LØNG PIPE USING QUADRATIC ISØPARAMETRIC ELEMENTS
8 SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-13-2
9 DISP = ALL
10 STRESS = ALL
11 SPC = 200
12 SUBCASE 1
13 LABEL = PRESSURE LØAD
14 LØAD = 400
15 SUBCASE 2
16 LABEL = THERMAL LØAD
17 TEMP(LØAD) = 500
18 BEGIN BULK
19 ENDDATA

1	2	3	4	5	6	7	8	9	10
CIHEX2	1	200	1	2	3	10	15	14	+HEX-1
+HEX-1	13	9	4	5	17	16	6	7	+HEX-11
+HEX-11	8	12	20	19	18	11			
CNGRNT	1	2							
CØRD2C	10	0	.0	.0	.0	.0	.0	100.0	+CRD-1
+CRD-1	100.0	.0	.0						
GRDSET		10				10	456		
GRID	1		4.0	-14.0	.0				
MAT1	300	3.+7		.3	7.535-4	1.428-5	.0		
PIHEX	200	300		4					
PLØAD3	400	-10.0	1	13	6	2	25	18	
SPC1	200	2	1	THRU	8				
TEMP	500	1	100.0	4	100.0	6	100.0		
TEMPD	500	.0							

```
Card
 No.

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  2   UMF       1977    11330
  3   APP       DISPLACEMENT
  4   SØL       1,0
  5   TIME      3
  6   CEND

  7   TITLE = LØADS ØN A LØNG PIPE USING CUBIC ISØPARAMETRIC ELEMENTS
  8   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 1-13-3
  9   DISPLACEMENT = ALL
 10   STRESS = ALL
 11   SPC = 200
 12   SUBCASE 1
 13   LABEL = PRESSURE LØAD
 14   LØAD = 80
 15   SUBCASE 2
 16   LABEL = THERMAL LØAD
 17   TEMP(LØAD) = 90
 18   BEGIN BULK
 19   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CIHEX3	10	60	1	2	3	4	5	6	+HEX-31
+HEX-31	7	8	9	10	11	12	13	14	+HEX-32
CØRD2C	111	0	.0	.0	.0	.0	.0	50.0	+COR1
+CØR1	50.0	.0	.0						
GRDSET		111				111	456		
GRID	1		4.0	.0	.0				
MAT1	70	3.+7		.3	7.535-4	1.428-5	.0		
PIHEX	60	70		4					
PLØAD3	80	-10.0	10	30	1				
SPC1	200	2	1	2	3	4	13	14	+SPC-A2
+SPC-A2	17	18	21	22	23	24	7	8	+SPC-B2
TEMP	90	1	100.0	12	100.0	11	100.0		
TEMPD	90	.0							

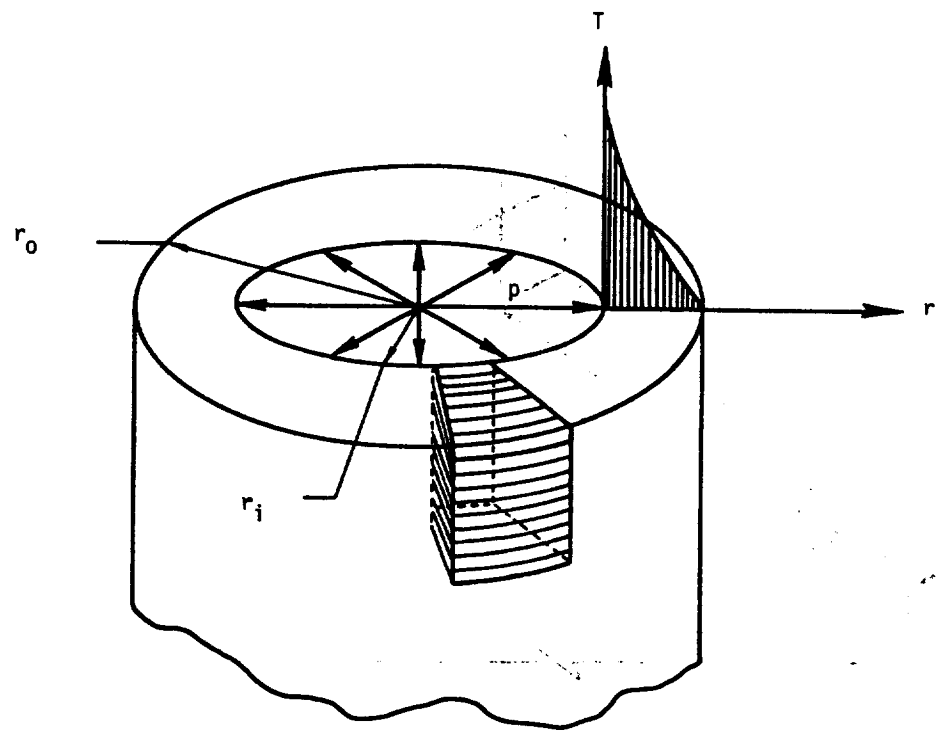


Figure 1. Long pipe with pressure and thermal loads.

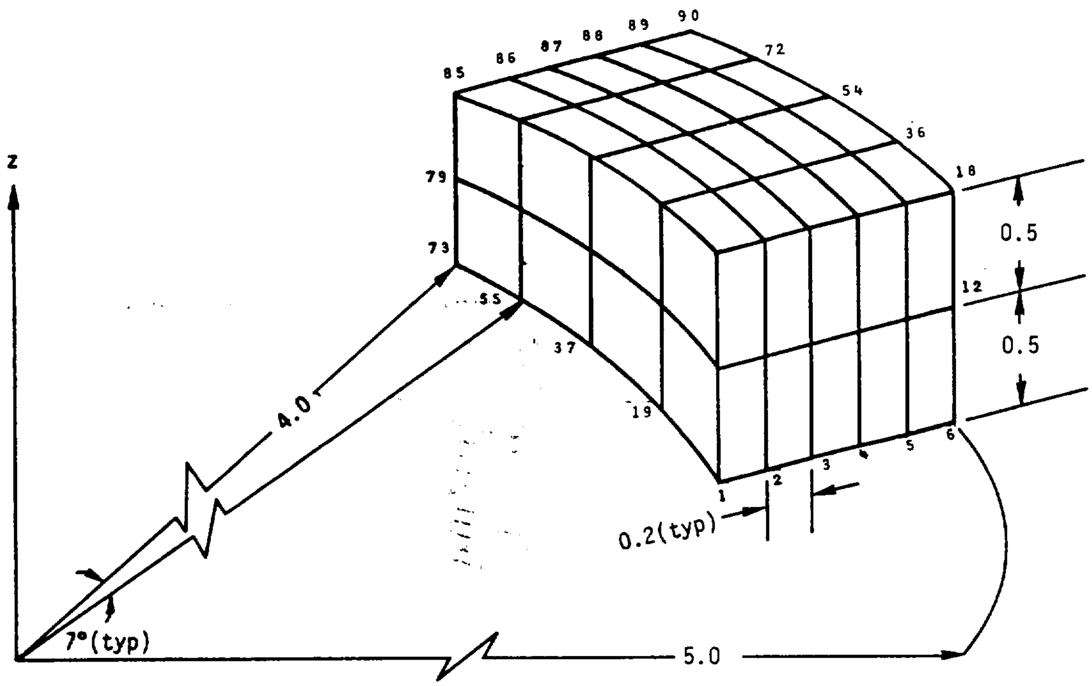


Figure 2. Model of section using forty IHEX1 elements.

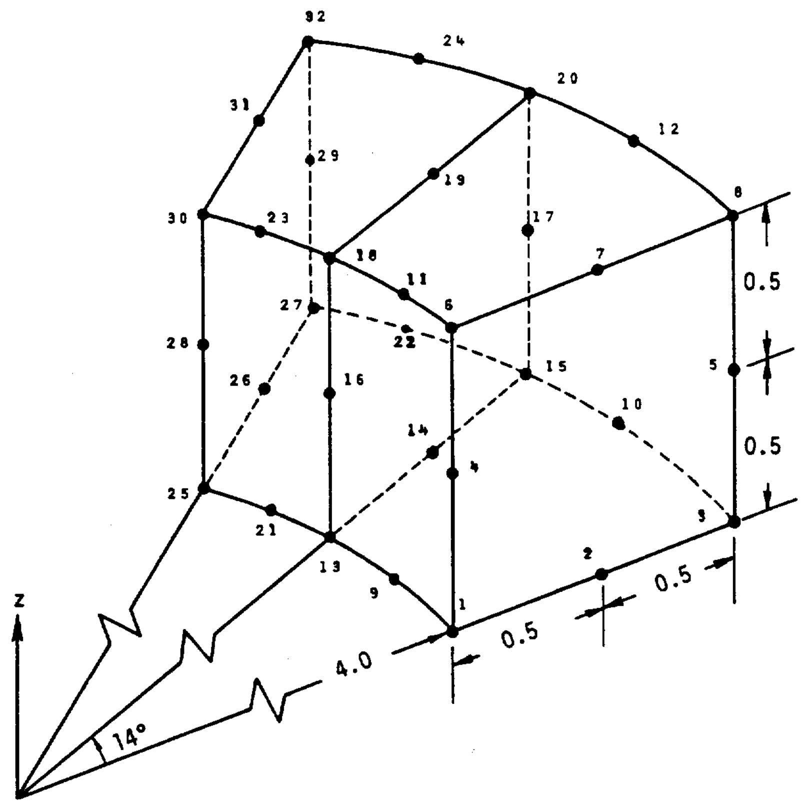


Figure 3. Model of section using two IHEX2 elements.

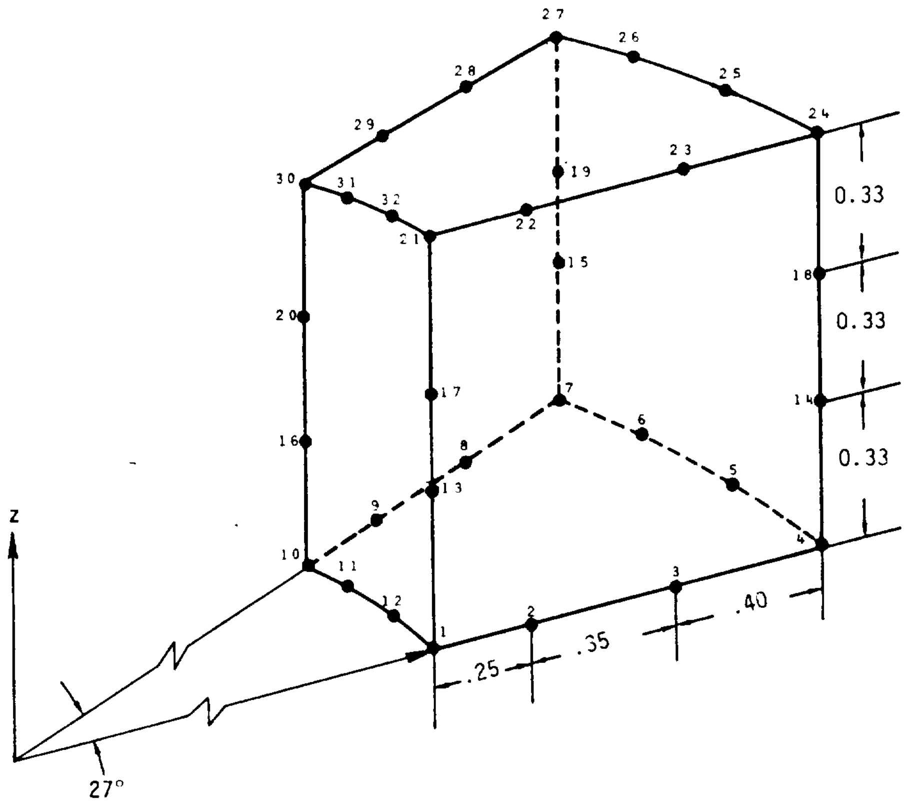
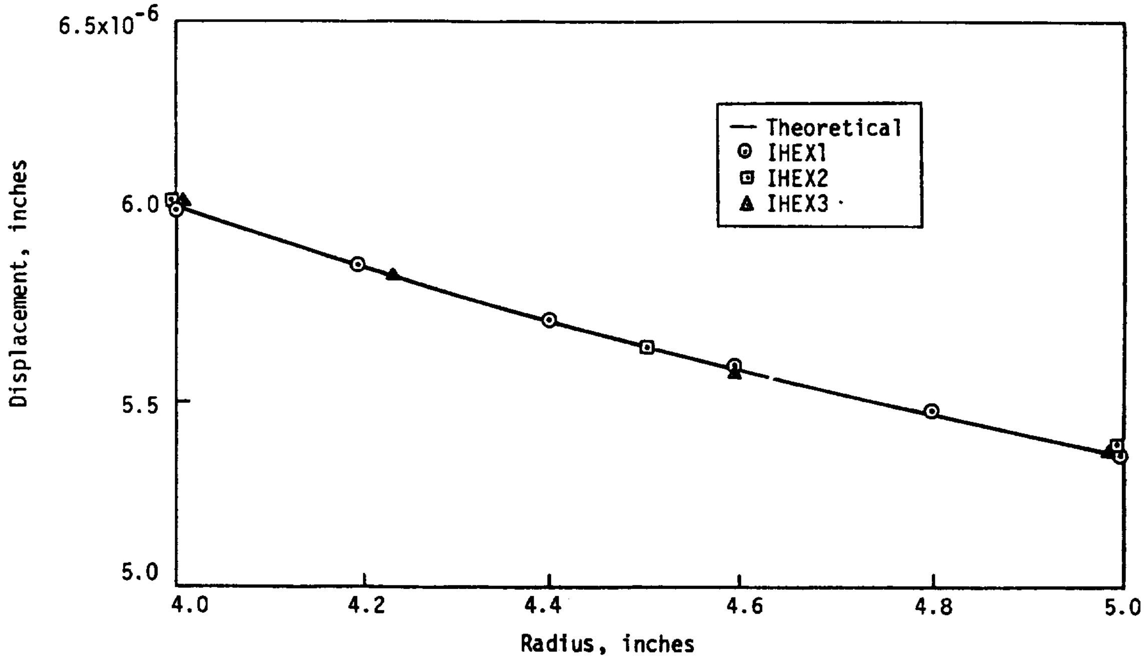
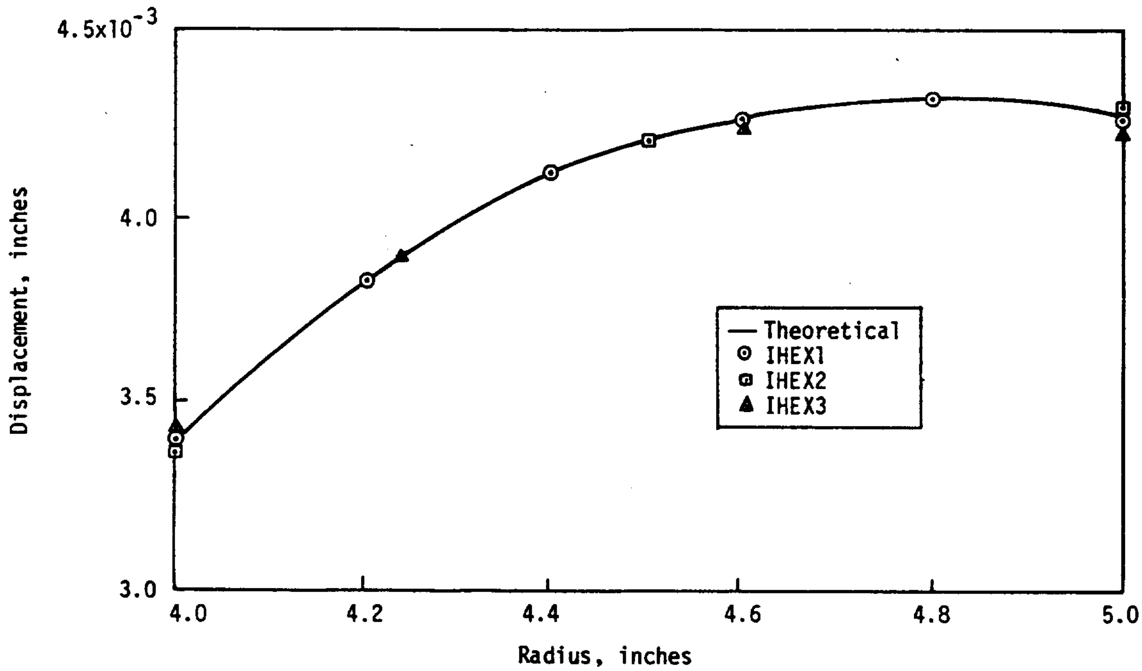


Figure 4. Model of section using one IHEX3 element.



(a) Radial deflections, pressure load.



(b) Radial deflections, thermal load.

Figure 5. Deflection comparisons.

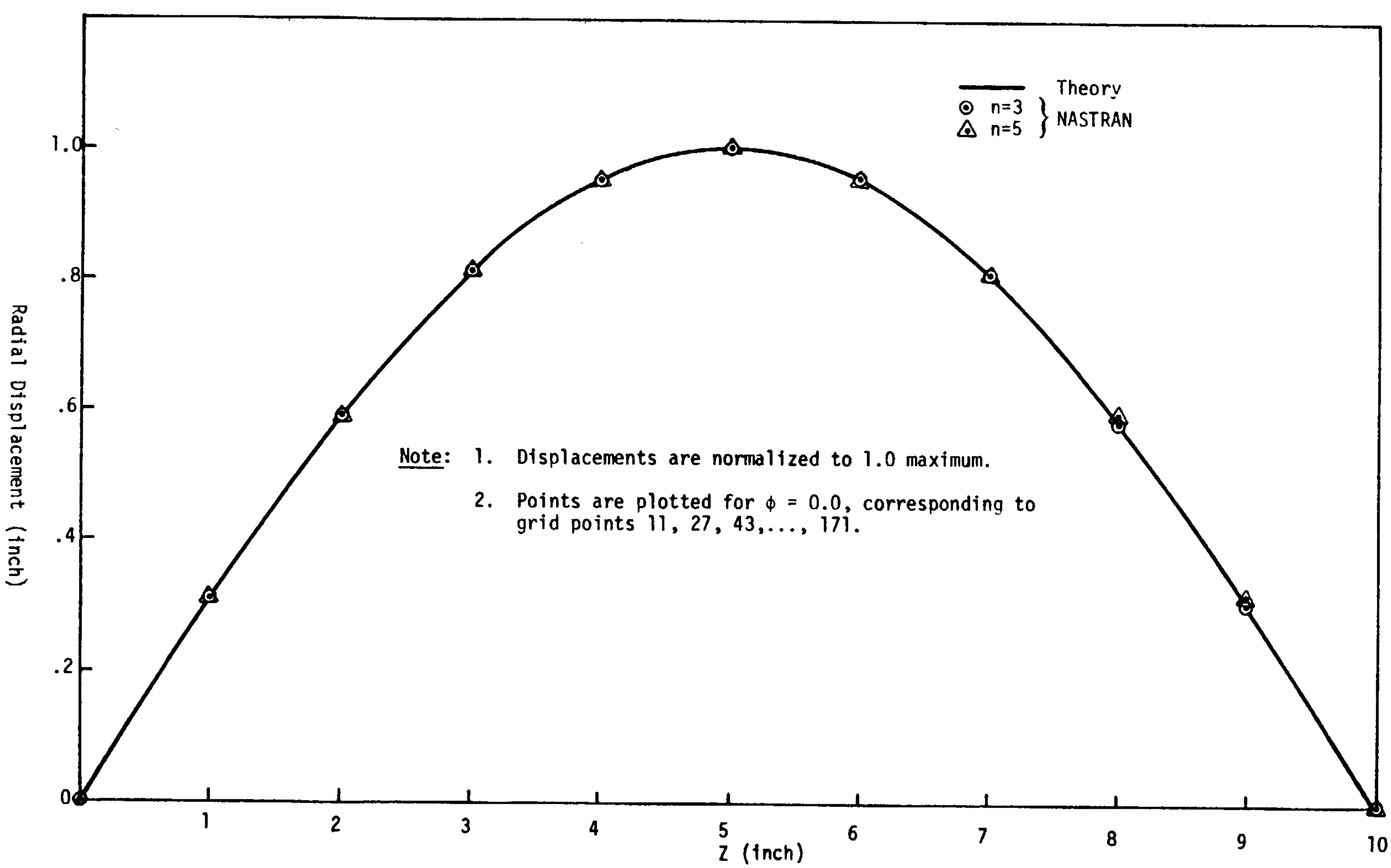


Figure 3. Radial displacement for harmonic 3 and 5 normal modes.

RIGID FORMAT No. 8, Frequency Response Analysis - Direct Formulation

Frequency Response of a 10x10 Plate (8-1-1)

Frequency Response of a 20x20 Plate (8-1-2)

Frequency Response of a 10x10 Plate (INPUT, 8-1-3)

Frequency Response of a 20x20 Plate (INPUT, 8-1-4)

A. Description

This problem illustrates the use of the direct method of determining structural response to steady-state sinusoidal loads. The applied load is given in terms of complex numbers which reflect the amplitudes and phases at each selected frequency. The steady-state response of the structure at each frequency is calculated in terms of complex numbers which reflect the magnitudes and phases of the results. Both configurations are duplicated via the INPUT module to generate the QUAD1 elements.

The particular model for this analysis is a square plate composed of quadrilateral plate elements as shown in Figure 1. The exterior edges are supported on hinged supports and symmetric boundaries are used along x = 0 and y = 0. The applied load is sinusoidally distributed over the panel and increases with respect to frequency. Although the applied load excites only the first mode, the direct formulation algorithm does not use this shortcut and solves the problem as though the load were completely general.

B. Input

1. Parameters:

   $a = b = 10$ - length and width of quarter model

   $t = 2.0$ - thickness

   $E = 3.0 \times 10^7$ - Young's Modulus

   $\nu = 0.3$ - Poisson's Ratio

   $\mu = 13.55715$ - nonstructural mass per area

2. Loads:

   The frequency dependent pressure function is:

$$P(x,y,f) = F(f) \cos \frac{\pi x}{2a} \cos \frac{\pi y}{2b} \quad , \tag{1}$$

where

$$F(f) = 10. + 0.3f \quad . \tag{2}$$

3. Constraints:

Only vertical motions and bending rotations are allowed. The exterior edges are hinged supports. The interior edges are planes of symmetry. This implies:

along $x = 0$, $\theta_y = 0$

along $y = 0$, $\theta_x = 0$

along $x = a$, $u_z = \theta_x = 0$

along $y = b$, $u_z = \theta_y = 0$

all points, $u_x = u_y = \theta_z = 0$

C. Theory

The excitation of the plate is orthogonal to the theoretical first mode. An explanation of the equations are given in Reference 8. The equations of response are:

$$u_z(f) = \frac{F(f)}{(2\pi)^2 \mu (f_1^2 - f^2)} , \tag{3}$$

where f_1 is the first natural frequency (10 cps).

D. Results

The following table gives the theoretical and NASTRAN results:

Frequency	$u_{z,1} \times 10^4$		
cps	Theory	10x10 NASTRAN	20x20 NASTRAN
0	1.868	1.874	1.869
8	6.435	6.49	6.45
9	12.489	12.69	12.53
10	∞	-824.92	-3284.4
11	-11.833	-11.67	-11.79

E. Driver Decks and Sample Bulk Data

```
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  1     ID        DEM8011,NASTRAN
  2     UMF       1977    80110
  3     APP       DISPLACEMENT
  4     SØL       8,1
  5     TIME      12
  6     CEND

  7     TITLE = FREQUENCY RESPØNSE ØF A 10X10 PLATE
  8     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 8-1-1
  9         SPC = 37
 10         DLØAD = 8
 11         FREQUENCY = 8
 12     ØUTPUT
 13         SET 1 = 1,4,7,11  45,55,  78,88,  111,114,117,121
 14         DISPLACEMENT(SØRT2,PHASE) = 1
 15         SPCFØRCE(SØRT2,PHASE) = 1
 16     BEGIN BULK
 17     ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	2	THRU	109					
CQUAD1	1	23	1	2	13	12	.00		
DAREA *	37		1		3		2.500000	0E-01	
FREQ	8	0.0	8.0	9.0	10.0	11.0			
GRDSET							126		
GRID	1		.0	.0	.0				
MAT1	8	3.0+7		.300					
PQUAD1	23			8	.6666667			13.55715	
RLØAD1	8	37			1				
SPC1	37	4	1	2	3	4	5	6	+41001H
+41001H	7	8	9	10	11				
TABLED1	1								+T1
+T1	.0	10.0	100.0	40.0	ENDT				

```
Card
 No.

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  2    UMF       1977    80120
  3    APP       DISPLACEMENT
  4    SØL       8,1
  5    TIME      30
  6    CEND

  7    TITLE = FREQUENCY RESPØNSE ØF A 20X20 PLATE
  8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 8-1-2
  9    $
 10        SPC = 37
 11        DLØAD = 8
 12        FREQUENCY = 8
 13    ØUTPUT
 14        SET 1 = 1,7,13,21,  169,189,  295,315,  421,427,433,441
 15        DISPLACEMENT(SØRT2,PHASE) = 1
 16        SPCFØRCE(SØRT2,PHASE) = 1
 17    BEGIN BULK
 18    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CNGRNT	1	2	THRU	419					
CQUAD1	1	23	1	2	23	22	.00		
DAREA *	37		1		3		2.500000	0E-01	
FREQ	8	0.0	8.0	9.0	10.0	11.0			
GRDSET							126		
GRID	1		.0	.0	.0				
MAT1	8	3.0+7		.300					
PQUAD1	23			8	.6666667			13.65715	
RLØAD1	8	37			1				
SPC1	37	4	1	2	3	4	5	6	+41001H
+41001H	7	8	9	10	11	12	13	14	+41002H
TABLED1	1								+T1
+T1	.0	2.5	100.0	10.0	ENDT				

```
Card
 No.

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  2     UMF       1977    80130
  3     APP       DISPLACEMENT
  4     SØL       8,1
  5     DIAG      14
  6     ALTER     1
  7     PARAM     //C,N,NØP/V,N,TRUE=-1 $
  8     INPUT,    ,,,,/G1,G2,,G4,/C,N,3/C,N,1 $
  9     EQUIV     G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE $
 10     ENDALTER
 11     TIME      12
 12     CEND

 13     TITLE = FREQUENCY RESPONSE OF A 10X10 PLATE
 14     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 8-1-3
 15         SPC = 10010
 16         DLØAD = 8
 17         FREQUENCY = 8
 18     ØUTPUT
 19         SET 1 = 1,4,7,11  45,55,  78,88,  111,114,117,121
 20         DISPLACEMENT(SØRT2,PHASE) = 1
 21         SPCFØRCE(SØRT2,PHASE) = 1
 22     BEGIN BULK
 23     ENDDATA

 24            10        10       1.0       1.0       126       0.0       0.0
 25             4         5        35        34         0         0
```

1	2	3	4	5	6	7	8	9	10
DAREA *	37		1		3		2.500000	0E-01	
FREQ	8	0.0	8.0	9.0	10.0	11.0			
MAT1	8	3.0+7		.300					
PQUAD1	101			8	.6666667			13.55715	
RLØAD1	8	37			1				
TABLED1	1								+T1
+T1	.0	10.0	100.0	40.0	ENDT				

Card No.

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 2   UMF       1977    80140
 3   APP       DISPLACEMENT
 4   SØL       8,1
 5   DIAG      14
 6   ALTER     1
 7   PARAM     //C,N,NØP/V,N,TRUE=-1 $
 8   INPUT,    ,,,,/G1,G2,,G4,/C,N,3/C,N,1 $
 9   EQUIV     G1,GEØM1/TRUE / G2,GEØM2/TRUE / G4,GEØM4/TRUE $
10   ENDALTER
11   TIME      30
12   CEND

13   TITLE = FREQUENCY RESPONSE ØF A 20X20 PLATE
14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 8-1-4
15       SPC = 20020
16       DLØAD = 8
17       FREQUENCY = 8
18   ØUTPUT
19       SET 1 = 1,7,13,21,  169,189,  295,315,  421,427,433,441
20       DISPLACEMENT(SØRT2,PHASE) = 1
21       SPCFØRCE(SØRT2,PHASE) = 1
22   BEGIN BULK
23   ENDDATA

24        20        20        0.5       0.5       126       0.0       0.0
25         4         5         35        34
```

1	2	3	4	5	6	7	8	9	10
DAREA *	37		1		3		2.500000	0E-01	
FREQ	8	9.0	8.0	9.0	10.0	11.0			
MAT1	8	3.0+7		.300					
PQUAD1	101			8	.6666667			13.55715	
RLØAD1	8	37			1				
TABLED1	1								+T1
+T1	.0	2.5	100.0	10.0	ENDT				

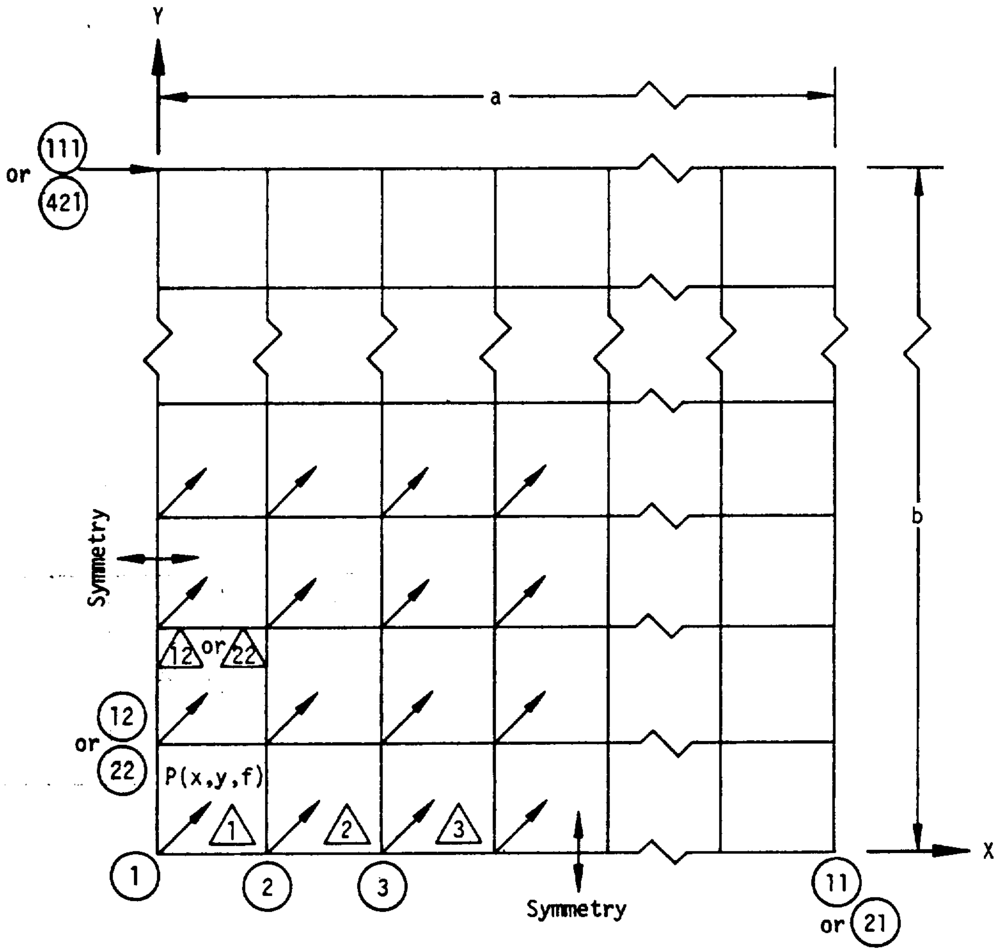

Figure 1. 10 x 10 or 20 x 20 Plate, quarter model.

RIGID FORMAT No. 9, Transient Analysis - Direct Formulation

Transient Analysis with Direct Matrix Input (9-1-1)

A. Description

This problem demonstrates the capability of NASTRAN to perform transient analysis on a system having nonsymmetric stiffness, damping and mass matrices. The problem also illustrates the use of time step changes, selection of printout intervals, application of loads, initial conditions, and a simple curve plot package.

The matrices and loads used are actually the product of a transformation matrix and diagonal matrices. The resulting answers are easily calculated while the input matrices are of general form. The matrix equation solved is

$$[M]\{\ddot{u}\} + [B]\{\dot{u}\} + [K]\{u\} = \{P(t)\} \quad . \tag{1}$$

The problem is actually four disjoint single degree of freedom problems which have been transformed to a general matrix problem. Figure 1 illustrates the problems schematically.

The resulting diagonal matrices are premultiplied by the matrix:

$$[X] = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \quad . \tag{2}$$

The answers for the disjoint problem above will be the same as for the general matrix problem since the general case:

$$[X]\Big([M_o]\{\ddot{u}\} + [B_o]\{\dot{u}\} + [K_o]\Big)\{u\} = [X]\{P\} \quad , \tag{3}$$

has the same results as the disjoint case:

$$[M_o]\{\ddot{u}\} + [B_o]\{\dot{u}\} + [K_o]\{u\} = \{P\} \quad . \tag{4}$$

B. Input

1. The actual matrix input is:

$$[M] = \begin{bmatrix} 20 & -1.5 & 0 & 0 \\ -10 & 3.0 & -4 & 0 \\ 0 & -1.5 & 8 & 0 \\ 0 & 0.0 & -4 & 0 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & -15 & 0 & 0 \\ 0 & 30 & -24 & 0 \\ 0 & -15 & 28 & -2 \\ 0 & 0 & -24 & 4 \end{bmatrix}$$

$$[K] = \begin{bmatrix} 2000 & 0 & 0 & 0 \\ -1000 & 0 & -100 & 0 \\ 0 & 0 & 200 & -20 \\ 0 & 0 & -100 & 40 \end{bmatrix}$$

2. The initial conditions are:

$$u_{10} = 0 \qquad \dot{u}_{10} = 10.0$$

$$u_{11} = 0 \qquad \dot{u}_{11} = 0.5$$

$$u_{12} = 0 \qquad \dot{u}_{12} = 0$$

$$u_{13} = -10.0 \qquad \dot{u}_{13} = 0$$

3. At t = 1.0 a step load is applied to each point. The load on the uncoupled problems is:

$$P_o = \begin{Bmatrix} 0 \\ 1.5 \\ 4.0 \\ 20 \end{Bmatrix}$$

The transformed load is:

$$\{P\} = [X]\{P_o\} = \begin{Bmatrix} -1.5 \\ -1.0 \\ -13.5 \\ 36.0 \end{Bmatrix}$$

C. Theory

The results are responses of single degree of freedom systems. Equations are given in Reference 12, Chapter 9.

$0 < t < 1.0$, $\Delta t = .005$

$$u_{10} = \sin 10t \qquad \dot{u}_{10} = 10 \cos 10t$$

$$u_{11} = 0.05(1 - e^{-10t}) \qquad \dot{u}_{11} = 0.5e^{-10t}$$

$$u_{12} = 0 \qquad \dot{u}_{12} = 0$$

$$u_{13} = -10e^{-10t} \qquad \dot{u}_{13} = 100e^{-10t}$$

$t > 1.0$, $\Delta t = .015$

$$u_{10} = \sin 10t$$

$$u_{11} = 0.05(1 - e^{-10t}) + 0.1(t - 1.1 + .1e^{-10(t-1)})$$

$$u_{12} = 0.04\left\{1 - e^{-3t}[\cos 4(t-1) + \tfrac{3}{4}\sin 4(t-1)]\right\}$$

$$u_{13} = -10e^{-10t} + 1 - e^{-10(t-1)}$$

D. Results

Figures 2 through 5 are tracings of the NASTRAN plots of the functions. The deviations of the NASTRAN results and the theoretical response are due to the selection of time steps. For instance point 11 has a time constant equal to two time steps. The initial error in velocity due to the first step causes the displacement error to accumulate. Using a smaller time step has resulted in much better results.

E. Driver Decks and Sample Bulk Data

Card No.

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ID        DEM9011,NASTRAN
UMF       1977    90110
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SØL       9,1
TIME      5
CEND
TITLE = TRANSIENT ANALYSIS WITH DIRECT MATRIX INPUT
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 9-1-1
TSTEP = 32
IC = 32
DLØAD = 32
K2PP=KCØMP
M2PP=MCØMP
B2PP=BCØMP
ØUTPUT
    SVELØ = ALL
    DISP(SØRT2) = ALL
    ØLØAD(SØRT2) = ALL
PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 9-1-1
ØUTPUT(XYØUT)
PLØTTER = SC
         CAMERA = 3
SKIP BETWEEN FRAMES = 1
    TCURVE = * * * *  EPØINT       DISPLACEMENT(INCHES)  * * * * * * *
    XTITLE =                TIME (SECØNDS)
$
         YVALUE PRINT SKIP = 1
         XDIVISIØNS = 25
         XVALUE PRINT SKIP = 1
$* * * * * * * * * * * * * * * * * FULL FRAME PLØTS * * * * * * * * * *
         YGRID LINES = YES
         XGRID LINES = YES
         YDIVISIØNS = 22
$
    YTITLE = EPØINT 10      DISPLACEMENT *INCH*
XYPLØT DISP / 10(T1)
$
         YDIVISIØNS = 20
    YTITLE = EPØINT 11      DISPLACEMENT *INCH*
XYPLØT DISP / 11(T1)
$
    YTITLE = EPØINT 12      DISPLACEMENT *INCH*
XYPLØT DISP / 12(T1)
$
    YTITLE = EPØINT 13      DISPLACEMENT *INCH*
XYPLØT DISP / 13(T1)
BEGIN BULK
ENDDATA
```

1	2	3	4	5	6	7	8	9	10
DAREA	1	10		-1.5	11		-1.0		
DELAY	1	10		1.0	11		1.0		
DMIG	BCØMP	0	1	1	2				
EPØINT	10	11	12	13					
TABLED1	1								+T1
+T1	-1.0	.0	.0	.0	.00	1.0	100.0	1.0	+T2
TIC	32	10		.0	10.				
TLØAD1	32	1	1		1				
TSTEP	32	200	.005	10					+S1
+S1		100	.015	5					

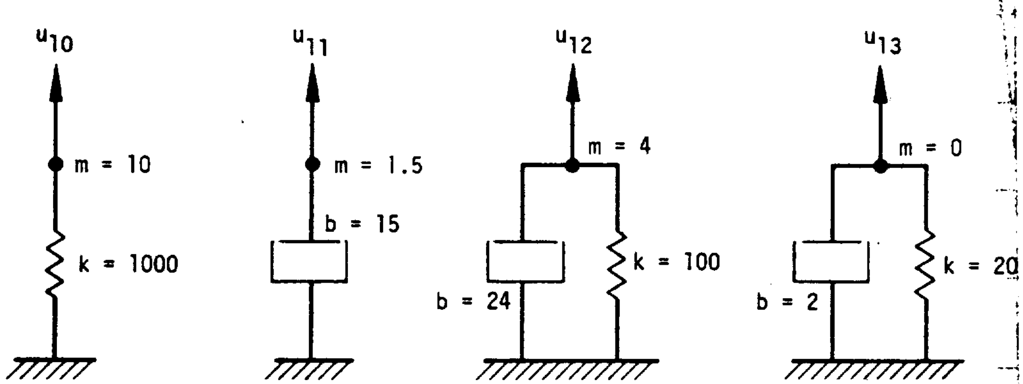


Figure 1. Disjoint equivalent systems.

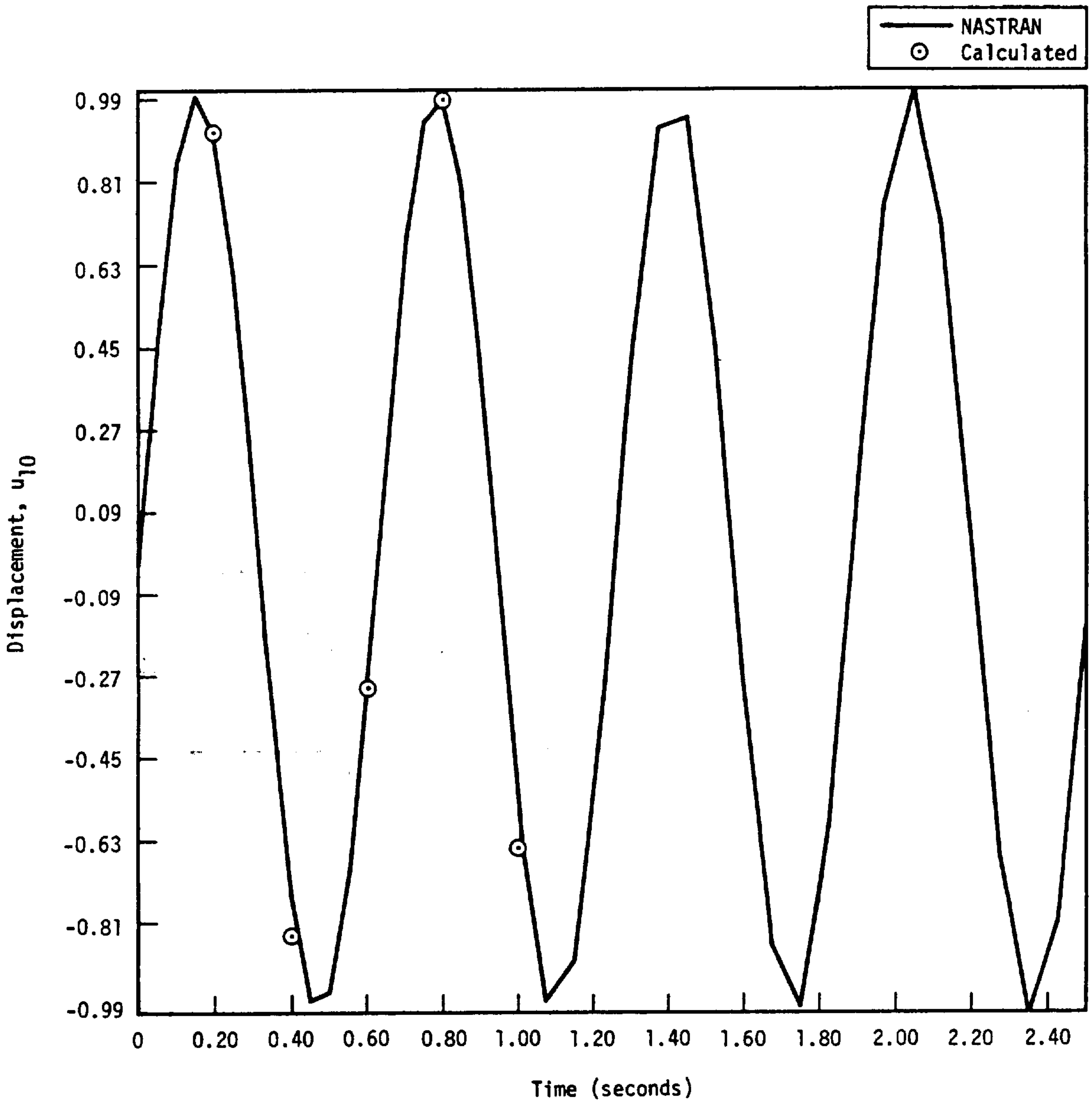


Figure 2. Point 10, displacement.

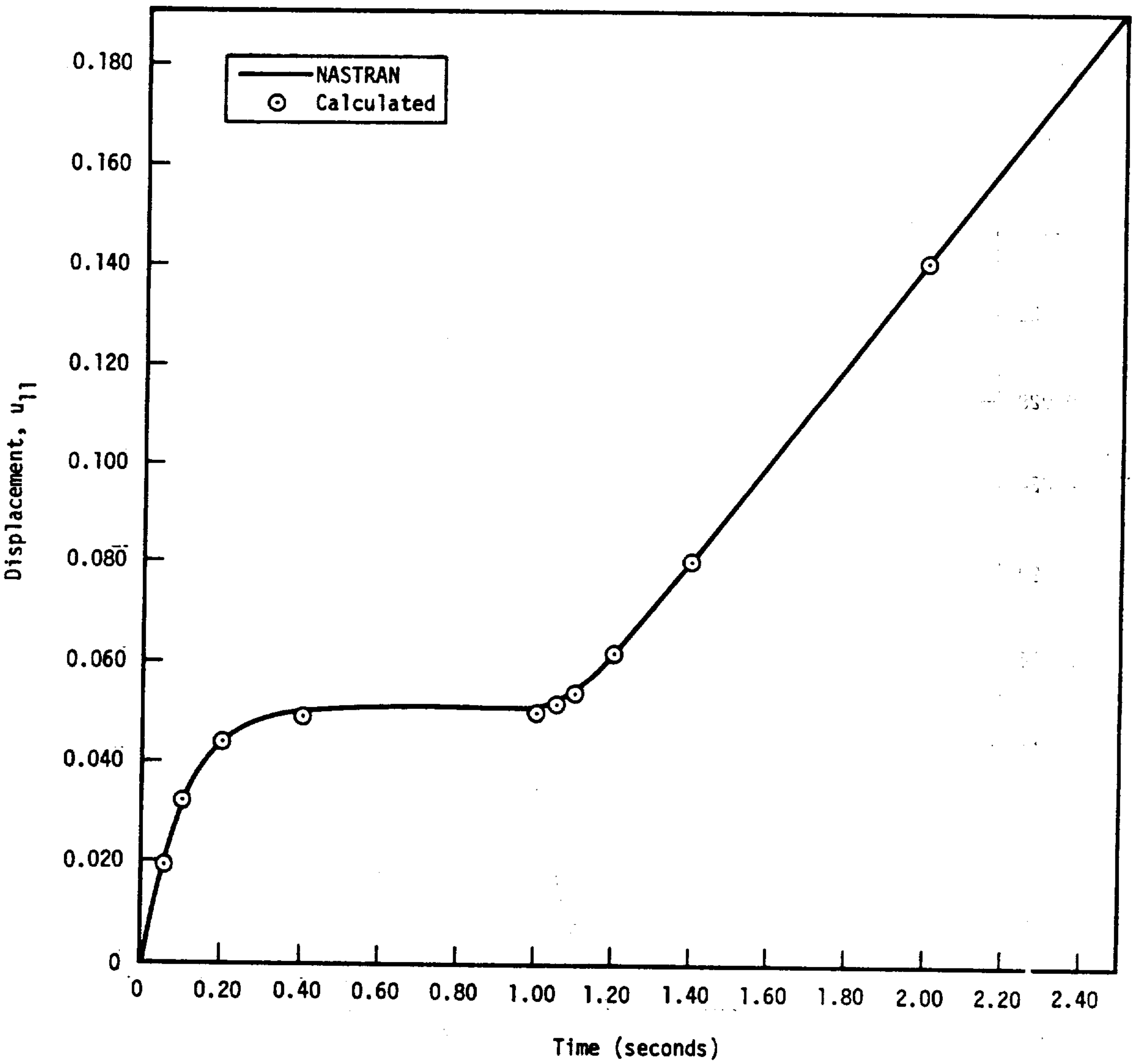


Figure 3. Point 11, displacement.

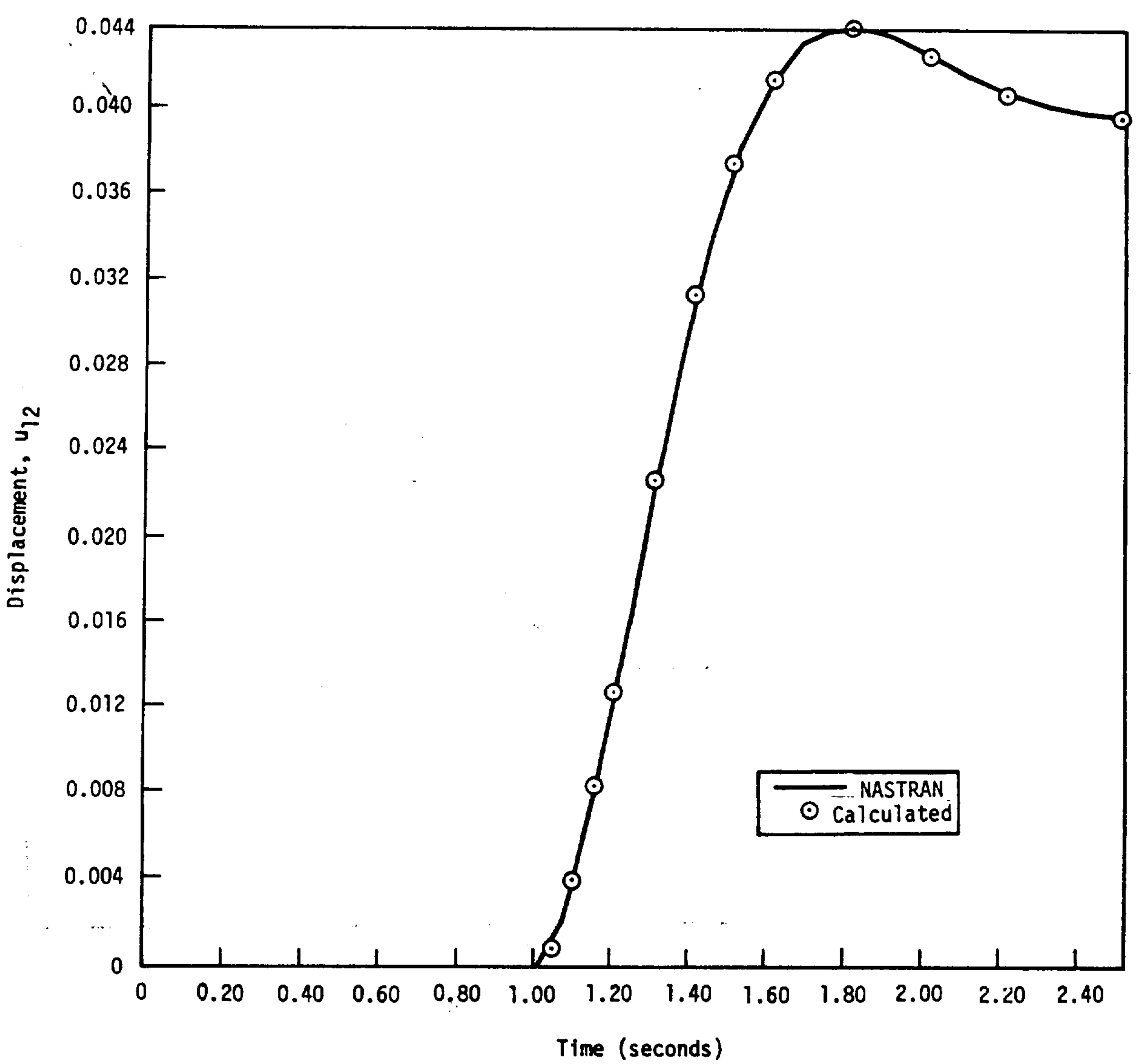


Figure 4. Point 12, displacement.

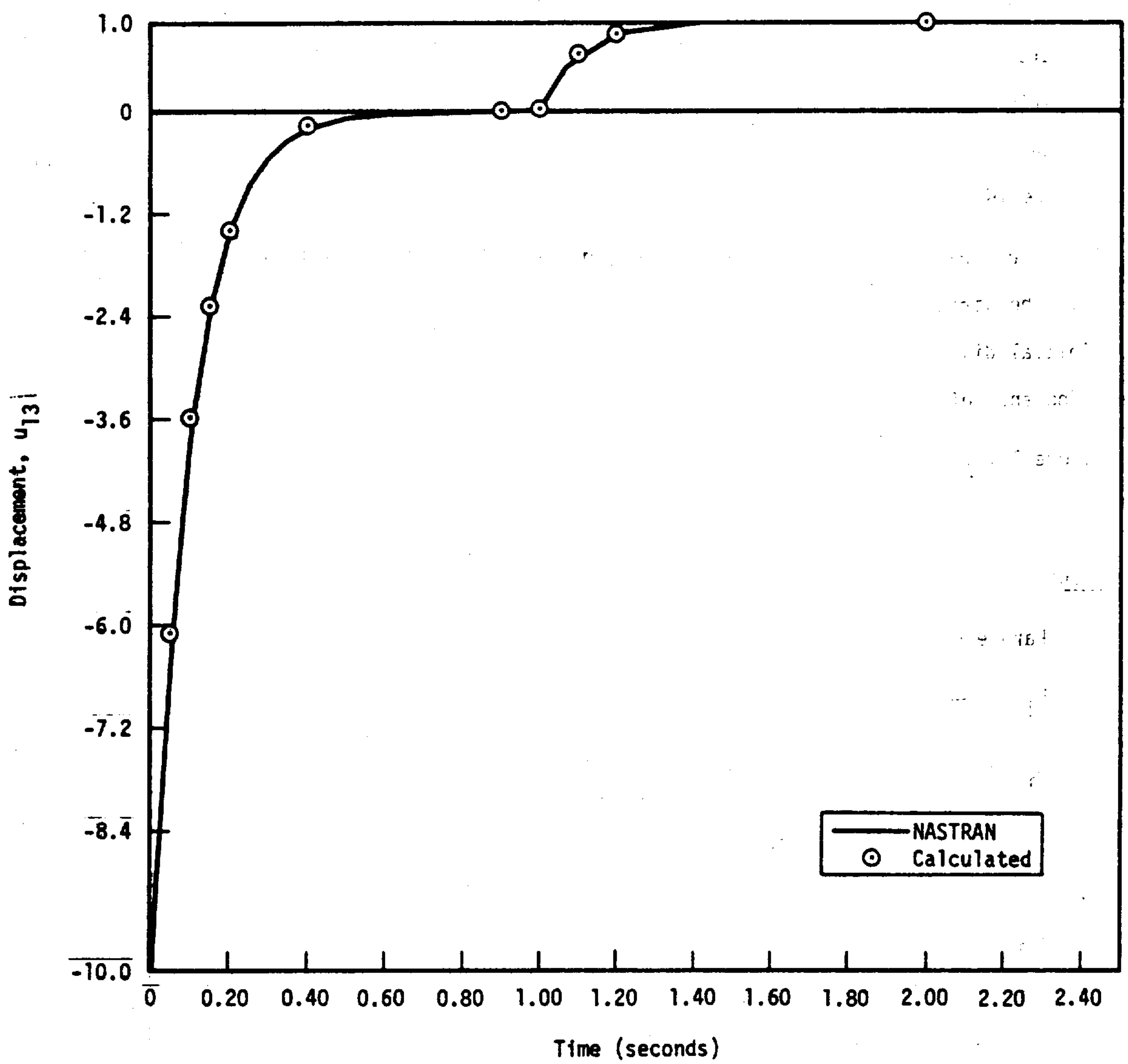


Figure 5. Point 13, displacement.

RIGID FORMAT No. 9, Transient Analysis - Direct Formulation

Transient Analyisis of a 1000 Cell String, Traveling Wave Problem (9-2-1)

Transient Analysis of a 1000 Cell String, Traveling Wave Problem (INPUT, 9-2-2)

A. Description

This problem illustrates the ability of NASTRAN to perform time integration studies using the structural matrices directly. At each time step the applied loads, the structural matrices, and the previous displacements are used to calculate a new set of displacements, velocities, and accelerations. Initial displacements and velocities are also allowed for all unconstrained coordinates. The INPUT module is used to generate the scalar springs and masses.

The structural model consists of a 1000 cell string under constant tension modeled by scalar elements. The string is given an initial condition at one end consisting of a triangular shaped set of initial displacements. The wave will then travel along the string, retaining its initial shape. The ends of the string are fixed causing the wave to reflect with a sign reversal.

Figure 1 illustrates the problem and the scalar element model for each finite increment of length.

B. Input

1. Parameters:

$k_i = \frac{T}{\Delta x} = 10^7$ - scalar spring rates

$m_i = \mu\Delta x = 10$ - scalar masses

$N = 1000$ - number of cells

where

T is the tension

Δx is the incremental length

μ is the mass per unit length

2. Loads:

The initial displacements are;

$u_2 = .2$ $u_{12} = 1.8$

$u_3 = .4$ $u_{13} = 1.6$

$u_4 = .6$.

. .

. $u_i = 0, \quad i > 21$

$u_{11} = 2.0$ $u_{21} = 0.0$

C. Theory

As shown in Reference 11. Chapter 6, the wave velocity c is,

$$c = \pm\sqrt{\frac{T}{\mu}} = \pm\Delta x\sqrt{\frac{k_i}{m_i}} = \pm 1000 \text{ points/unit time} \quad (1)$$

The initial displacement may be divided into two waves, traveling in opposite directions. The first wave travels outward; the second wave travels toward the fixed support and reflects with a sign change.

D. Results

The theoretical and NASTRAN results are compared in Figure 2, when both waves have traveled their complete width.

E. Driver Decks and Sample Bulk Data

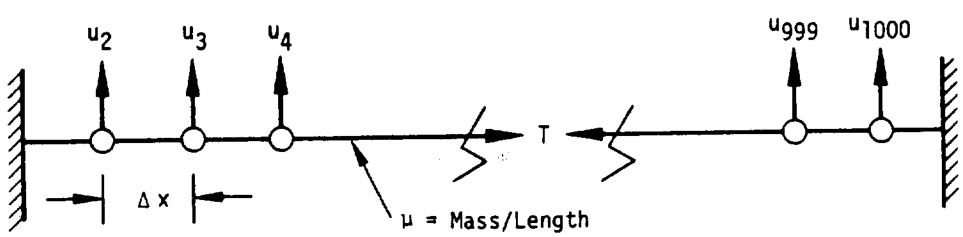
```
Card
 No.

 0     NASTRAN FILES=UMF
 1     ID         DEM9021,NASTRAN
 2     UMF        1977     90210
 3     TIME       16
 4     APP        DISP
 5     SØL        9,1
 6     CEND

 7     TITLE = TRANSIENT ANALYSIS ØF A 1000 CELL STRING
 8     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 9-2-1
 9     LABEL = TRAVELING WAVE PRØBLEM
10         TSTEP = 9
11         IC = 9
12     ØUTPUT
13     SET 1 = 2,4,5,6,10,12,14,16,18,20,22,24,26,28,30,40,50, 100,200,500
14         DISPLACEMENT = 1
15         VELØCITY = 1
16     BEGIN BULK
17     ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CELAS3	1	101	0	2	2	101	2	3	
CMASS3	40002	301	2	0					
PELAS	101	1.0+7		10.0					
PMASS	301	10.000							
TIC	9	2		.2					
TSTEP	9	50	.5-3	1					

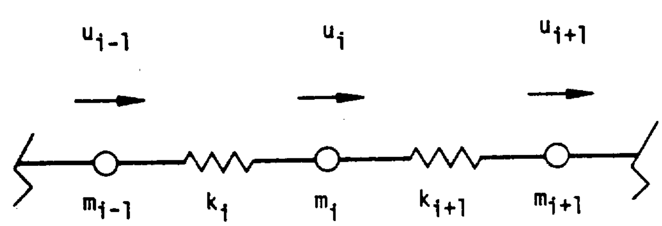
Card No.

```
 0   NASTRAN FILES=UMF
 1   ID        DEM9022,NASTRAN
 2   UMF       1977     90220
 3   ALTER     1
 4   PARAM     //C,N,NØP/V,N,TRUE=-1 $
 5   INPUT,    ,,,,/,G2,,,/C,N,5 $
 6   EQUIV     G2,GEØM2/TRUE $
 7   ENDALTER
 8   TIME      16
 9   APP       DISP
10   SØL       9,1
11   DIAG      14
12   CEND

13   TITLE = TRANSIENT ANALYSIS ØF A 1000 CELL STRING
14   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 9-2-2
15   LABEL = TRAVELING WAVE PRØBLEM
16       TSTEP = 9
17       IC = 9
18   ØUTPUT
19   SET 1 = 2,4,5,6,10,12,14,16,18,20,22,24,26,28,30,40,50, 100,200,500
20       DISPLACEMENT = 1
21       VELØCITY = 1
22   BEGIN BULK
23   ENDDATA

24        1000    1.0E7     0.0     10.0
```

1	2	3	4	5	6	7	8	9	10
TIC	9	2		.2					
TSTEP	9	50	.5-3	1					



1000 Cell String



Finite Element Model

Figure 1. Representations of dynamic string.

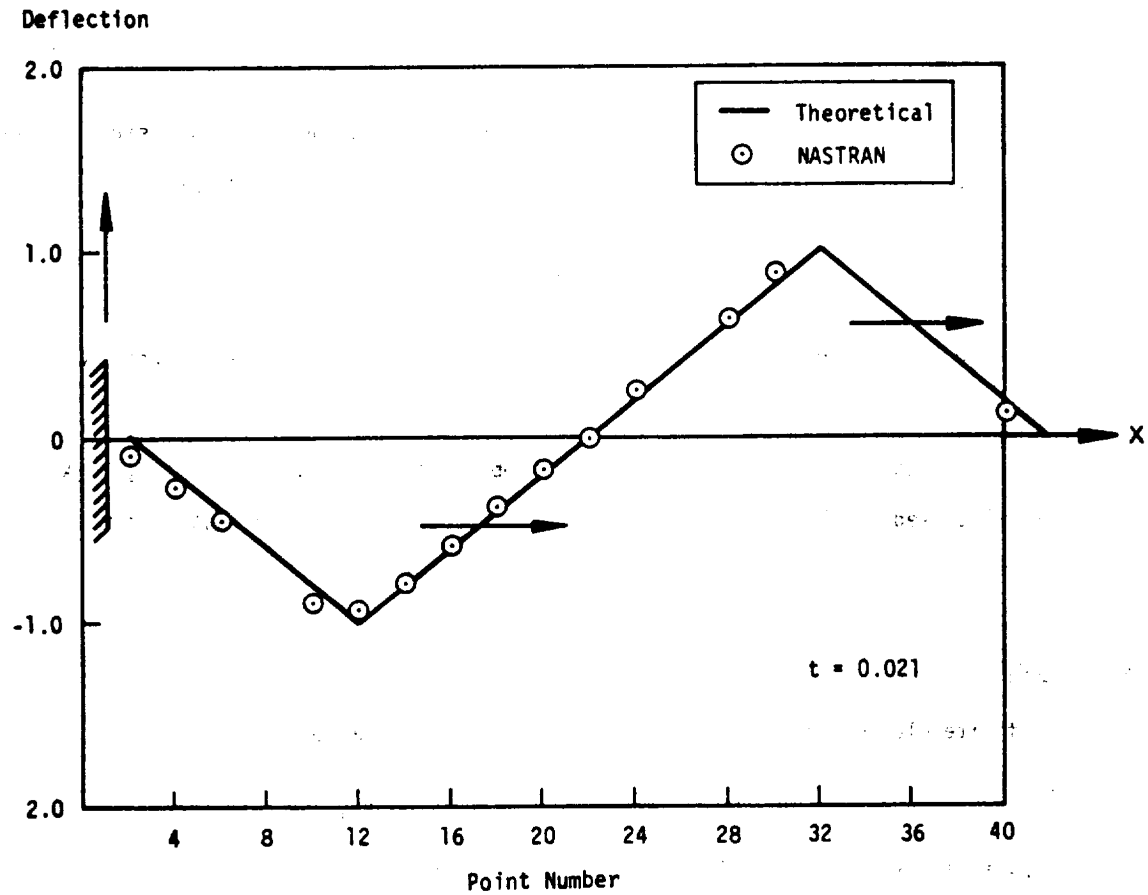


Figure 2. Traveling Wave on string.

Transient Analysis of a Fluid-Filled Elastic Cylinder (9-3-1)

A. Description

The fluid-filled shell, used for analysis of the third harmonic, in Demonstration Problem No. 7-2-1 is subjected to a step change in external pressure at t = 0 of the form

$$p = p_o \sin \frac{\pi z}{\ell} \cos n\phi \quad .$$

The fluid is assumed incompressible in order to obtain an analytical solution with reasonable effort. The harmonic used is n = 3.

In addition to the cards of Demonstration Problem No. 7-2-1, DAREA, PRESPT, TLØAD2, and TSTEP cards are also used. Selected displacements and pressures are plotted against time.

B. Input

The finite element model is shown in Figures 1 and 2. Parameters used are:

B	(Bulk modulus of fluid - incompressible)
$\rho_f = 1.8 \times 10^{-2}$ lb-sec^2/in^4	(Fluid mass density)
$\rho_s = 6.0 \times 10^{-2}$ lb-sec^2/in^4	(Structure mass density)
$E = 1.6 \times 10^5$ lb/in^2	(Young's modulus for structure)
$G = 6.0 \times 10^4$ lb/in^2	(Shear modulus for structure)
$a = 10.0$ inch	(Radius of cylinder)
$\ell = 10.0$ inch	(Length of cylinder)
$h = 0.01$ inch	(Thickness of cylinder wall)
$p_o = 2.0$	(Pressure load coefficient)

C. Theory

The theory was derived with the aid of Reference 16 as in Demonstration Problem No. 7-2-1. Since the fluid is incompressible, it acts on the structure like a pure mass. Neglecting the bending stiffness, the equation of force on the structure is:

$$p_s = (m + m_f)\,\ddot{w} + \frac{1}{a}\frac{\partial^2 F}{\partial z^2} \quad , \tag{1}$$

where:

p_s is the loading pressure on the structure (positive outward).

$m = \rho_s h$ is the mass per area of the structure.

m_f is the apparent mass of the fluid.

w is the normal displacement (positive outward)

The function F is defined by the equation,

$$\nabla^4 F = \frac{Eh}{a}\frac{\partial^2 w}{\partial z^2} \quad . \tag{2}$$

The spatial functions of pressure, displacement, and function F may be written in the form:

$$\begin{aligned} p_s &= p_o \sin\frac{\pi z}{\ell}\cos n\phi \quad , \\ w &= w_o \sin\frac{\pi z}{\ell}\cos n\phi \quad , \\ F &= F_o \sin\frac{\pi z}{\ell}\cos n\phi \quad . \end{aligned} \tag{3}$$

where p_o, w_o, and F_o are variables with respect to time only.

Substituting Equations 3 into Equation 2 we obtain:

$$F_o = -\frac{Eh}{a}\left(\frac{\ell}{\pi}\right)^2 \frac{w_o}{\left[1 + \left(\frac{n\ell}{\pi a}\right)^2\right]^2} \quad . \tag{4}$$

Substituting Equations 3 and 4 into Equation 1 we obtain:

$$p_o = (m + m_f)\,\ddot{w}_o + \frac{Eh}{a^2\left[1 + \left(\frac{n\ell}{\pi a}\right)^2\right]^2}\,w_o \quad . \tag{5}$$

The incompressible fluid is described by the differential equation:

$$\nabla^2 p = 0 \quad . \tag{6}$$

Applying the appropriate boundary conditions to Equation 6 results in the pressure distribution:

$$p = p_r \sin\frac{\pi z}{\ell}\,\cos(n\phi)\,I_n\left(\frac{\pi r}{\ell}\right) \quad , \tag{7}$$

where I_n is the modified Bessel function of the first kind and p_r is an undetermined variable. The balance of pressure and flow at the boundary of the fluid, with no structural effects, is described by the equations:

$$p_o = -\,p_r\,I_n\left(\frac{\pi a}{\ell}\right) \quad , \tag{8}$$

$$\rho_f \ddot{w} = -\left.\frac{\partial p}{\partial r}\right|_{r=a} \quad . \tag{9}$$

Substituting Equations 3 and 7 into Equation 9 results in:

$$\rho_f\,\ddot{w}_o = -\frac{\pi}{\ell}\,I_n'\left(\frac{\pi a}{\ell}\right)p_r \quad . \tag{10}$$

Eliminating P_r with Equations 8 and 10 gives the expression for apparent mass, m_f:

$$p_o = I_n\left(\frac{\pi a}{\ell}\right)\frac{\rho_f \ddot{w}_o}{\frac{\pi}{\ell}\,I_n'\left(\frac{\pi a}{\ell}\right)} = m_f\,\ddot{w}_o \quad . \tag{11}$$

Substituting the expression for m_f from Equation 11 into Equation 5 results in a simple single degree of freedom system. When the applied loading pressure is a step function at $t = 0$,

$$w = \frac{p_o}{k}\ (1 - \cos \omega t)\ \sin \frac{\pi z}{\ell}\ \cos n\phi \quad , \qquad (12)$$

where

$$\omega = \sqrt{\frac{k}{m_T}} \quad ,$$

and

$$k = \frac{Eh}{a^2\left[1 + \left(\frac{n\ell}{\pi a}\right)^2\right]^2} \quad ,$$

and

$$m_T = m + m_f = \rho_s h + \rho_f \frac{\ell}{\pi} \frac{I_n\left(\frac{\pi a}{\ell}\right)}{I_n'\left(\frac{\pi a}{\ell}\right)} \quad .$$

D. Results

A transient analysis was performed for the case $n = 3$ on the model and various displacements and pressures were output versus time up to one second. The theoretical frequency is calculated to be 1.580 Hertz and the period is 0.633 seconds. The displacements at two points on the structure (Point 91 is located at $\phi = 0$, $z = 5.0$; Point 94 is located at $\phi = 18°$, $z = 5.0$) are plotted versus time in Figure 3.

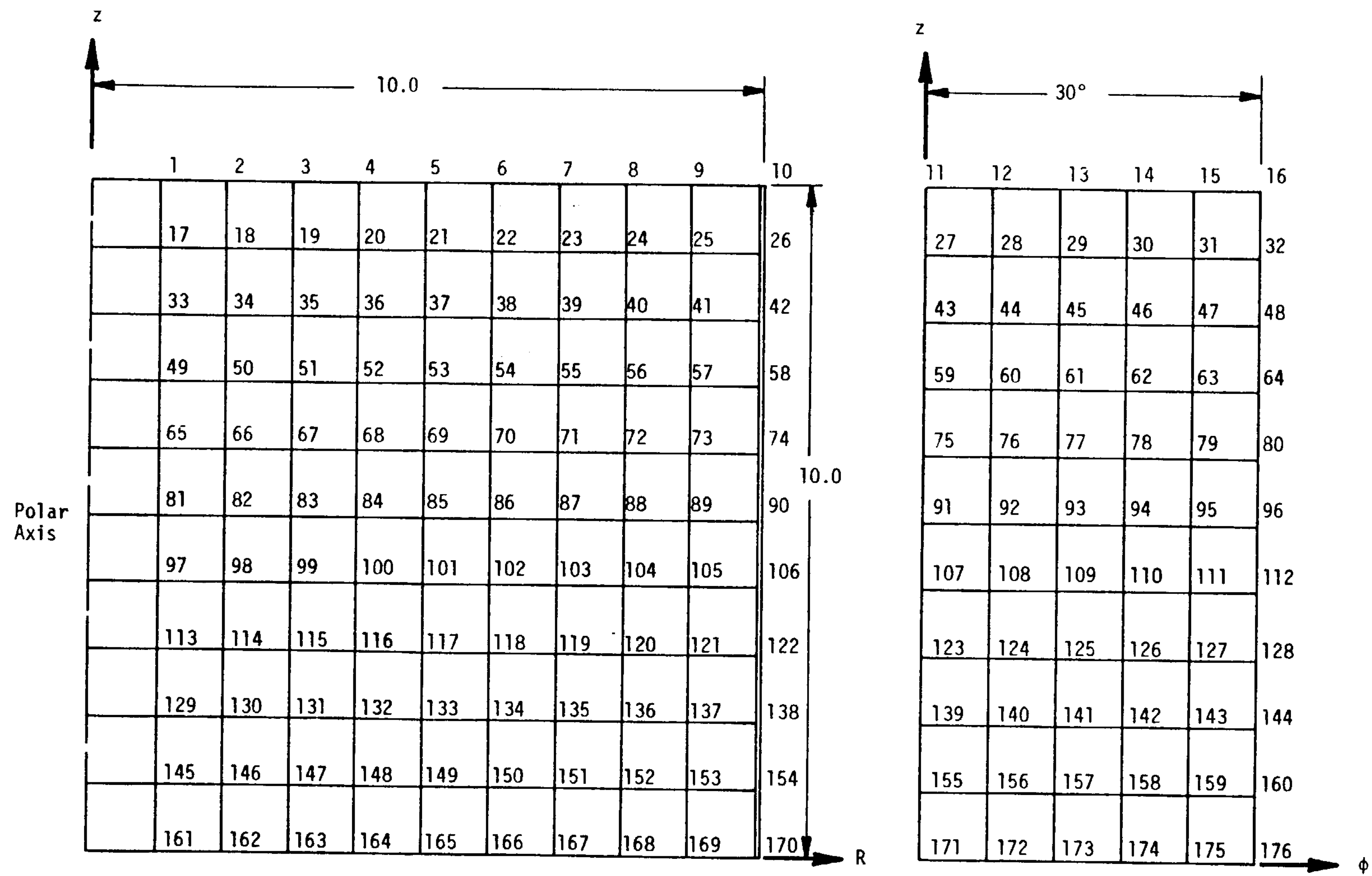
The maximum error for the first full cycle occurs at the end of the cycle. The ratio of the error to maximum displacement is 4.75%. Changes in the time step used in the transient integration algorithm did not affect the accuracy to any great extent. The most probable causes for error were the mesh size of the model and the method used to apply the distributed load. The applied load was calculated by multiplying the pressure value at the point by an associated area. The "consistent method" of assuming a cubic polynomial displacement and integrating would eliminate the extraneous response of higher modes. The method chosen in this problem, however, is typical of actual applications.

E. Driver Decks and Sample Bulk Data

Card
No.

```
NASTRAN FILES=(UMF,PLT2)
ID       DEM9031,NASTRAN
UMF      1977    90310
APP      DISPLACEMENT
SØL      9,3
TIME     30
CEND

TITLE = TRANSIENT ANALYSIS ØF A FLUID-FILLED ELASTIC CYLINDER.
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 9-3-1
LABEL = THIRD HARMØNIC ANALYSIS.
    TSTEP = 10
    ØLØAD = 10
    SPC = 3
    AXISYMMETRIC = FLUID
ØUTPUT
    HARMONICS = 3
    SET 100 = 10,11, 26,27,42,43,  58,59, 74,75, 81 THRU 96,
       106,107, 122,123, 138,139, 154,155, 170,171
    DISPLACEMENT = 100
PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 9-3-1
ØUTPUT(XYPLØT)
    XPLØTTER = SC 4020
    XTGRID = YES
    YTGRID = YES
    XBGRID = YES
    YBGRID = YES
    XDIVISIØNS = 10
  XTITLE =                                          TIME (SECØNDS)
  YTTITLE = R DISP       -INCHES-
  YBTITLE = R DISP       -INCHES-
$
TCURVE = PLØTTED  *TØP GRID 91(Z=5,A=0), *BØTTØM GRID 110(Z=5,A=18)
  XYPLØT DISP   /91(T1,), 110(,T1)
TCURVE = PLØTTED GRID(A=0,18) *TØP - 59,62(Z=7) *BØTTØM 123,126(Z=3)
  XYPLØT DISP   /59(T1,), 62(T1,), 123(,T1),126(,T1)
$
  YTTITLE =  PRESSURE     *LB/INCH*
  YBTITLE =  PRESSURE     *LB/INCH*
TCURVE = PLØTTED PRESPT (Z=5,A=0)  *TØP 5301(R=3)  *BØTTØM 5801(R=8)
  XYPLØT DISP  /5301(T1,), 5801(,T1)
TCURVE = PLØTTED PRESPT (R=5,A=0,Z=3,5,7)*TØP 3501,5501  *BØT 7501,5501
  XYPLØT DISP  /5501(T1,T1), 3501(T1,), 7501(,T1)
  YTITLE = R DISP          -INCH-
TCURVE = PLØTTED DISP AT MIDPØINT(Z=5.), ANGLE = 0.0 and 18.0 DEGREES.
  XYPLØT DISP  /  91(T1), 110(T1)
  YTITLE = HARMØNIC PRESSURE
TCURVE = PLØTTED RINGFL (R=5,Z=5)  * 85
  XYPLØT DISP / 4000085 (T1)
$
BEGIN BULK
ENDDATA
```

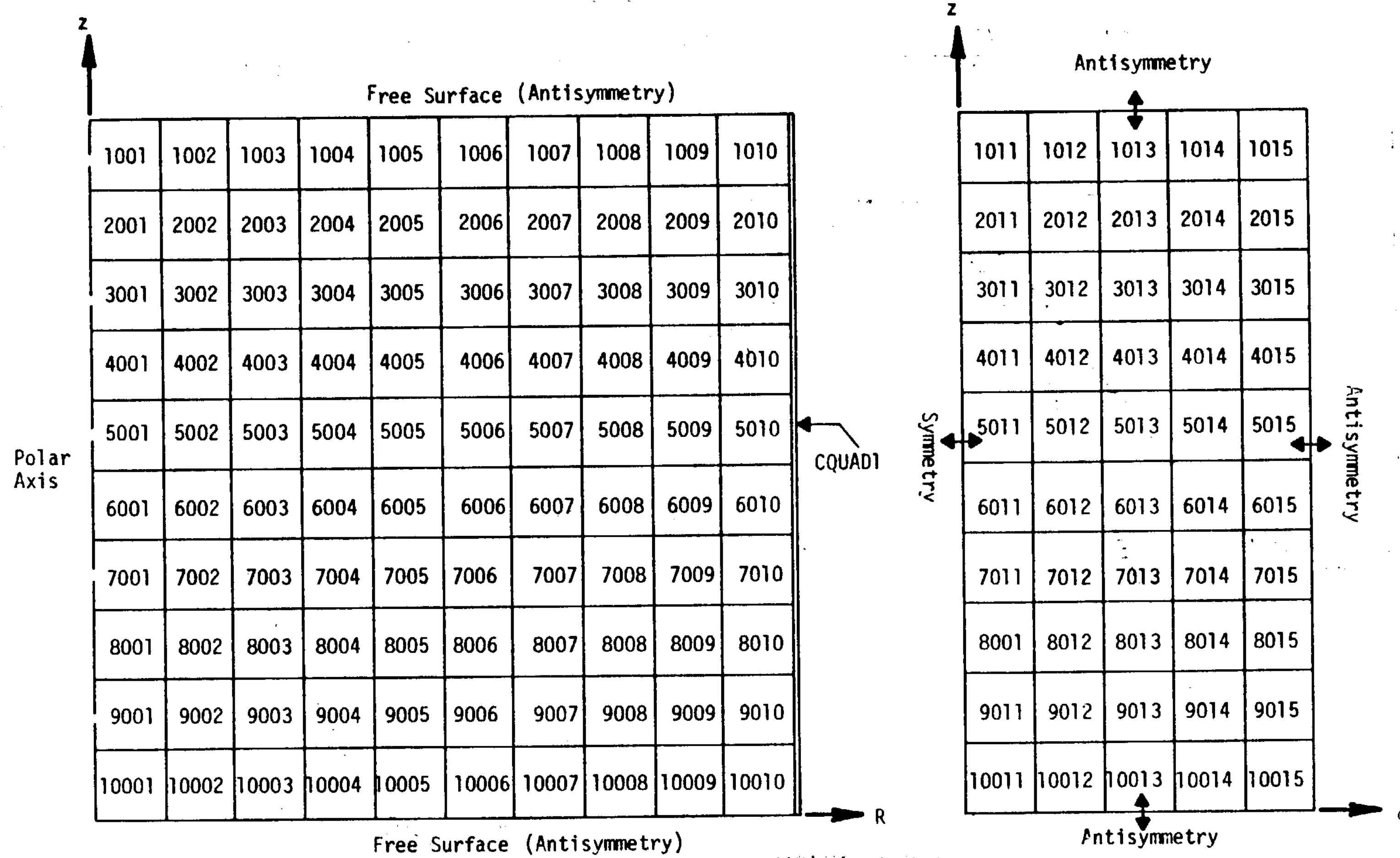
1	2	3	4	5	6	7	8	9	10
AXIF	1	.0	1.8-2	.00	NØ				+AXIF
+AXIF	3								
BDYLIST		10	26	42	58	74	90	106	+BDY-1
+BDY-1	122	138	154	170					
CFLUID2	1001	17	1	1	17				
CFLUID4	1002	18	2	1	17				
CØRD2C	1		.0	.0	.0	.0	.0	1.0	+CØRD2C
+CØRD2C	1.0	.0	.0	.0					
CQUAD1	1011	1	27	28	12	11			
DAREA	1	27	1	.32345	28	1	.61525		
FLSYM	12	S	A						
FSLIST		AXIS	1	2	3	4	5	6	+FSL-1
+FSL-1	7	8	9	10					
GRIDB	11			0.0		1	4	10	
MAT1	2	1.6+5	6.0+4		6.0-2				
PQUAD1	1	2	.01	2	8.3333-8				+PQUAD1
+PQUAD1	.0	.005							
PRESPT	21		1501	+0.0					
RINGFL	1	1.00000		10.0000	2	2.00000		10.0000	
SPC1	3	126	11	12	13	14	15	16	
TLØAD2	10	1			.0	1.0	.0	.0	
TSTEP	10	50	.02	2					



RINGFL Identification Number - Fluid

GRIDB Identification Number - Structure

Figure 1. Transient analysis model.



CFLUID Identification Number - Fluid

CQUAD1 Identification Number - Structure

Figure 2. Transient analysis model.

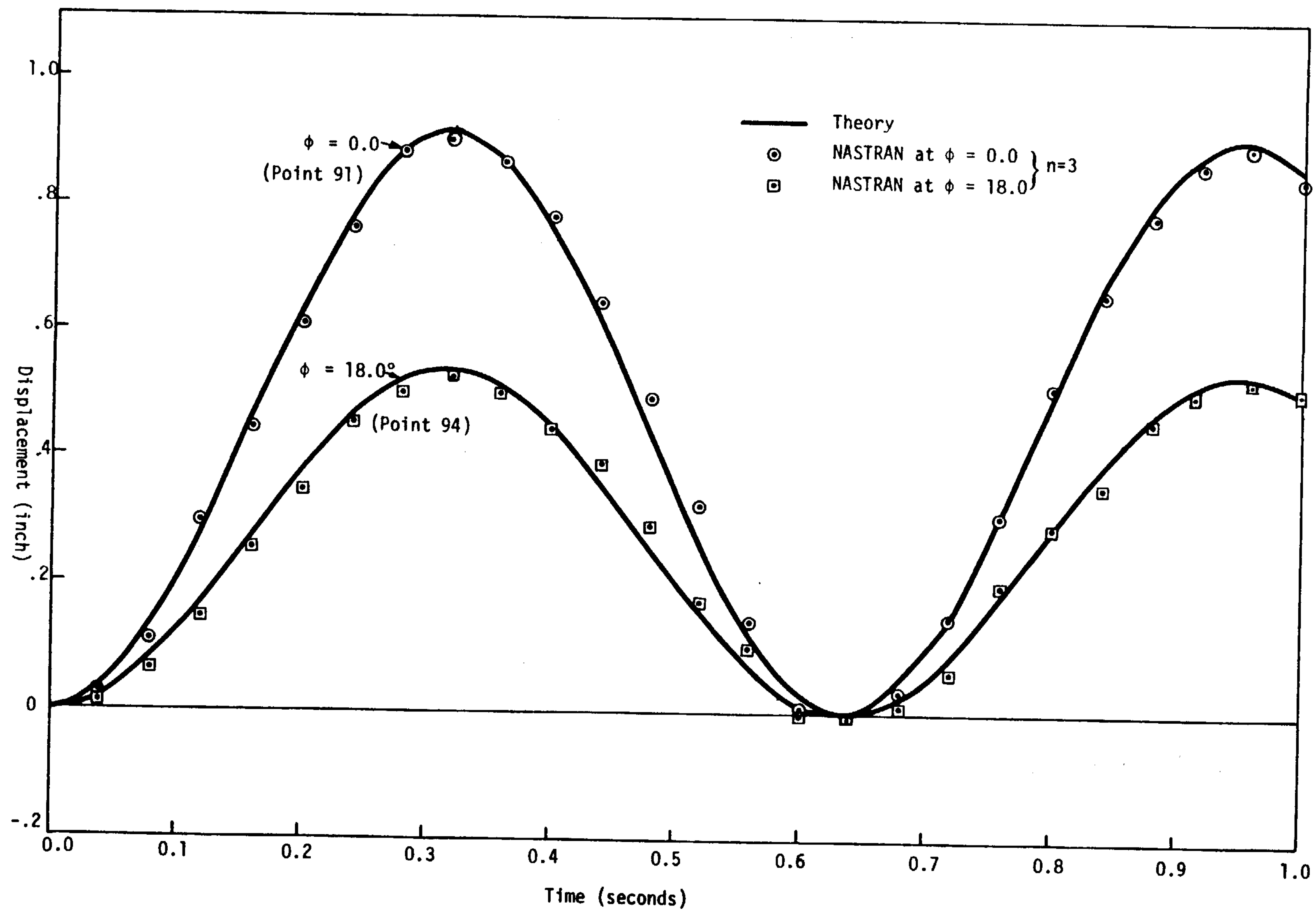


Figure 3. Displacement at midpoint ($z = 5.0$).

RIGID FORMAT No. 9 (APP HEAT), Linear Transient Heat Transfer Analysis
Plate with Suddenly Applied Flux and Edge Temperature (9-4-1)

A. Description

The time history of the temperature in a long thin plate initially at zero degrees is analyzed using NASTRAN's transient heat analysis capability. At time t=0 a heat flux is applied on one surface of the plate and simultaneously the temperature along the edges is increased. These temperatures are maintained at a value by using a large heat flux through a good conductor to ground. The problem is one dimensional since it is assumed that no temperature variation exists along the length or through the thickness. Since the plate is symmetric about the center plane, only one half of the plate is modeled.

B. Input

The plate is shown in Figure 1 and the idealized NASTRAN model, shown in Figure 2, is represented by five RØD elements going from the centerplane to the edge. The conductor-ground arrangement is modeled by an ELAS2 element and an SPC card referenced in Case Control. The injected heat flux at the edge is specified using DAREA and TLØAD2 cards which are referenced in Case Control through a DLØAD card. The surface heat flux is specified on a QBDY1 card and references the TLØAD2 card. The time step intervals at which the solution is generated are given on the TSTEP card. The initial temperature conditions are specified on the TEMPD card and referenced in Case Control by an IC card. The heat capacity and conductivity are given on the MAT4 card.

C. Theory

The analytic solution is

$$T(x,t) = \begin{aligned} &0.5 \left[1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} e^{-(2n+1)^2 t} \cos(2n+1)\pi x/2 \right] + \\ &50. \left[(1-x^2) - \frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} e^{-(2n+1)^2 t} \cos(2n+1)\pi x/2 \right] \quad . \end{aligned} \tag{1}$$

D. Results

A comparison of theoretical and NASTRAN results is given in Table 1.

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E. Driver Decks and Sample Bulk Data

```
Card
 No.

 0    NASTRAN FILES=UMF
 1    ID        DEM9041,NASTRAN
 2    UMF       1977     90410
 3    APP       HEAT
 4    SØL       9,1
 5    TIME      10
 6    CEND

 7    TITLE = LINEAR TRANSIENT HEAT ANALYSIS ØF A PLATE
 8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 9-4-1
 9    TEMP(MATERIAL) = 60
10    SPC = 21
11    IC = 60
12    DLØAD = 70
13    TSTEP = 80
14    SET 21 = 21
15    ØUTPUT
16    THERMAL = ALL
17    ØLØAD = ALL
18    SPCF = 21
19    BEGIN BULK
20    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CELAS2	28	3.0+8	20	1	21	1			
CHBDY	31	2	LINE	10	12				
CRØD	11	1	10	12	13	1	12	14	
DAREA	70	20	0	1.5+8					
GRID	10		.0	.0	.0				
MAT4	1	1.0	2.4674						
PHBDY	2		1.0						
PRØD	1	1	1.0						
QBDY1	70	1-0.0	31	33	35	37	39		
SPC	21	21	1						
TEMPD	60	.0							
TLØAD2	70	70	0		.0	100.0			
TSTEP	80	100	.05	2					

Table 1. Theoretical and NASTRAN temperatures.

		GRID(X)					
		10(0.)	12(.2)	14(.4)	16(.6)	18(.8)	20(1.)
t = 0	Theory*	0.	0.	0.	0.	0.	0.
	NASTRAN	0.	0.	0.	0.	0.	0.
t = 1	Theory*	31.282	30.222	26.952	21.204	12.562	.500
	NASTRAN	30.641	29.612	26.433	20.826	12.362	.500
t = 2	Theory*	43.430	41.776	36.780	28.344	16.316	.500
	NASTRAN	43.117	41.478	36.527	28.160	16.218	.500
t = 3	Theory*	47.916	46.026	40.396	30.971	17.696	.500
	NASTRAN	47.755	45.890	40.280	30.887	17.652	.500
t = ∞	Theory	50.500	48.500	42.500	32.500	18.500	.500

* n = 0 term only.

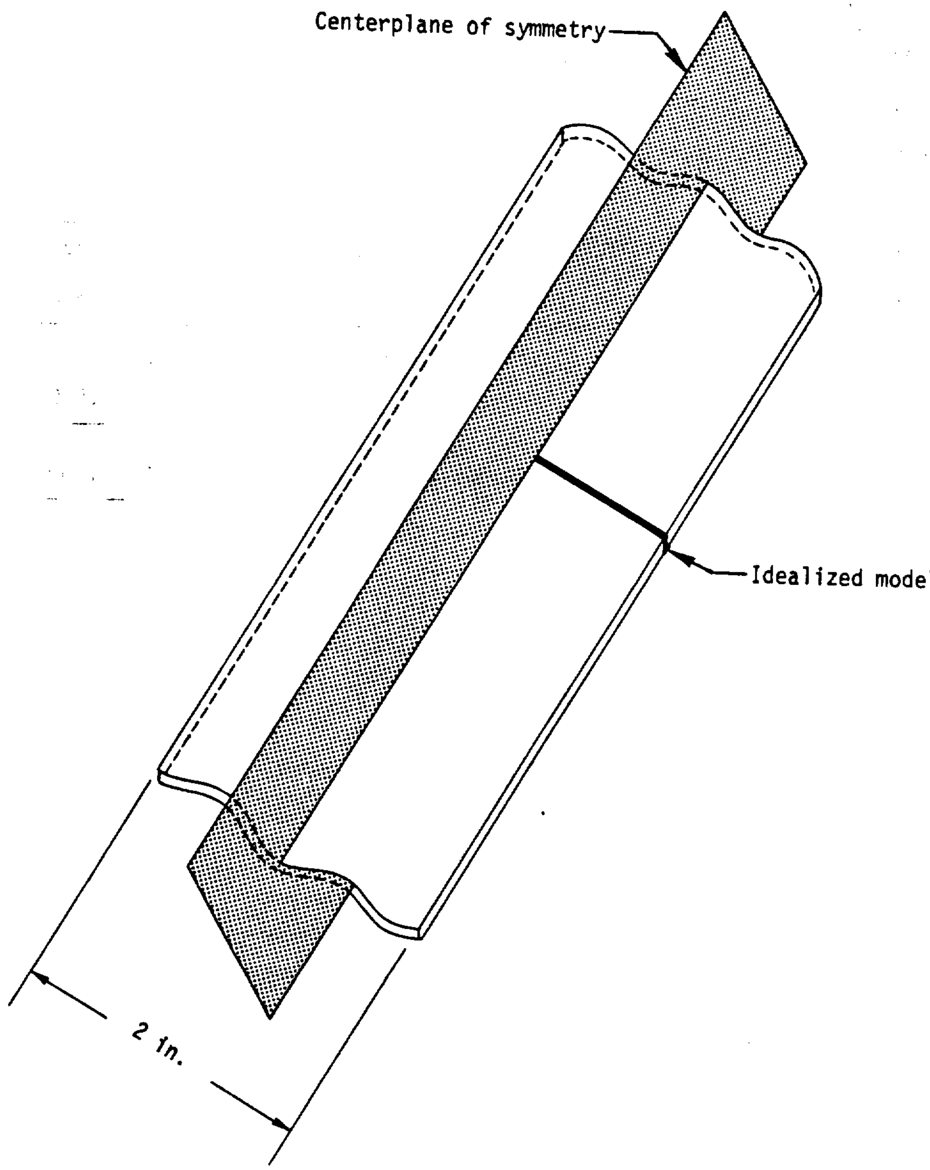

Figure 1. Long thin plate.

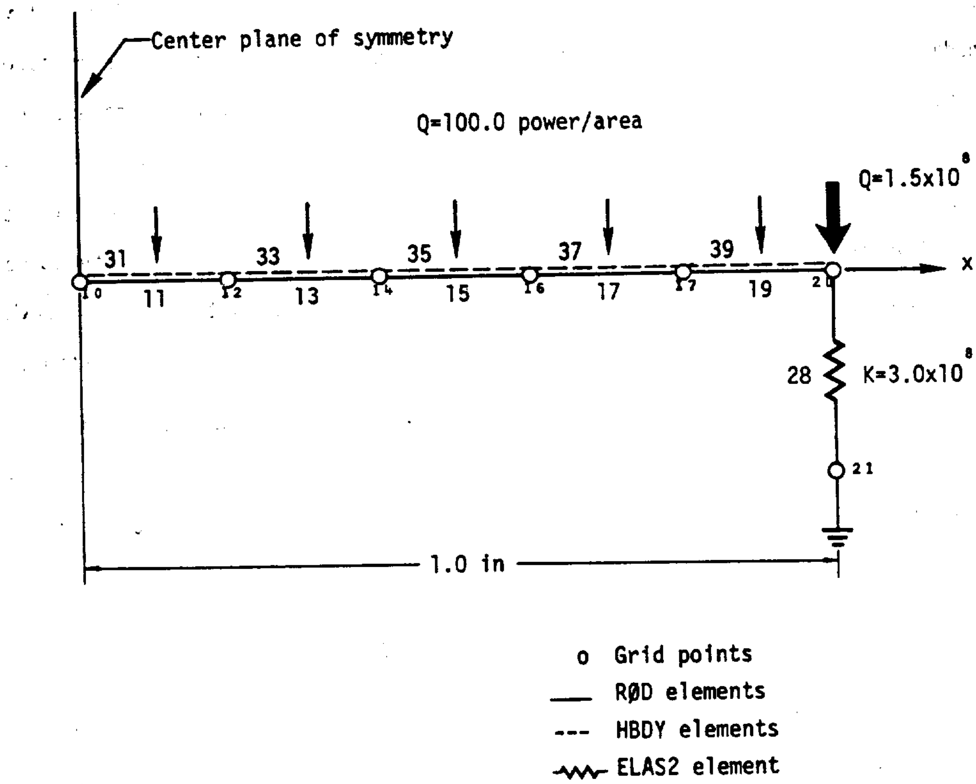

Figure 2. Idealized NASTRAN model.

RIGID FORMAT No. 10, Complex Eigenvalue Analysis - Modal Formulation

Rocket Guidance and Control Problem (10-1-1)

A. Description

This problem, although a simplified model, contains all of the elements used in a linear control system analysis. The flexible structure, shown in Figure 1, consists of three sections: two sections are constructed of structural finite elements; the third section is formulated in terms of its modal coordinates. A sensor is located at an arbitrary point on the structure and connected to a structural point with multipoint constraints. The measured attitude and position of the sensor point is used to generate a control voltage for the gimbal angle of the thrust nozzle. The nozzle control is in itself a servomechanism consisting of an amplifier, a motor, and a position and velocity feedback control. The nozzle produces a force on the structure due to its mass and the angle of thrust. The motion of any point on the structure is dependent on the elastic motions, free-body motions, and large angle effects due to free-body rotation.

The guidance and control system is shown in block diagram form in Figure 2. The definitions for the variables and coefficients along with values for the coefficients are given in Table 1. The use of the Transfer Function data card (TF) allows the direct definition of the various relations as shown in Figure 2.

B. Theory

1. A section of the structure is defined by its modal coordinates by using a modification of the method given in the NASTRAN Theoretical Manual. The algorithm is given as follows:

   Define ξ_i, $i = 1, n$ - modal deflections scalar points

   u_r - grid point components used as nonredundant supports for modal test. These may or may not be connected to the rest of the structure.

   u_c - grid point components to be connected to the remaining structure (not u_r points)

   x_i, $i = 1, n$ - rigid body component degrees of freedom for the nonzero modes

   The relations between these variables are defined by using multipoint constraints with the following relationships:

a) $$\{u_c\} = [\phi_{ci}]\{\xi_i\} + [D_{cr}]\{u_r\} \quad , \tag{1}$$

where ϕ_{ci} is the angular deflection of point u_c for mode i. D_{cr} is the deflection of point u_c when the structure is rigid and point u_r is given a unit deflection.

b) $$\{x_i\} = [K_i]^{-1}[H]^T\{u_r\} = [G]\{u_r\} \quad , \tag{2}$$

where $[K_i]$ is a diagonal matrix. Each term K_i, the modal stiffness, is defined as:

$$K_i = m_i\omega_i^2 \qquad (\omega_i \neq 0) \quad , \tag{3}$$

where m_i is the modal mass and ω_i is the natural frequency in radians per second. [H] is determined by the forces on the support points due to each nonzero eigenvector:

$$P_r = -\sum_i H_{ri}\,\xi_i \qquad (\omega_i \neq 0) \quad . \tag{4}$$

c) Scalar masses and springs are connected to each modal coordinate as shown by Figure 3(a).

d) The structure to be added in this problem consists of a simply supported uniform beam as shown in Figure 3(b). The support points, u_r, are y_{16} and y_{19}. The additional degree of freedom to be connected is $u_c = \theta_{16}$. Four modes are used in the test problem. The following data is used to define and connect the modal coordinates of this substructure.

The mode shapes are

$$\phi_n(x) = \sin\frac{n\pi x}{\ell} \quad . \tag{5}$$

The modal frequencies, masses, and stiffness in terms of normal beam terminology are

$$\omega_n = \frac{n^2\pi^2}{\ell^2}\,\frac{EI}{\rho A} \quad , \qquad (n = 1, 2, 3, 4) \tag{6}$$

$$m_n = \frac{\rho A\ell}{2} \quad , \tag{7}$$

and $$K_n = \frac{n^4\pi^4 EI}{2\ell^3} \quad . \tag{8}$$

The forces of support for each mode are

$$P_y(16) = \sum - \frac{EI\pi^3}{\ell^3} n^3 \quad , \tag{9}$$

and

$$P_y(19) = \sum(-1)^n \frac{EI\pi^3}{\ell^3} n^3 \quad . \tag{10}$$

The motion θ_{16} is defined by multipoint constraints:

$$\theta_{16} = \frac{1}{\ell}(y_{19} - y_{16}) + \sum_n \frac{n\pi}{\ell} \xi_n \quad . \tag{11}$$

The free-body components of the modes are defined, using multipoint constraints, as:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = -\left(\frac{2\ell^3}{\pi^4 EI}\right)\left(\frac{EI\pi^3}{\ell^3}\right) \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix} \begin{Bmatrix} y_{16} \\ y_{19} \end{Bmatrix} \quad . \tag{12}$$

2. The mass of the nozzle would normally be included with the structural modeling. However, to demonstrate the flexibility of the Transfer Function data, it is modeled as part of the guidance system as shown in Figure 4(a).

Defining the angle of thrust, γ, to be measured relative to the deformed structure, the forces which result are

$$T = (I_{no} + x_n^2 m_i)(\ddot{\gamma} + \ddot{\theta}_1) - m_n x_n \ddot{y}_1 \quad , \tag{13}$$

and

$$F_y = m_n y_1 - x_n m_n (\ddot{\gamma} + \ddot{\theta}_1) - F_n \gamma \quad . \tag{14}$$

Using the thrust force, F_n, as a constant, the transfer functions are

$$I_n s^2 \gamma - T + I_n s^2 \theta_1 - x_n m_n s^2 y_1 = 0 \quad , \tag{15}$$

$$m_n s^2 y_1 - (x_n m_n s^2 + F_n)\gamma - x_n m_n s^2 \theta_1 = 0 \quad , \tag{16}$$

and

$$(0)\theta_1 + T = 0 \quad , \tag{17}$$

where

$$I_n = I_{no} + x_n^2 m_n = 500 \quad . \tag{18}$$

3. The large angle motion must be included in the analysis since it contributes to the linear terms. The equations of motion of the structure are formed relative to a coordinate system parallel to the body. As shown in Figure 4(b), the accelerations are coupled when the body rotates.

Since the axial acceleration, $\ddot{x}$, is constant throughout the body, the vertical acceleration at any point, to the first order, is

$$\ddot{y}_{abs} = \ddot{y}_{rel} + \ddot{x}\theta_1 = \ddot{y}_{rel} + \ddot{y}_\theta \quad . \tag{19}$$

An extra degree of freedom y_θ is added to the problem and coupled by the equations:

$$m\ddot{y}_\theta = F_n \theta_1 \quad , \tag{20}$$

and

$$y_{abs} = y_{rel} + y_\theta \quad . \tag{21}$$

4. The center of gravity (point 101) and the sensor location (point 100) are rigidly connected to the nearest structural point with multipoint constraints. For instance the sensor point is located a distance of 4.91 from point 8 as shown in Figure 4(c).

It is desired to leave point 101 as an independent variable point. Therefore point 8 is defined in terms of point 101 by the equations:

$$y_8 = y_{101} + 4.91\theta_{101} \quad , \tag{22}$$

and

$$\theta_8 = \theta_{101} \quad . \tag{23}$$

C. Results

A comparison of the NASTRAN complex roots and those derived by a conventional analysis described below are given in Table 2. The resulting eigenvectors were substituted into the equations of motion to check their validity. The equations of motion for a polynomial solution may be written in terms of the rigid body motions of the center of gravity plus the modal displacements. The equations of motion using Laplace transforms are

$$ms^2\, y_{cg} = F_n(\theta_1 + \gamma) \quad , \tag{24}$$

and

$$Is^2\, \theta_{cg} = -F_n\, x_1\, \gamma \quad . \tag{25}$$

The inertia forces of the nozzle on the structure may be ignored.

The motion of the nozzle, as explained in section B2, is

$$\left(\frac{s^2}{\beta} + \tau s + 1\right)\gamma \cong (a + bs)y_s + (c + ds)\theta_s - \frac{s^2}{\beta}\theta_1 + \frac{s^2 m_n x_n}{\beta I_n} y_1 \quad , \tag{26}$$

where γ is defined as the relative angle between the nozzle and the structure.

The flexible motions at the sensor point, y_s and θ_s, may be defined in terms of the modal coefficients and the rigid motions of the center of gravity:

$$y_s = y_{cg} + x_2\,\theta_{cg} + \sum_i \phi_{100,i}\,\xi_i \quad , \tag{27}$$

and

$$\theta_s = \theta_{cg} + \sum_i \phi'_{100,i}\,\xi_i \quad . \tag{28}$$

The motions of the nozzle point, in terms of the modal and center of gravity motions are

$$\theta_1 = \theta_{cg} + \sum_i \phi'_{1,i}\,\xi_i \quad , \tag{29}$$

and

$$y_1 = y_{cg} - x_1\,\theta_{cg} + \sum_i \phi_{1,i}\,\xi_i \quad . \tag{30}$$

The modal displacements are due primarily to the vertical component of the nozzle force. Their equation of motion is

$$m_i(s^2 + \omega_i^2)\xi_i = F_n\gamma \quad , \tag{31}$$

where

$\phi_{j,i}$ is the deflection of point j for mode i ,

$\phi'_{j,i}$ is the rotation of point j for mode i ,

m_i is the modal mass of mode i ,

ω_i is the natural frequency of mode i ,

and ξ_i is the modal displacement of mode i .

Using two flexible modes the characteristic matrix of the problem is given in Figure 5. The determinant of the matrix forms a polynomial of order 10. The roots of this polynomial were obtained by a standard computer library routine and are presented in Table 2 as the analytical results. The rigid body solution is also presented.

The differences between the two sets of answers is due to the differences in models. The NASTRAN model produces errors due to the finite difference approximation and the number of modes chosen to model the third stage. The polynomial solution produces errors due to the approximations used in the equations of motion as applied to control system problems.

As a further check the first eigenvalue ($\lambda = -1.41$) was substituted into the matrix given in Figure 5 and the matrix was normalized by dividing each row by its diagonal value. The NASTRAN eigenvector was multiplied by the matrix, resulting in an error vector which theoretically should be zero. Dividing each term in the error vector by its corresponding term in the eigenvector resulted in very small error ratios.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0    NASTRAN FILES=UMF
  1    ID         DEM10011,NASTRAN
  2    UMF        1977      100110
  3    TIME       5
  4    APP        DISPLACEMENT
  5    SØL        10,1
  6    DIAG       14
  7    ALTER      103 $
  8    MATGPR     GPLD,USETD,SILD,PHIA // C,N,H / C,N,A  $
  9    ENDALTER
 10    CEND

 11    TITLE = CØMPLEX EIGENVALUE ANALYSIS ØF A RØCKET CØNTRØL SYSTEM
 12    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 10-1-1
 13        LABEL = FLEXIBLE STRUCTURE CASE
 14    MPC = 101
 15        METHØD = 2
 16        TFL = 20
 17        CMETHØD = 11
 18    ØUTPUT
 19    SET 1 = 1,100,101,1010 THRU 1090
 20        SVECTØR(SØRT1,PHASE) = ALL
 21        DISPLACEMENT(SØRT1,PHASE) = 1
 22    BEGIN BULK
 23    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
BARØR					.0	10.0	.0	1	
CBAR	1	10	1	2					
CELAS4	1001	2.0261+7	1001		1002	32.417+7	1002		
CMASS4	2001	2.5+3	1001	2001	2002	2.5+3	1002	2002	
CØNM2	101	1		3333.333					
EIGC	11	DET	MAX						+EC
+EC	-2.0	-1.0	-2.0	10.0	10.0	6	6		
EIGP	11	.0	.0	2					
EIGR	1	INV	.0	1.0	1	2	2		+E1
+E1	MASS								
EPØINT	1010	1011	1030	1040	1050	1060	1070	1080	
GRDSET							1345		
GRID	1		.0	.0	.0				
MAT1	1	10.4+6	4.0+6						
MPC	3	16	6	-1.0	1001		.0628318		+161
+161		1002		.1256637	1003		.1884935		+162
MPCADD	101	100	3						
PARAM	GRDPNT	101							
PBAR	10	1	4.0+2	6.0+4	6.0+4				
SEQGP	100	10.5	101	7.5					
SPØINT	1001	1002	1003	1004	2001	2002	2003	2004	
SUPØRT	101	2	101	6					
TF	20	1	2	.0	.0	50.0			+T6
+T6	1	6	.0	.0	-150.0				+T61

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Table 1. Variables and Parameters

Extra Point Number	Symbol	Description
1010	e_y	Voltage describing y
1011	e_θ	Voltage describing θ
1020	E_{yc}	Control voltage for y (Input)
1021	$E_{\theta c}$	Control voltage for θ (Input)
1030	E_γ	Attitude error function
1040	ε_γ	Nozzle position error
1050	E_m	Voltage for Nozzle servo
1060	T	Torque for Nozzle servo
1070	γ	Nozzle Thrust angle relative to structure
1080	y_θ	Position increment due to attitude

Parameters	Value	Description
K_s	1.0	Servo amplifier gain
K_m	500	Servo gain
τ	.1414	Nozzle angular velocity feedback
x_n	3.0	Distance from nozzle C.G. to Gimbal axis
I_n	500.0	Inertia of Nozzle about gimbal axis
F_n	4.25×10^6	Thrust of Nozzle
m_n	50	Nozzle mass
β_θ	100.0	Overall voltage-to-angle ratio
β_y	1.0	Overall voltage to postton ratio
a	.16	Position feedback coefficient
b	.28	Velocity feedback coefficient
c	15.0	Angle feedback coefficient
d	7.0	Angular velocity feedback coefficient
m	8.5×10^4	Mass of structure

Table 2. Comparison of Complex Roots for NASTRAN Modeling vs. Simplified Polynomial Expansion

Rigid Body Model		2 Flexible Modes Model	
NASTRAN	POLYNOMIAL	NASTRAN	POLYNOMIAL
-.540 ± .821i	-.522 ± .802i	-.507 ± .819i	-.494 ± .801i
-1.68 ± 0i	-1.74 ± 0i	-1.41 ± 0i	-1.46 ± 0i
+.751 ± 5.96i	+.774 ± 5.98i	+.520 ± 3.82i	+.522 ± 3.83i

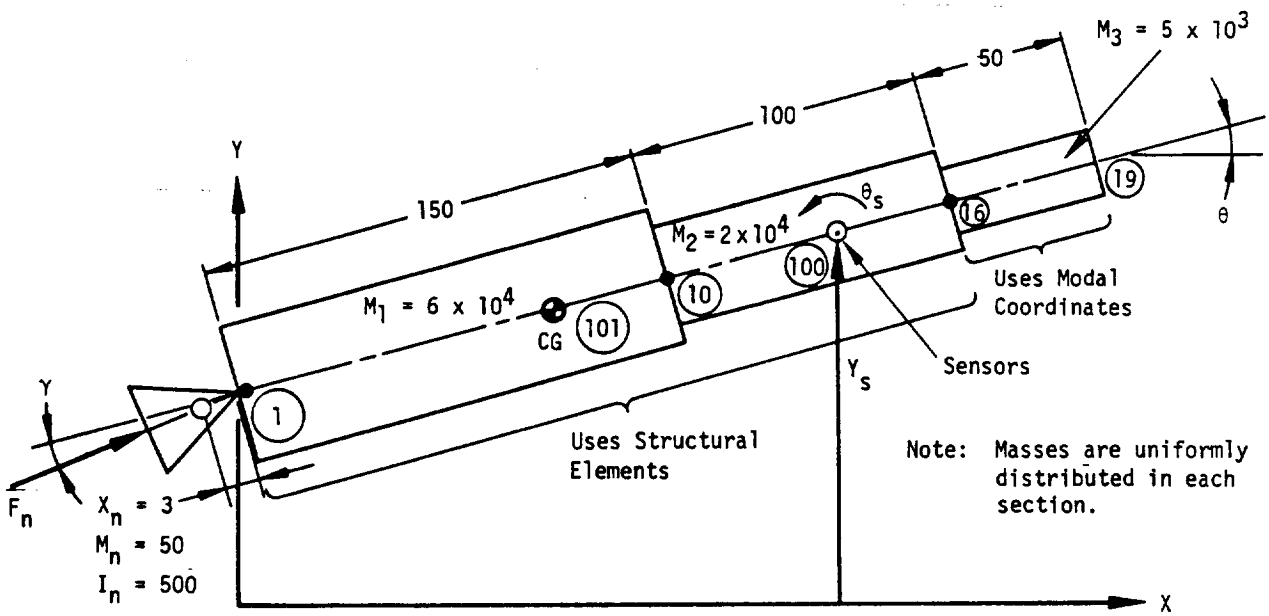


Figure 1. Rocket structural model.

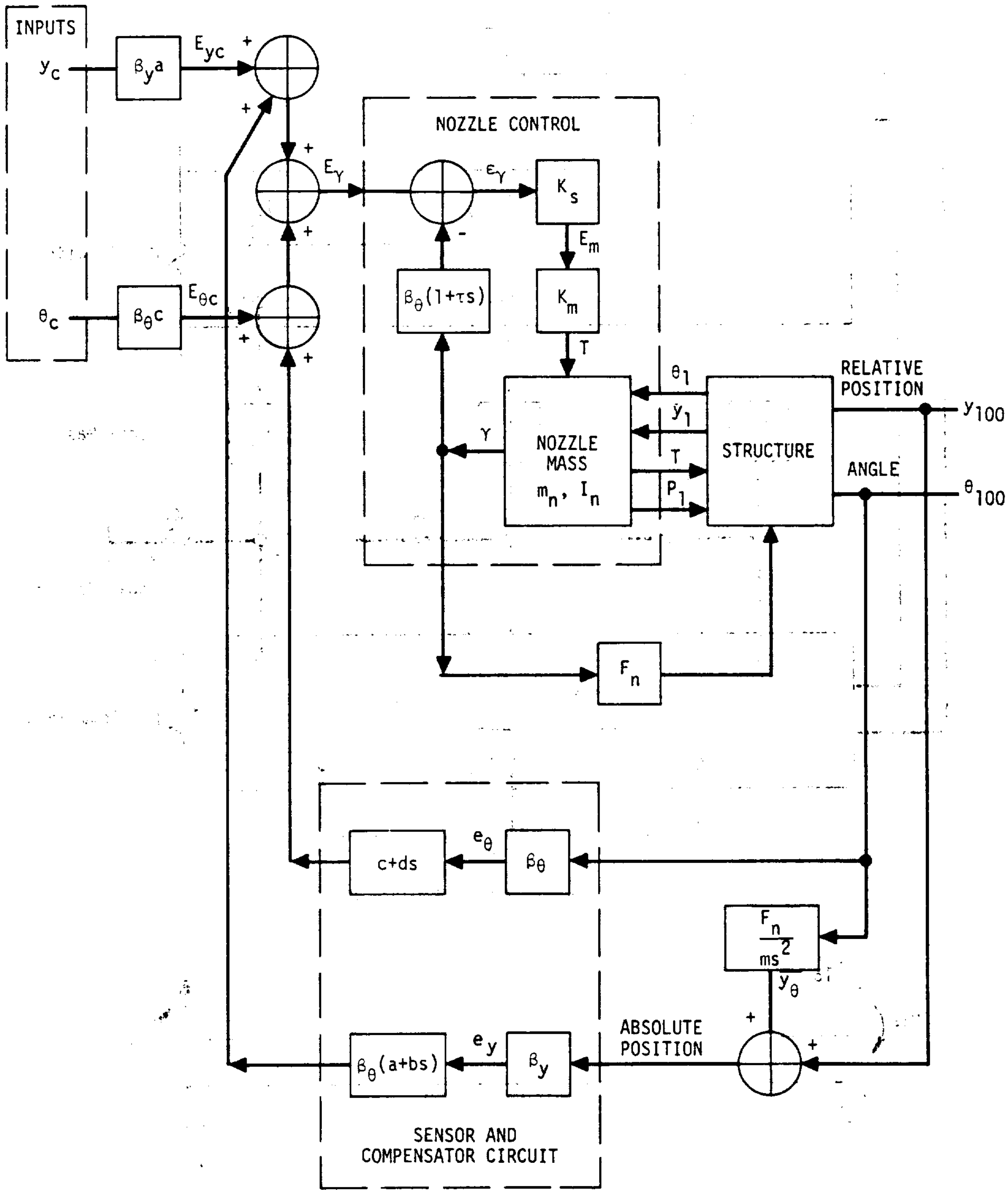
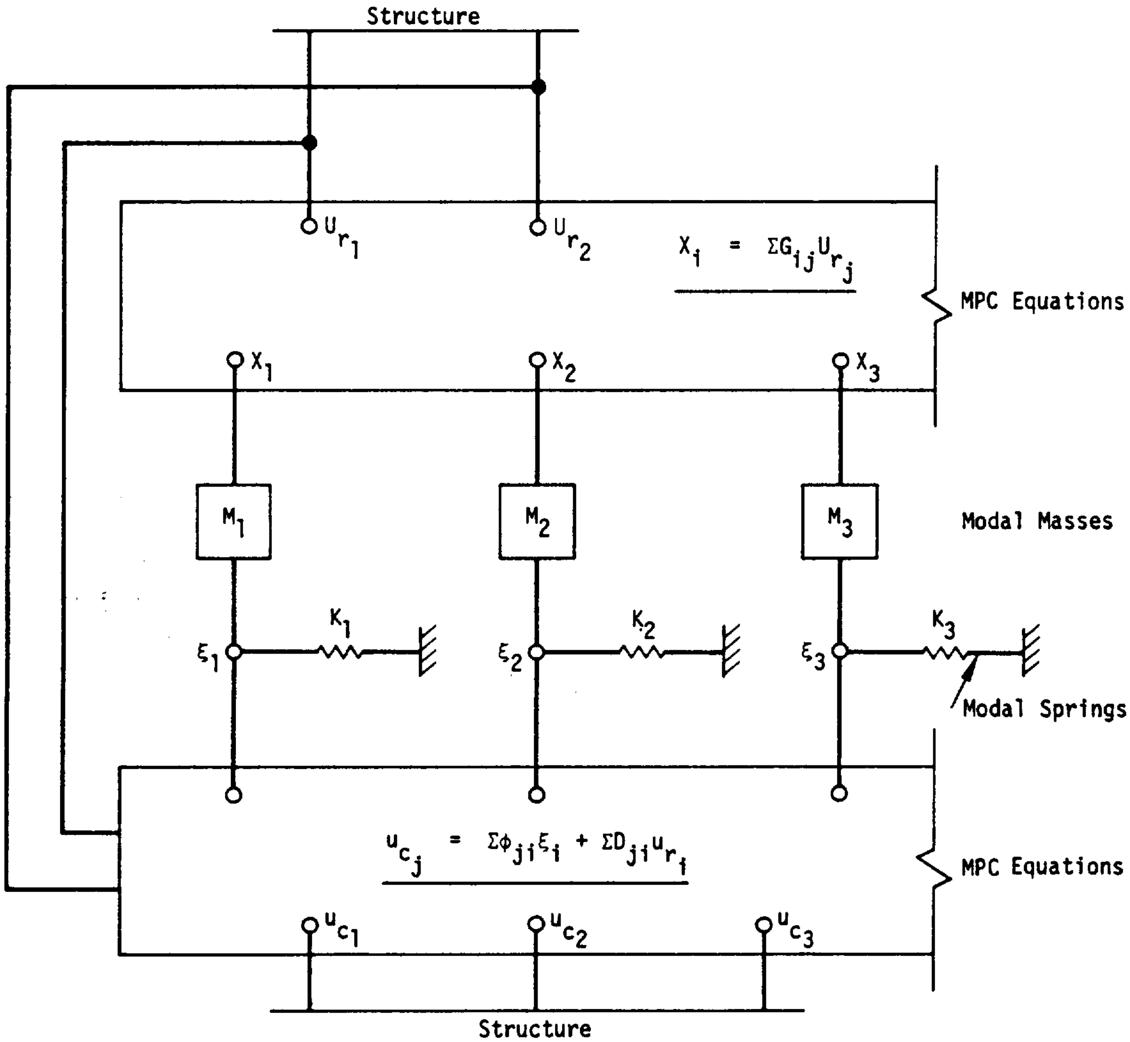
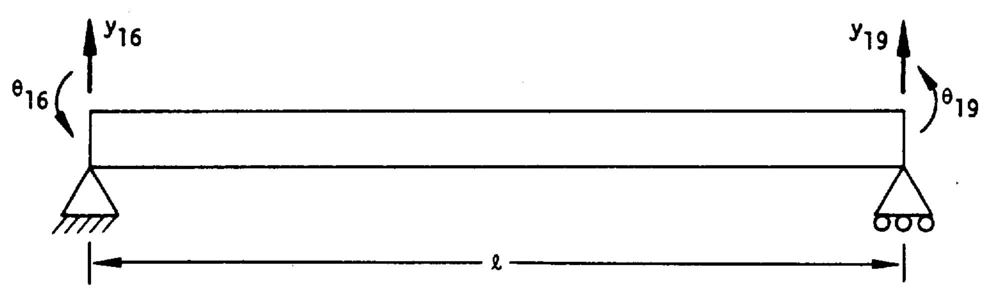


Figure 2. Overall system diagram.

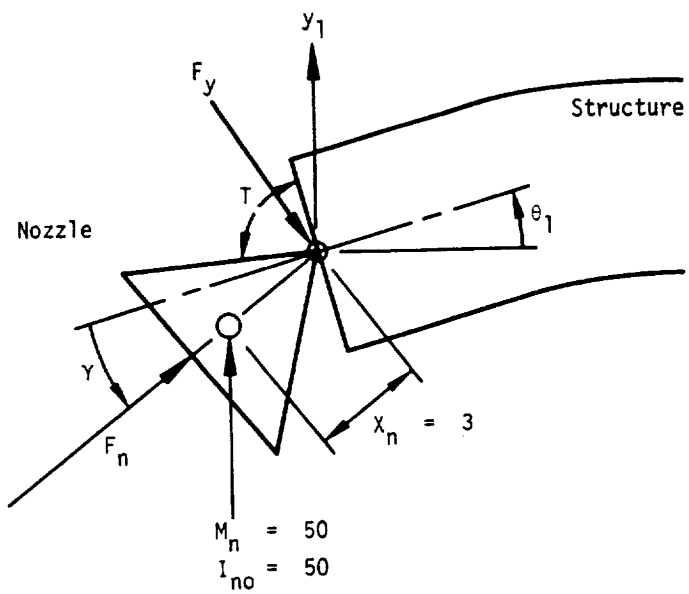


(a) Diagram for input of modal data

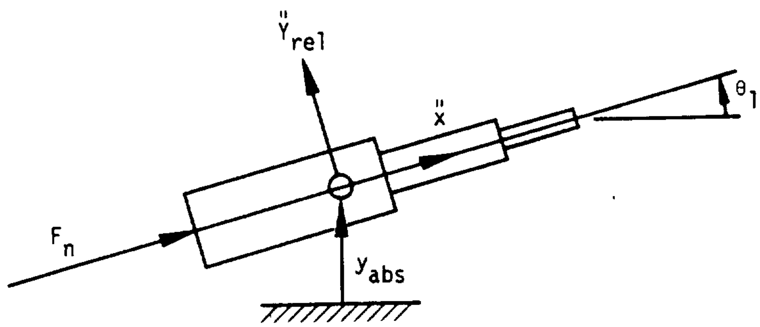


(b) Structure used for modal data

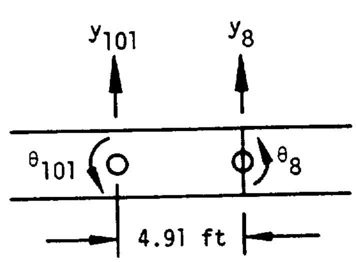
Figure 3. Modal data input diagrams.



(a) Nozzle displacements and forces



(b) Relative motion due to large angles



(c) Relationship for multi-point constraints

Figure 4. Modeling diagrams.

y_{cg}	θ_{cg}	γ	ξ_1	ξ_2
s^2	$-\frac{F_n}{m}$	$-\frac{m_n x_n s^2 + F_n}{m}$	$-\frac{F_n \phi'_{1,1}}{m}$	$-\frac{F_n \phi'_{1,2}}{m}$
0	s^2	$\frac{1}{I}(I_n + m_n x_n x_1)s^2$ $+ x_1 \frac{F_n}{I}$	0	0
$-\frac{m_n x_n}{I_n} s^2$ $-(bs + a)\beta$	$1 + \frac{m_n x_n x_1}{I_n} s^2$ $-(d + x_2 b)\beta s$ $-(c + x_2 a)\beta$	$s^2 + \beta\tau s + \beta$	$(\phi'_{1,1} - \frac{m_n x_n}{I_n} \phi_{1,1})s^2$ $-\beta(b\phi_{s,1} + d\phi'_{s,1})s$ $-\beta(a\phi_{s,1} + c\phi'_{s,1})$	$(\phi'_{1,2} - \frac{m_n x_n}{I_n} \phi_{1,2})s^2$ $-\beta(b\phi_{s,2} + d\phi'_{s,2})s$ $-\beta(a\phi_{s,2} + c\phi'_{s,2})$
0	0	$-\phi_{1,1}F_n$	$m_1(s^2 + \omega_1^2)$	0
0	0	$-\phi_{1,2}F_n$	0	$m_2(s^2 + \omega_2^2)$

Figure 5. Matrix of equations of motion, analytic approach.

RIGID FORMAT No. 10 (APP AERØ), Aeroelastic Analysis
Aeroelastic Flutter Analysis of a 15° Swept Wing (10-2-1)

A. Description

This problem illustrates the use of the aeroelastic analysis to determine flutter frequencies and mode shapes for an untapered wing having 15° sweep and an aspect ratio of 5.34 as shown in Figure 1.

B. Input

Bulk data cards used include CAERØ1, PAERØ1, SPLINE2, SET1, AERØ, MKAERØ1, FLUTTER, and FLFACT as illustrated in User's Manual Section 1.11.

C. Theory

Reference 22 specifies the reduced frequency k = .1314 (p.17), frequency ratio $\omega/\omega_\alpha = 0.51$ (p.35) and torsion frequency $\omega_\alpha = 1488$ (p.17).

The flutter velocity is found from

$$V = \frac{b\omega}{k} = \frac{\frac{REFC}{2} \times \omega_\alpha \times \frac{\omega}{\omega_\alpha}}{k} = 5980 \text{ in/sec}, \tag{1}$$

where REFC is the reference length input on the AERØ bulk data card.

The flutter frequency is found from

$$f = \frac{\omega_\alpha \times \frac{\omega}{\omega_\alpha}}{2\pi} = 121 \text{ Hz} \tag{2}$$

D. Results

The results obtained are compared with both theoretical results using the modified strip analysis method and with experimental results. The flutter velocity is in good agreement. (See Figure 2.)

Frequencies are automatically output while mode shapes used in the modal formulation are obtained using an ALTER to the Rigid Format following the Real Eigenvalue Analysis Module.

Mode shapes for all points in the model may be obtained by checkpointing the problem using the Normal Mode Analysis (Rigid Format 3) and subsequently restarting using the Aeroelastic Analysis.

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E. Driver Decks and Sample Bulk Data

Card
No.

```
 0   NASTRAN FILES=UMF
 1   ID         DEM10021,NASTRAN
 2   UMF        1977      100210
 3   APP        AERØ
 4   SØL        10,0
 5   TIME       10
 6   DIAG       14
 7   ALTER      94 $
 8   MATGPR     GPL,USET,SIL,PHIA//C,N,FE/C,N,A $
 9   ENDALTER $
10   DIAG 18
11   CEND

12   TITLE = AERØELASTIC FLUTTER ANALYSIS ØF A FIFTEEN DEGREE SWEPT WING
13   SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 10-2-1
14   LABEL = K VALUES .200(*)  .167(0)  .143(1)  .125(2)  .111(3)  .100(4)
15   ECHØ = BØTH
16   SPC = 1
17   METHØD = 10
18   CMETHØD = 20
19   FMETHØD = 30
20   ØUTPUT(XYØUT)
21   XTITLE = VELØCITY
22   YTTITLE = DAMPLING (G)
23   YBTITLE = FREQUENCY (F)
24   TCURVE = V-G AND V-F DATA PØINTS
25   CURVELINESYMBØL = -1
26   XYPAPERPLØT VG / 1(G,F) 2(G,F) 3(G,F) 4(G,F) 5(G,F) 6(G,F)
27   BEGIN BULK
28   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
AERØ	0	1.3+4	2.0706	1.145-7					
CAERØ1	101	1	1	6	4			1	+CA101
+CA101	-1.	-.26795	0.0	2.0706	-1.	5.45205	0.0	2.0706	
CBAR	1	1	1	2	0.0	0.0	1.	1	
CMASS2	12	2.8-6	2	5					
CØRD2R	1		0.0	0.0	0.0	0.0	0.0	1.	+C1
+C1	.96593	-.25882	0.0						
EIGC	20	HESS	MAX						+EC
+EC									
EIGR	10	GIV	.3	.1		6			+ER
+ER	MAX								
FLFACT	1	.967							
FLUTTER	30	K	1	2	3	L	3		
GRDSET		1				1	126		
GRID	1		0.0	.0	0.0				
MAT1	1	10.4+6	3.9+6		2.61-4				
MKAERØ1	.45								+MK
+MK	0.0001	.1	.2						
PAERØ1	1								
PARAM	CØUPMASS	1							
PARAM	LMØDES	3							
PBAR	1	1	7.175-2	9.83-6		36.8-6			
SET1	100	1	THRU	11					
SPC1	1	345	1						
SPLINE2	100	101	101	124	100	0.0	1.	1	+SP
+SP	0.0	0.0							

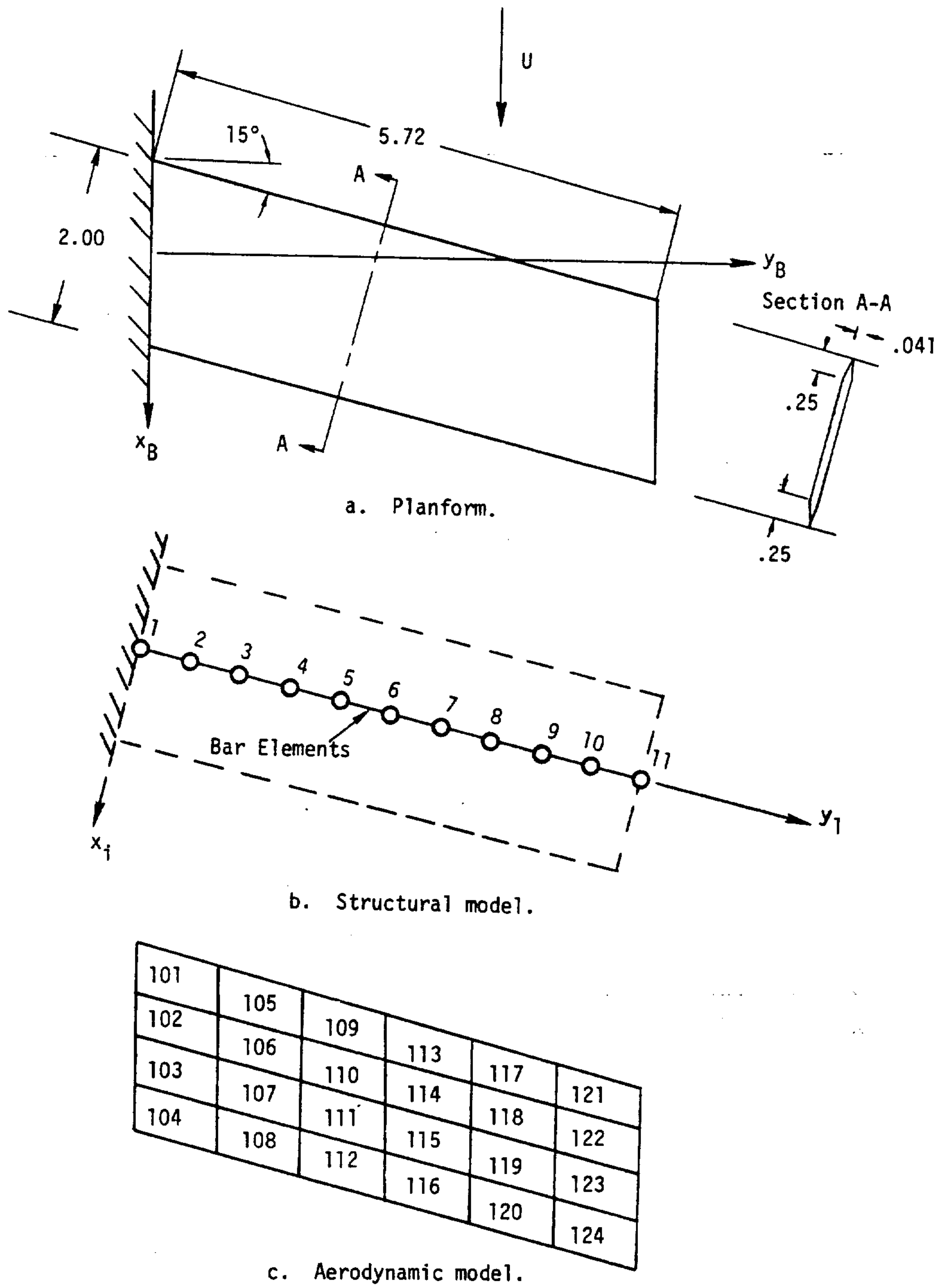

Figure 1. Fifteen degree sweep model.

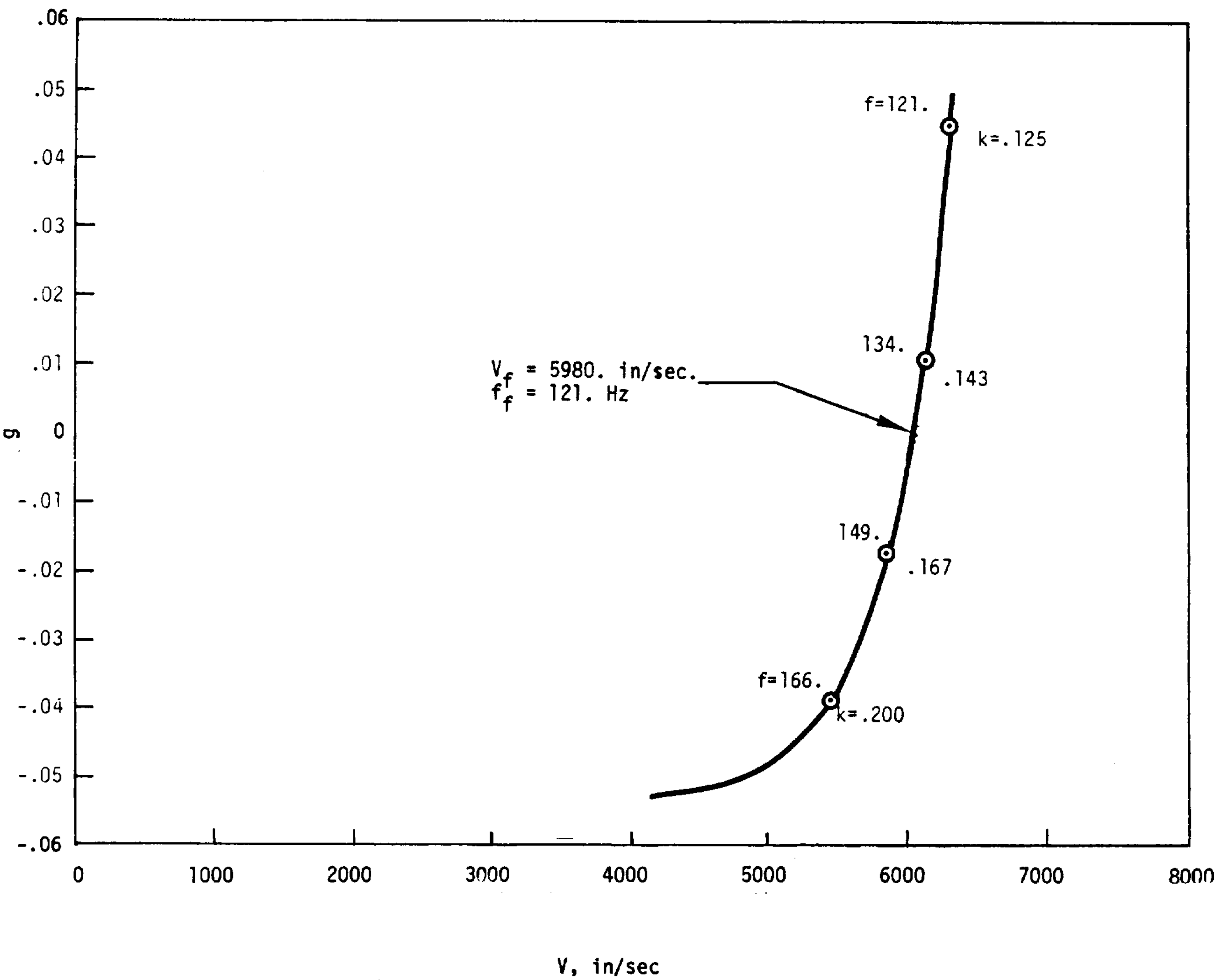

Figure 2. V-g results for fifteen degree sweep model.

RIGID FORMAT No. 11, Frequency Response - Modal Analysis

Frequency Response and Random Analysis of a Ten-Cell Beam (11-1-1)

A. Description

This problem demonstrates the frequency response solution of a structure using uncoupled modal formulation. With modal formulation, the structural degrees of freedom used in the solution are the uncoupled modal displacements. The solution equations are simple and efficient. The saving in time, however, is offset by the operations necessary to extract the modes, transform the loads to modal coordinates, and transform the modal displacements to structural displacements.

This problem also illustrates the various methods of applying frequency response loads. Loads may be input as complex numbers, with phase lag angles and/or time lag factors. The loads may be added together for each subcase.

The structure to be solved consists of a beam with simple supports on the end as described in Figure 1. The parameters selected produce natural frequencies of 50, 200, 450 and 800 cps. The applied loads for the three subcases are applied to the center with variations in phase angles, time lags and input formats. The first two subcases use three loaded points which, in essence, simulate a load on the center.

Included in the structural representation is a "general element" representing the first two cells of the ten-cell beam. The flexibility matrix, [Z], of the element represents the displacements of grid points 2 and 3 when point 1 is fixed. The rigid body matrix, [S], represents the rigid body motions of points 2 and 3 when point 1 is displaced in the x, z, or θ_y directions.

The random analysis data consists of a flat power spectral density function ("white noise") for the three loading subcases. The first subcase spectral density is connected to the third subcase spectral density, simulating two interdependent probability functions. The XY-plotter is used to plot the displacement and acceleration power spectral density function of grid 6 (center of the beam). The displacement autocorrelation function is also plotted for the same point. All values are tabulated in the printout. The NASTRAN power spectural density results are compared against a simplified analytic calculation in Figure 2.

A static analysis restart of the frequency response problem is demonstrated. Gravity and element enforced deformation loads are used with a change in the single-point constraints.

B. Input

1. Parameters:

$\ell = 20$ - length

$I_1 = .083$ - bending inertia

$A = 21.18922$ - cross sectional area

$E = 10.4 \times 10^6$ - modulus of elasticity

$\rho = .2523 \times 10^{-3}$ - mass density

$M = \underline{\rho A \ell}$ - total mass

2. Constraints:

$u_y = \theta_x = \theta_z = 0$ - all points

$u_{x1} = u_{z1} = u_{z11} = 0$ - frequency response

$u_{x1} = u_{z1} = u_{x11} = u_{z11} = 0$ - static analysis

3. Modal Data:

Interval: $40 < f < 1000$ cps

Normalization: Modal Mass = 1.0

Number of modes used in formulation: 4

Modal Damping ratio: $g = 4 \times 10^{-4} f$

4. Loads, Frequency Response:

The loading functions for subcase 1 are:

$$P_{z,5} = 50$$

$$M_{y,5} = -100$$

$$P_{z,6} = 50 + \underbrace{100(\cos 60° + i \sin 60°)}_{\text{SET 6}}$$

$$P_{z,7} = 50$$

$$M_{y,7} = 100$$

The loading for subcase 2 is:

$$P_{z,5} = 50$$

$$M_{y,5} = -100$$

$$P_{z,6} = 50 + \underbrace{100(\cos 2f° - i \sin 2f°)}_{\text{SET 7, } \tau = .005555}$$

$$P_{z,7} = 50$$

$$M_{y,7} = 100$$

The load for subcase 3 is:

$$P_{z,6} = 2[75 + 50i(\cos 30° - i \sin 30°)] = 200 + 86.6i$$

Note: At f = 30cps the three subcases are nearly identical.

5. Random Analysis Data

The nonzero factors for the three subcases are:

$$\left.\begin{array}{l} S_{11} = 50 \\ S_{13} = S_{31} = 50 \\ S_{22} = 100 \\ S_{33} = 50 \end{array}\right\} \quad 0 < f < 100$$

$$S_{ij} = 0 , \quad f > 100$$

The time lags selected for the autocorrelation function calculations are:

$$\tau = 0.0, 0.001, 0.002, \ldots\ldots, 0.1$$

6. Static Loads for Restart

The problem is run first as a frequency response analysis. It is restarted as a static analysis with the following loads:

Gravity vector: $g_z = 32.2$

Element Deformation: $\delta_{10} = 0.089045$ (expansion)

C. Theory

1. The theoretical eigenvalue data, according to Reference 8 is

$$f_n = \frac{n^2\pi^2}{(2\pi)\ell^2}\sqrt{\frac{EI}{A}} = 50,\ 200,\ 450,\ 800\ \ldots \qquad \text{(natural frequencies)} \quad , \qquad (1)$$

$$m_n = 1.0 \qquad \text{(modal mass)} \quad , \qquad (2)$$

and

$$\phi_n(x) = \left[\int_0^\ell \rho A \sin^2 \frac{n\pi x}{\ell}\, dx\right]^{-\frac{1}{2}} \sin\left(\frac{n\pi x}{\ell}\right) = \sqrt{\frac{2}{M}}\ \sin\left(\frac{n\pi x}{\ell}\right) \qquad \text{(mode shape)} \ . \qquad (3)$$

2. The theoretical frequency response at the center point is essentially the response of the first mode which is

$$u_6(\omega) = \frac{\sum_j \phi_{1,6}\, P_j(\omega)\, \phi_{1,j}}{m_1(\omega_1^2 - \omega^2 + ig\omega\omega_1)} \qquad (j = \text{degree of freedom number}) . \ (4)$$

At the first natural frequency of 50 cps, the response will be nearly equal to the response of the first mode. The response at the center point for the three subcases are·

$$u_6^1 = u_6^3 = \frac{94.764 + 41.033i}{(50-f^2) + if} \quad , \qquad \text{(Subcases 1 and 3)} \qquad (5)$$

and

$$u_6^2 = \frac{23.691(3 + 2\cos 2f - 2i \sin 2f)}{(50 - f^2) + if} \ . \qquad \text{(Subcase 2)} \qquad (6)$$

3. The random analysis is explained in Reference 15. The power spectral response coefficients for the three subcases are given by the matrix:

$$[S_\ell] = 100 \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 1.0 & 0 \\ 0.5 & 0 & 0.5 \end{bmatrix} . \qquad (7)$$

If $\{H_j\}$ is the vector of the responses of a point, j, to the three loading cases, the power spectral response, S_j, is

$$S_j = \{\bar{H}_j\}^T[S]\{H_j\} \qquad [\bar{H}_j \text{ is the complex conjugate}) \quad , \qquad (8)$$

or $$S_j = 100[0.5|H_{1j}|^2 + 0.5(\bar{H}_{1j} H_{3j} + \bar{H}_{3j} H_{1j}) + |H_{2j}|^2 + 0.5|H_{3j}|^2] \quad . \tag{9}$$

Since $H_{1j} = H_{3j}$, then:

$$S_j = 200|H_{1j}|^2 + 100|H_{2j}|^2 \quad . \tag{10}$$

The mean square response in obtained by integrating the power spectral density over the frequency. In this particular case the frequency increments are uniform and the mean square response is simply

$$E_i = \sum_i \pi[S_j(f_{i+1}) - S_j(f_i)]\Delta f \quad . \tag{11}$$

The analytic solution for the displacement spectral density response of the center point due to the first mode is:

$$S_j(f) = \frac{200(1.066 \times 10^4) + 100(.5613 \times 10^3)(13 + 12\cos 2f)}{[(50^2 - f^2)^2 + f^2]} = \frac{2.862 \times 10^6 + .6735 \times 10^6 \cos 2f}{(f^4 - 4999f^2 + 50^4)} \quad . \tag{12}$$

The mean deviation, σ_j, is

$$\sigma_j = \sqrt{\frac{E_i}{2\pi(f_n - f_o)}} \quad , \tag{13}$$

where f_n and f_o are the upper and lower frequency limits.

4. The results of the static analysis restart are

a) The gravity load produces normal displacements (in the z direction) and element moments as follows:

$$u_z(x) = \frac{\rho A g x}{24EI} (\ell^3 - 2\ell x^2 + x^3) \quad , \tag{14}$$

and $$M_1(x) = \frac{\rho A g}{2} (x^2 - \ell x) \quad . \tag{15}$$

b. The element deformation produces the following axial forces and displacements:

$$F_x = AE\frac{\delta 10}{\ell} \quad , \tag{16}$$

and

$$u_x = -\frac{F_x}{AE}x \quad (x < 18) \quad . \tag{17}$$

D. Results

The response at the center point for Subcases 1 and 3 are

f	u_6 (one mode)	u_6 (NASTRAN)
0	.0413 @ 23.42°	.0429 @ 22.9°
30	.0646 @ 22.34°	.0668 @ 21.8°
50	2.066 @ 293.42°	2.074 @ 281.5°

The response at the center point for Subcase 2 is

f	u_6 (one mode)	u_6 (NASTRAN)
0	.047 @ 0°	.049 @ 0°
30	.0646 @ -22.34°	.0668 @ -23.97°
50	1.565 @ 233.4°	1.577 @ 223.0°

The results from Equation 12 are compared with the NASTRAN results in Figure 2. Equation 13 can be checked by summing the NASTRAN results.

In numerical terms, the displacements of the center point ($x = \frac{\ell}{2}$) are

Theoretical	NASTRAN
$u_{x6} = 4.452 \times 10^{-2}$	4.435×10^{-2}
$u_{z6} = 4.155 \times 10^{-4}$	4.121×10^{-4}

The element forces at the center of the beam are:

Theoretical	NASTRAN
$F_{x5} = -.9811 \times 10^6$	$-.9848 \times 10^6$
$M_6 = -8.607$	-8.607

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E. Driver Decks and Sample Bulk Data

Card No.

```
NASTRAN FILES=(UMF,PLT2,NPTP)
ID        DEM11011,NASTRAN
UMF       1977      110110
CHKPNT    YES
APP       DISPLACEMENT
SØL       11,3
DIAG      14
TIME      6
ALTER     112 $
MATPRN    PHIA,,,,// $
ENDALTER
CEND

MAXLINES = 50000
TITLE = FREQUENCY RESPØNSE AND RANDØM ANALYSIS ØF A 10 CELL BEAM
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-1-1
    SPC = 11
    METHØD = 2
    FREQUENCY = 508
    RANDØM = 11
    SDAMPING = 11
ØUTPUT
    SET 2 = 5,10
    SET 6 = 6
    SET 10 = 6,11
    DISP(SØRT2,PHASE) = 10
    ACCELER(SØRT2,PHASE) = 10
    ØLØAD = 6
    ELFØRCE(SØRT2,PHASE) = 2
SUBCASE 1
    LABEL = THREE PØINTS LØADED WITH TWØ SETS
    DLØAD = 506
SUBCASE 2
    LABEL = ØNE PØINT LØADED WITH TWØ SETS AND TIME DELAYS
    DLØAD = 507
SUBCASE 3
    LABEL = ØNE PØINT LØADED WITH TWØ TABULAR LØADS
    DLØAD = 510
$
$ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
$
$
PLØTID = NASTRAN PRØBLEM NØ. 11-1-1
ØUTPUT(XYØUT)
PLØTTER = SC
    CAMERA = 3
    SKIP BETWEEN FRAMES = 1
    XGRID LINE = YES
    YGRID LINE = YES
  XTITLE =                          FREQUENCY (HERTZ)
  YTITLE = S
  TCURVE = PØWER SPECTRAL DENSITY ØF PØINT 6 DISPLACEMENT
  XYPLØT,XYPRINT DISP PSDF /  6(T3)
$
  TCURVE = PØWER SPECTRAL DENSITY ØF PØINT 6 ACCELERATIØN
  XYPLØT ACCELERATION PSDF / 6(T3)
$
  XTITLE =                          TIME LAG (SECØNDS)
```

```
Card
 No.

 57      YTITLE = R
 58      TCURVE = AUTØCØRRELATIØN FUNCTIØN FØR PØINT 6 DISPLACEMENT
 59      XYPLØT,XYPRINT DISP AUTØ /  6(T3)
 60    BEGIN BULK
 61    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CBAR	3	1	3	4	20.	.0	1.	1	
CØNM2*	11		1				5.34604-	3	*M1
*M1	.0								
DAREA	2	5	5	-100.					
DELAY	1	6	3	.5555-2					
DLØAD	506	1.	1.	5	1.	6			
DPHASE	1	6	3	30.					
EIGR	2	INV	40.0	1000.0	3	5			+EG
+EG	MASS								
FREQ1	508	.0	5.0	40					
GENEL	1101		2	1	2	3	2	5	+1
+1	3	1	3	3	3	5			+2
GRDSET							246		
GRID	1		.0	.0	.0				
MAT1	1	10.4+6	4.+6		.2523-3				
PARAM	GRDPNT	0							
PARAM	LMØDES	4							
PBAR	1	1	21.18922	.083	.083				
RANDPS	11	1	1	.5		11			
RANDT1	11	100	.0	.1					
RLØAD1	5101	510			5101				
RLØAD2	5	2			1				
SPC	1	1	13		11	13			
TABDMP1	11								+DAMP
+DAMP	.0	.0	50.0	.02	ENDT				
TABLED1	1								+TAUU
+TAUU	.0	1.	100.	1.	ENDT				
TABRND1	11								+TR
+TR	-1.0	.0	.0	100.0	100.0	100.0	100.0	.0	+TR2

```
Card
 No.

  0     NASTRAN FILES=ØPTP
  1     ID        DM11011A,RESTART
  2     APP       DISPLACEMENT
  3     SØL       1,9
  4     DIAG      14
  5     TIME      5
  6     CEND

  7     TITLE = 10 CELL BEAM   RESTART WITH ENFØRCED DEFØRMATIØN, GRAVITY LØAD
  8     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-1-1A
  9     LABEL = RIGID FORMAT SWITCH FROM 11 TO 1
 10         SPC = 1
 11     DEFØRM = 1102
 12     LØAD = 1101
 13         ØUTPUT
 14              ECHØ = BØTH
 15              DISPLACEMENTS = ALL
 16              ØLØAD = ALL
 17              ELFØRCE = ALL
 18     BEGIN BULK
```

Card No.	1	2	3	4	5	6	7	8	9	10
19	DEFØRM	1102	10	.089045						
20	GRAV	1101		32.2	0.0	0.0	0.0			
21	ENDDATA									

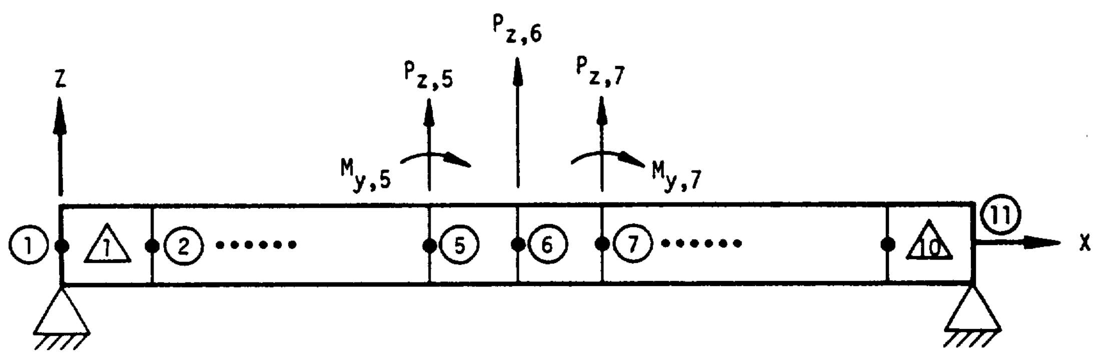

Figure 1. 10 cell beam.

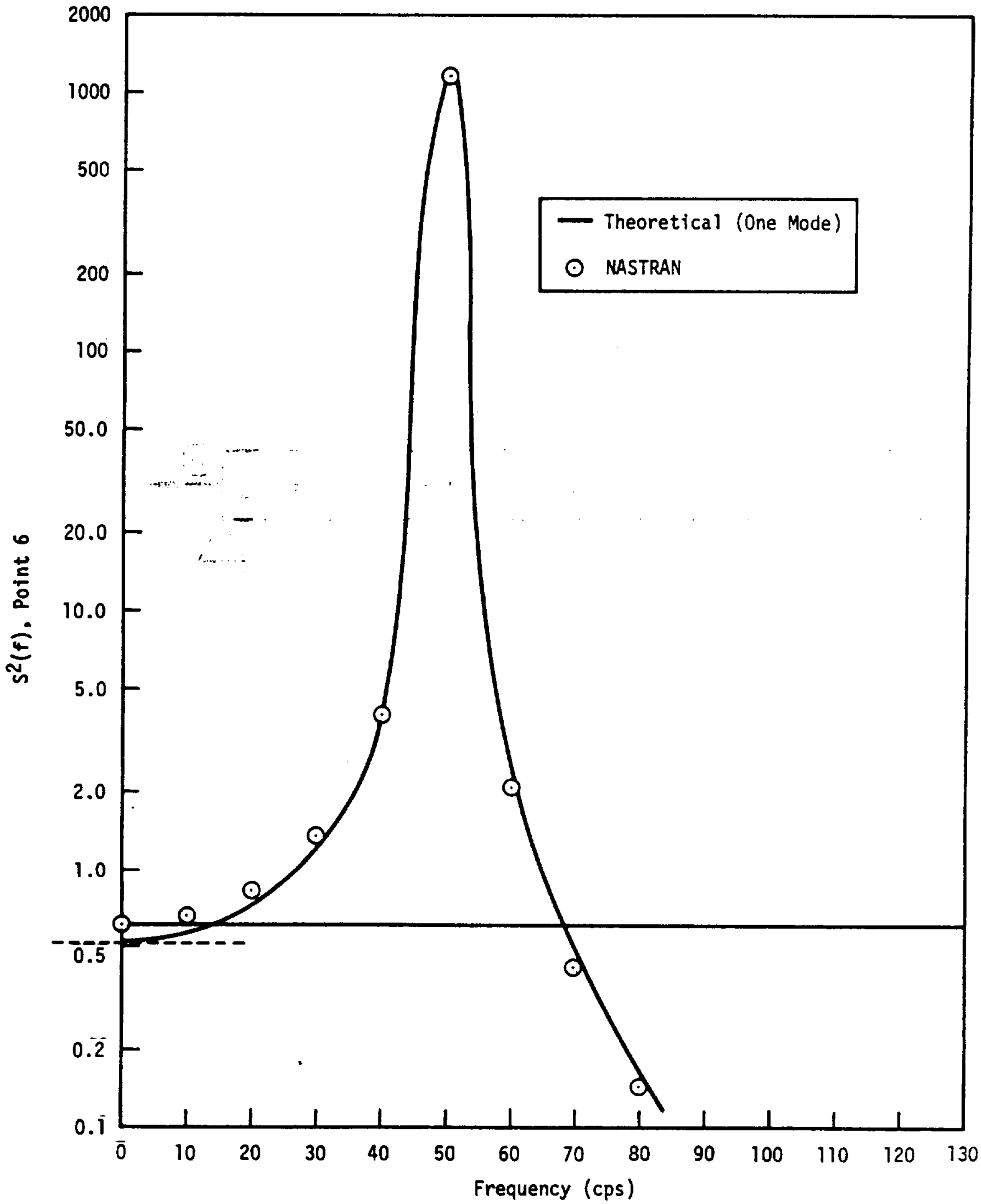

Figure 2. Power spectral density of center point displacement.

RIGID FORMAT No. 11, Frequency Response Analysis - Modal Formulation

Frequency Response of a 500-Cell String (11-2-1)
Frequency Response of a 500-Cell String (INPUT, 11-2-2)

A. Description

This problem illustrates the solution of a large frequency response problem using modal coordinates. When large numbers of frequency steps are used, or the problem is very large, the relative efficiency of the modal formulation is more attractive than the direct formulation. The structural model consists of scalar points, springs, and masses which simulate the transverse motions of a string under tension, T, with a mass per length of μ. The model and its finite element representation is shown in Figure 1. A duplicate model is obtained via the INPUT module to generate the scalar springs and masses.

Selected scalar point displacements and scalar element forces are plotted versus frequency. The magnitude and phase of the displacements are plotted separately, each on one-half of the plotter frame. The magnitude plots for the selected points are all drawn on a whole plotter frame for comparisons. The center spring element has the magnitude of its internal force plotted versus frequency.

B. Input

1. Parameters:

m_i = 10 - mass

K_i = 10^7 - spring rate

N = 500 - number of cells

where

$$K_i = \frac{T}{\Delta x}, \qquad m_i = \mu \Delta x$$

2. Loads

The load on each point is:

$$P_i(\omega) = \Delta x p_x = 10\pi^3$$

where p_x is the load per length of string.

The steady state frequency response is desired from .1 to 10 cycles per second in 15 logrithmic increments.

3. Real Eigenvalue Data

Method: FEER

Center of neighborhood: 10.5

Normalization: maximum deflection

Number of modes used in formulation: 20

C. Theory

The analysis of the string is given in Reference 11, Chapter 6. The response, ξ_n, of mode number n is given by the equation:

$$\xi_n = \frac{\int_0^{\ell} P(x) \sin\left(\frac{n\pi x}{\ell}\right) dx}{\left[\int_0^{\ell} \mu \sin^2\left(\frac{n\pi x}{\ell}\right) dx\right] [\omega_n^2 - \omega^2]} \quad , \tag{1}$$

where ω_n, the natural frequencies, are $\frac{n\pi}{N}\sqrt{\frac{K_i}{m_i}}$ for the theoretical continuous string.

For a uniform Load:

$$\int_0^{\ell} P(x) \sin\left(\frac{n\pi x}{\ell}\right) dx = \frac{2p_x \ell}{n\pi} = \frac{2P_i N}{n\pi} = \frac{10^4 \pi^2}{n} \quad , \tag{2}$$

and

$$\int_0^{\ell} \mu \sin^2\left(\frac{n\pi x}{\ell}\right) dx = \frac{\mu\ell}{2} = \frac{Nm_i}{2} = 2.5 \times 10^3 \quad . \tag{3}$$

The displacement of the center point is:

$$u\left(\frac{\ell}{2}\right) = \sum \xi_n \sin \frac{n\pi}{2} = \xi_1 - \xi_3 + \xi_5 - \xi_7 + \ldots \tag{4}$$

D. Results

At f = 0.1, the response due to 20 modes is:

$u(\frac{\ell}{2})$ = .97895 (Theory)

u_{251} = .97888 (NASTRAN)

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0     NASTRAN FILES=(UMF,PLT2)
  1     ID        DEM11021,NASTRAN
  2     UMF       1977      110210
  3     APP       DISPLACEMENT
  4     TIME      26
  5     SØL       11,1
  6     CEND

  7     TITLE = FREQUENCY RESPØNSE ØF A 500 CELL STRING
  8     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-2-1
  9         METHØD = 10
 10         FREQ = 11
 11         DLØAD = 11
 12     ØUTPUT
 13         SET 1 = 51, 101, 151, 201, 251, 301, 351, 401, 451
 14         SET 2 = 1 THRU 5
 15              DISPLACEMENT(PHASE,SØRT2) = 1
 16             SDISPLACEMENT(PHASE,SØRT2) = 2
 17     PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-2-1
 18     ØUTPUT(XYØUT)
 19     PLØTTER = SC
 20         CAMERA = 3
 21         SKIP BETWEEN FRAMES = 1
 22       CURVE LINE AND SYMBØLS = 1
 23       XLØG = YES
 24       YTLØG = YES
 25       XTGRID = YES
 26       XBGRID = YES
 27       YTGRID = YES
 28       YBGRID = YES
 29       XTITLE =                           FREQUENCY (HERTZ)
 30       YTTITLE = MAGNITUDE  *INCH*
 31       YBTITLE = PHASE  *DEGREE*
 32     $
 33     $ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
 34     $
 35         TCURVE = * * * * * *  SPØINT  5 1   * * * * * * * * * * * * *
 36     XYPLØT DISP  /   51(T1RM,T1TP)
 37         TCURVE = * * * * * *  SPØINT  1 0 1   * * * * * * * * * * * *
 38     XYPLØT DISP  /  101(T1RM,T1IP)
 39         TCURVE = * * * * * *  SPØINT  1 5 1   * * * * * * * * * * * *
 40     XYPLØT DISP  /  151(T1RM,T1IP)
 41         TCURVE = * * * * * *  SPØINT  2 0 1   * * * * * * * * * * * *
 42     XYPLØT DISP  /  201(T1RM,T1IP)
 43         TCURVE = * * * * * *  SPØINT  2 5 1   * * * * * * * * * * * *
 44     XYPLØT DISP  /  251(T1RM,T1IP)
 45     $
 46     $ *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
 47     $
 48       YLØG = YES
 49       YTITLE = MAGNITUDE  *INCH*
 50       XGRID LINES = YES
 51       YGRID LINES = YES
 52       TCURVE = * * * * *  SUPERPØSITIØN ØF SPØINT 51, 101, 151, 201, 251 * *
 53     XYPLØT DISP / 51(3), 101(3), 151(3), 201(3), 251(3)
 54       YLØG = NØ
 55       YTITLE = REAL PART  *PØUNDS*
 56       TCURVE = * * * * * * *   FØRCE IN STRING ELEMENT 251  * * * * * * * *
```

Card
No.

57 XYPLØT, XYPRINT ELFØRCE RESPØNSE / 251(2)
58 $
59 BEGIN BULK
60 ENDDATA

1	2	3	4	5	6	7	8	9	10
CELAS3	1	101	0	2	2	101	2	3	
CMASS3	40002	301	2	0					
DAREA	11	2		1.0	3		1.0		
EIGR	10	FEER	10.5			20			+FEER
+FEER	MAX								
FREQ2	11	.1	10.0	15					
PARAM	LMØDES	20							
PELAS	101	1.0+7							
PMASS	301	10.000							
RLØAD1	11	11			1				
TABLED1	1								*T1
*T1	-10.0		310.0227	67	100.0		310.0227	67	*T2

Card
No.

```
NASTRAN FILES=(UMF,PLT2)
ID       DEM11022,NASTRAN
UMF      1977     110220
ALTER    1
PARAM    //C,N,NØP/V,N,TRUE=-1$
INPUT,   ,,,,/,G2,,,/C,N,5 $
EQUIV    G2,GEØM2/TRUE $
ENDALTER
APP      DISPLACEMENT
TIME     26
SØL      11,1
DIAG     14
CEND

TITLE = FREQUENCY RESPØNSE ØF A 500 CELL STRING
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-2-2
METHØD = 10
    FREQ = 11
    DLØAD = 11
ØUTPUT
    SET 1 = 51, 101, 151, 201, 251, 301, 351, 401, 451
    SET 2 = 1 THRU 5
         DISPLACEMENT(PHASE,SØRT2) = 1
        SDISPLACEMENT(PHASE,SØRT2) = 2
PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-2-2
ØUTPUT(XYØUT)
PLØTTER = SC
         CAMERA = 3
         SKIP BETWEEN FRAMES = 1
  CURVE LINE AND SYMBØLS = 1
  XLØG = YES
  YTLØG = YES
  XTGRID = YES
  XBGRID = YES
  YTGRID = YES
  YBGRID = YES
  XTITLE =                                 FREQUENCY (HERTZ)
  YTTITLE = MAGNITUDE  *INCH*
  YBTITLE = PHASE  *DEGREE*
$
$ * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
$
    TCURVE = * * * * * *  SPØINT  5 1   * * * * * * * * * * * * * * * *
XYPLØT DISP  /   51(T1RM,T1IP)
    TCURVE = * * * * * *  SPØINT  1 0 1   * * * * * * * * * * *
XYPLØT DISP  /  101(T1RM,T1IP)
    TCURVE = * * * * * *  SPØINT  1 5 1   * * * * * * * * * * *
XYPLØT DISP  /  151(T1RM,T1IP)
    TCURVE = * * * * * *  SPØINT  2 0 1   * * * * * * * * * * *
XYPLØT DISP  /  201(T1RM,T1IP)
    TCURVE = * * * * * *  SPØINT  2 5 1   * * * * * * * * * * *
XYPLØT DISP  /  251(T1RM,T1IP)
$
$ *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
$
  YLØG = YES
  YTITLE = MAGNITUDE  *INCH*
  XGRID LINES = YES
```

```
Card
 No.

57      YGRID LINES = YES
58      TCURVE = * * * * *  SUPERPØSITIØN ØF SPØINT 51, 101, 151, 201, 251 * *
59    XYPLØT DISP / 51(3), 101(3), 151(3), 201(3), 251(3)
60      YLØG = NØ
61      YTITLE = REAL PART  *PØUNDS*
62      TCURVE = * * * * * * *   FORCE IN STRING ELEMENT 251   * * * * * * * *
63    XYPLØT, XYPRINT ELFØRCE RESPØNSE / 251(2)
64    $
65    BEGIN BULK
66    ENDDATA

67         500    1.0E7      0.0      10.0      0.0
```

1	2	3	4	5	6	7	8	9	10
DAREA	11	2		1.0	3		1.0		
EIGR	10	FEER	10.5			20			+FEER
+FEER	MAX								
FREQ2	11	.1	10.0	15					
PARAM	LMØDES	20							
RLØAD1	11	11			1				
TABLED1	1								*T1
*T1	-10.0		310.0227	67	100.0		310.0227	67	*T2

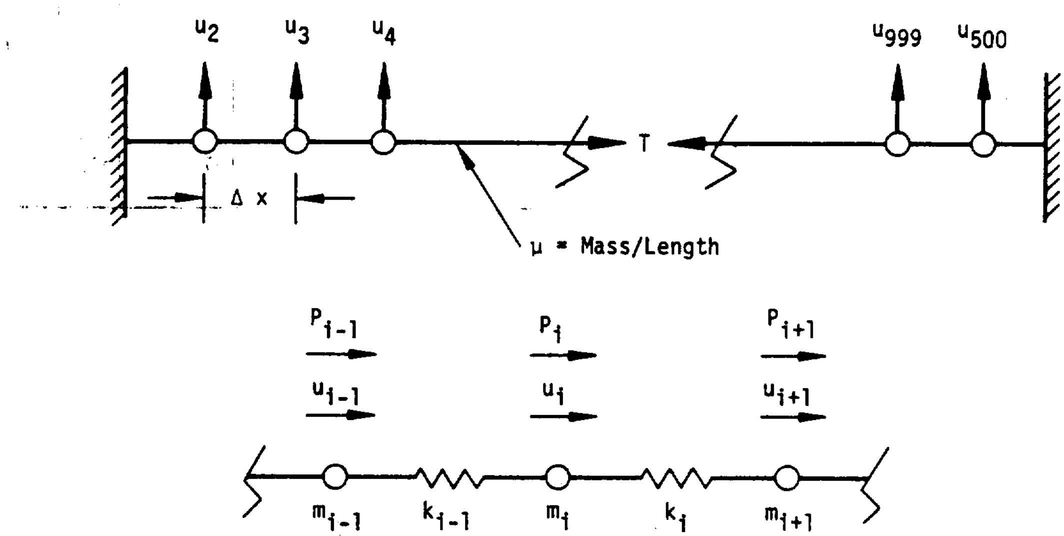

Figure 1. Representations of 500 cell string.

RIGID FORMAT No. 11 (APP AERØ), Aeroelastic Response

Jet Transport Wing Dynamic Analysis, Frequency Response (11-3-1)

Jet Transport Wing Dynamic Analysis. Transient Response (11-3-2)

A. Description

This example illustrates the use of the aeroelastic response analysis to perform frequency, random, and transient response calculations for a structure excited by aerodynamic loadings. This problem is also discussed in Section 1.11.5 of the User's Manual.

The structural model represents a wing and aileron configuration as shown in Figure 1. For this demonstration problem, the aileron is locked and the fuselage to which the wing is attached is a rigid body represented by grid point 11. Only out-of-plane motions are retained in the model. The wing is modeled with GENEL data defining the flexibility matrix, [Z], and a free-body matrix, [S]. The aileron also is modeled as a rigid body with the hinge line at point 8. The vertical flap deflection at point 12 is defined by an MPC equation.

The aerodynamic model consists of 42 doublet lattice aerodynamic boxes, forming one coupled group as shown in Figure 2. Three CAERØ1 aerodynamic elements are used to define the areas of uniform mesh on the wing. The aerodynamic degrees of freedom, implicitly defined by the CAERØ data, are coupled to the structure with surface splines defined on SPLINE2 data cards.

B. Input

Two separate analyses are performed with this structural model. Problem 11-3-1 performs a frequency response analysis for a smooth gust shape and generates spectral density output plots for a random gust magnitude. Problem 11-3-2 produces a transient response solution using a Fourier transform of the frequency response solution.

1. Parameters:

$V = 5183.2$	(Airstream velocity)
$M = 0.62$	(Airstream mach number)
$\rho = 1.1468 \times 10^{-7}$	(Air density)
$g = 0.06$	(Structural damping)

2. Constraints:

$\Theta_y = \Theta_z = 0$ Grid 11 (No fuselage isolation)

$u_x = u_y = \Theta_z = 0$ All Grids

$\Theta_x = \Theta_y = 0$ All Grids except 11 and 12

3. Loads:

Problem 11-3-1, Frequency Response Analysis

$$V_g = \frac{8360}{2}(1 - \cos 2\pi t) \quad (t < 1) \quad \text{Gust Velocity}$$

Problem 11-3-2, Transient Analysis

$$V_g = \begin{Bmatrix} 8360 & t < 1.0 \\ -16720 & t > 1.0 \end{Bmatrix} \quad \text{Gust Velocity}$$

C. <u>Theory</u>

No theoretical results are available to confirm the NASTRAN results.

D. <u>Results</u>

Shown in Figures 3 and 4 are plots of the fuselage plunge and aileron displacement as real and imaginary functions of frequency for Problem 11-3-1. Shown in Figures 5 and 6 are the same quantities obtained from Problem 11-3-2 results. For this case the data are plotted versus time.

E. Driver Decks and Sample Bulk Data

```
Card
 No.

  0    NASTRAN FILES=UMF
  1    ID      DEM11031,NASTRAN
  2    UMF     1977     110310
  3    APP     AERØ
  4    SØL     11,0
  5    TIME    3
  6    CEND

  7    TITLE = JET TRANSPØRT WING DYNAMIC ANALYSIS
  8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-3-1
  9    LABEL = SYMMETRIC RESPØNSE , STIFF AILERØN
 10    ECHØ = BØTH
 11    $
 12    $       MØDEL DESCRIPTIØN              JET TRANSPØRT WING EXAMPLE
 13    $                                  SYMMETRIC RESPØNSE TØ A RANDØM
 14    $                                  GUST WITH A STIFF AILERØN
 15    $
 16        SPC = 14 $ SYM , NØ PITCH
 17        MPC = 1
 18        METHØD = 10 $ GIVENS
 19        SDMAP = 2000
 20        FREQ = 40
 21        RANDØM = 1031 $  EMPIRICAL PSDF
 22    ØUTPUT
 23    $
 24    $       SØLUTIØN                       RANDØM ANALYSIS USING
 25    $                                  DØUBLET-LATTICE METHØD AERØDYNAMICS
 26    $                                  AT MACH. NØ. ØF .62
 27    $
 28        SET 1 = 1 , 2 , 12 $
 29        SET 2 = 1 , 9 THRU 12 , 1040
 30        SET 3 = 11
 31        SET 4 = 1001, 1022 , 1023 , 1040 , 1041 $
 32        SDISP(IMAG) = 1
 33        DISP(IMAG) = 2
 34        SPCF(IMAG) = 3
 35        AERØF = 4
 36    SUBCASE 1
 37        LABEL = RANDØM GUST ANALYSIS
 38        GUST = 3002
 39    $
 40    $         PRØDUCES XY PAPER PLØTS ØF MØDAL AND GRID PØINT DISPLACEMENTS
 41    $       AND WING RØØT BENDING MØMENTS
 42    $
 43    ØUTPUT(XYØUT)      $ FREQ RESP PACKAGE (CØMPLEX NUMBERS)
 44        CURVELINESYMBØL = 1
 45        XTITLE = FREQUENCY(HERTZ)       JET TRANSPØRT , FREQUENCY RESPØNSE
 46    YTITLE = MØDAL DEFLECTIØN
 47        TCURVE = FIRST MØDE (PLUNGE)
 48        XYPAPERPLØT   SDISP /   1(T1RM) , 1(T1IP)
 49        TCURVE =      SECØND MØDE (WING BENDING)
 50        XYPAPERPLØT   SDISP /   2(T1RM) , 2(T1IP)
 51        TCURVE =      TWELFTH MØDE (AILERØN)
 52        XYPAPERPLØT   SDISP /  12(T1RM) , 12(T1IP)
 53    YTITLE = PHYSICAL DEFLECTIØN
 54        TCURVE =      WING ( 3/4 CHØRD , 1/4 CHØRD , STA 458 )
 55        XYPAPERPLØT   DISP /  10(T3RM) , 10(T3IP) , 9(T3RM) , 9(T3IP)
 56        TCURVE =      FUSELAGE PLUNGE
 57        XYPAPERPLØT   DISP /  11(T3RM) , 11(T3IP)
```

```
Card
 No.

 58        TCURVE =      AILERØN DEFLECTIØN
 59        XYPAPERPLØT   DISP /  12(R2RM) , 12(R2IP)
 60        TCURVE =      AERØDYNAMIC BØX NEAR TIP , PITCH
 61        XYPAPERPLØT   DISP /  1040(R2RM) , 1040(R2IP)
 62        TCURVE =      WING RØØT BENDING MØMENT
 63    YTITLE = RØTATIONAL CØNSTRAINTS
 64        XYPAPERPLØT   SPCF /  11(R3RM) , 11(R3IP)
 65    $   RANDØM ANALYSIS ØUTPUT REQUESTS
 66        XTITLE = FREQUENCY (HERTZ)      JET TRANSPØRT , RANDØM ANALYSIS
 67        TCURVE =     PØWER SPECTRAL DENSITY FUNCTIØN
 68        YTITLE = FUSELAGE PLUNGE  (11T3)          , PSDF , GUST LØAD
 69        XYPAPERPLØT   DISP  PSDF         /  11(T3)
 70        YTITLE = WING TIP DISPLACEMENT (9T3)      , PSDF , GUST LØAD
 71        XYPAPERPLØT   DISP  PSDF         /   9(T3)
 72        YTITLE = WING RØØT BENDING MØMENT (11R3)  , PSDF , GUST LØAD
 73        XYPAPERPLØT  SPCF  PSDF          /  11(R3)
 74    BEGIN BULK
 75    ENDDATA
```

1	2	3	4	5	6	7	8	9	10
AEFACT	1	0.0	.09	.21	.33	.45	.56	.66	+AE1
+AE1	.74								
AERØ	1	8360.	131.232	1.1468-7	1				SYM
CAERØ1	1001	1000	0			1	4	1	+CA01
+CA01	78.75	0.	0.	225.	35.	500.	0.	100.	
CELAS2	3	5142671.	12	5					
CMASS2	2	13967.2	12	5					
CØNM1	1	11							+51
+51	17400.				4.37+7				+52
CØRD2R	1		0.0	0.0	0.0	0.0	0.0	-1.	+C1
+C1	-1.	0.	0.						
DAREA	9999	11	1	1.					DUMMY
EIGR	10	GIV	0.0	1.		12			+EIGR
+EIGR	MAX								
FREQ1	40	0.0	.25	39					
GENEL	432		1	3	2	3	3	3	+01
+01	4	3	5	3	6	3	7	3	+02
GRID	1		20.25	90.			12456		
GUST	3002	3002	1.1962-4	0.0	8360.				
MKAERØ1	.62								+MK
+MK	.02	0.10	0.50						
MPC	1	12	3	-1.0	8	3	1.5		+MPC1
+MPC1		7	3	-0.5	12	5	33.25		
PAERØ1	1000								
PARAM	GUSTAERØ	-1							
PARAM	LMØDES	12							
PARAM	MACH	.62							
PARAM	Q	4.00747							
PARAM	WTMASS	.0025907							
RANDPS	1031	1	1	1.		1032			
RLØAD1	3002	9999			1004				
SET1	14	1	THRU	11					
SPC	14	11	45						
SPLINE1	104	1022	1026	1039	15				
SPLINE2	101	1001	1001	1021	14	0.0	2.	0.	+SP1
+SP1	-1.0	-1.0							
SUPØRT	11	3							
TABDMP1	2000								+T2000
+T2000	0.	.06	10.	.06	ENDT				
TABLED1	1004								+T1004
+T1004	0.	0.	.01	1.	10.	1.	ENDT		
TABRND1	1032								+001
+001	0.00	2.8708+0	0.25	1.2641+0	0.50	4.7188-1	0.75	2.3080-1	+002

Card No.

```
NASTRAN FILES=UMF
ID        DEM11032,NASTRAN
UMF       1977    110320
APP       AERØ
SØL       11,0
TIME      3
CEND


TITLE = JET TRANSPØRT WING DYNAMIC ANALYSIS
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 11-3-2
LABEL = SYMMETRIC RESPØNSE . SQUARE EDGE GUST . TRANSIENT ANALYSIS
ECHØ = BØTH
$
$        MØDEL DESCRIPTIØN          JET TRANSPØRT WING EXAMPLE
$                                 SYMMETRIC RESPØNSE TØ A SQUARE
$                                 EDGE GUST WITH A STIFF AILERØN
$
    SPC = 14 $ SYM , NØ PITCH
    MPC = 1
    METHØD = 10 $ GIVENS
    SDAMP = 2000
    FREQ = 40
    TSTEP = 41
$$$$$$$ TWELVE MØDES AND FØRTY TWØ BØXES    AERØ CALC THREE K VALUES
    GUST = 1011 $ SQUARE
    DLØAD = 9999 $ NEEDED TØ FØRCE APPRØACH TRANSIENT GUST
ØUTPUT
$
$        SØLUTIØN                  TRANSIENT ANALYSIS USING
$                                  DØUBLET-LATTICE METHØD AERØDYNAMICS
$                                  AT MACH NØ. ØF 0.62
$
    SET 1 = 1 , 2 , 12 $
    SET 2 = 1 , 9 THRU 12 , 1040
    SET 3 = 11
    SDISP = 1
    DISP  = 2
    SPCF  = 3
$
$           PRØDUCES XY PAPER PLØTS ØF MØDAL AND GRID PØINT DISPLACEMENTS
$        AND WING RØØT BENDING MØMENT TIME HISTØRIES
$
ØUTPUT(XYØUT)     $ TRANSIENT PACKAGE (REAL NUMBERS)
    CURVELINESYMBØL = 1
    XTITLE = TIME(SECØNDS)       JET TRANSPØRT . SQUARE GUST
    TCURVE = FIRST MODE (PLUNGE)
YTITLE = MØDAL DEFLECTIØN
    XYPAPERPLØT    SDISP /   1(T1)
    TCURVE = SECØND MØDE (WING BENDING)
    XYPAPERPLØT    SDISP /   2(T1)
    TCURVE = TWELFTH MØDE (AILERØN)
    XYPAPERPLØT    SDISP /  12(T1)
YTITLE = PHYSICAL DEFLECTIØN
    TCURVE = WING ( 3/4 CHØRD . 1/4 CHØRD , STA 458 )
    XYPAPERPLØT     DISP /  10(T3) , 9(T3)
    TCURVE = FUSELAGE PLUNGE
    XYPAPERPLØT     DISP /  11(T3)
    TCURVE = AILERØN DEFLECTIØN
    XYPAPERPLØT     DISP /  12(R2)
```

```
Card
 No.

 58        TCURVE = AERØDYNAMIC BØX NEAR TIP  , PITCH
 59        XYPAPERPLØT    DISP /  1040(R2)
 60     YTITLE = RØTATIØNAL CØNSTRAINTS
 61        TCURVE = WING RØØT BENDING MØMENT
 62        XYPAPERPLØT    SPCF /  11(R3)
 63     BEGIN BULK
 64     ENDDATA
```

1	2	3	4	5	6	7	8	9	10
AEFACT	1	0.0	.09	.21	.33	.45	.56	.66	+AE1
+AE1	.74								
AERØ	1	8360.	131.232	1.1468-7	1				SYM
CAERØ1	1001	1000	0			1	4	1	+CAØ1
+CAØ1	78.75	0.	0.	225.	35.	500.	0.	100.	
CELAS2	3	5142671.	12	5					
CMASS2	2	13967.2	12	5					
CØNM1	1	11							+51
+51	17400.				4.37+7				+52
CØRD2R	1		0.0	0.0	0.0	0.0	0.0	-1.	+C1
+C1	-1.	0.	0.						
DAREA	1001	12	5	5142671.					
EIGR	10	GIV	0.0	1.		12			+EIGR
+EIGR	MAX								
FREQ1	40	0.0	.25	39					
GENEL	432		1	3	2	3	3	3	+01
+01	4	3	5	3	6	3	7	3	+02
GRID	1		20.25	90.			12456		
GUST	1011	1000	1.	0.0	8360.				
MKAERØ1	.62								+MK
+MK	.02	0.10	0.50						
MPC	1	12	3	-1.0	8	3	1.5		+MPC1
+MPC1		7	3	-0.5	12	5	33.25		
PAERØ1	1000								
PARAM	GUSTAERØ	-1							
PARAM	IFTM	0							
PARAM	LMØDES	12							
PARAM	MACH	.62							
PARAM	Q	4.00747							
PARAM	WTMASS	.0025907							
SET1	14	1	THRU	11					
SPC	14	11	45						
SPLINE1	104	1022	1026	1039	15				
SPLINE2	101	1001	1001	1021	14	0.0	2.	0.	+SP1
+SP1	-1.0	-1.0							
SUPØRT	11	3							
TABDMP1	2000								+T2000
+T2000	0.	.06	10.	.06	ENDT				
TABLED1	1003								+T1003
+T1003	0.	1.	1.	1.	1.	-1.	2.	-1.	+T1003A
TLØAD1	1000	1001			1003				
TSTEP	41	40	.1	1					

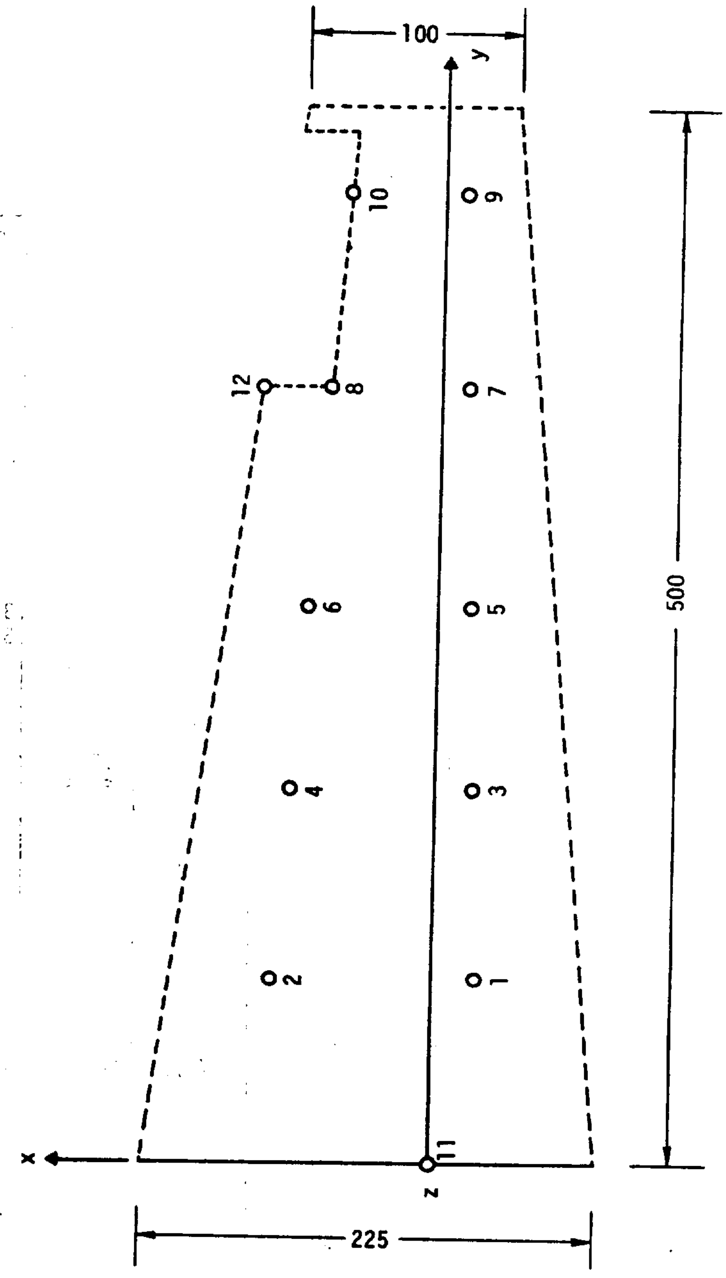

Figure 1. Structural definition of the transport wing.

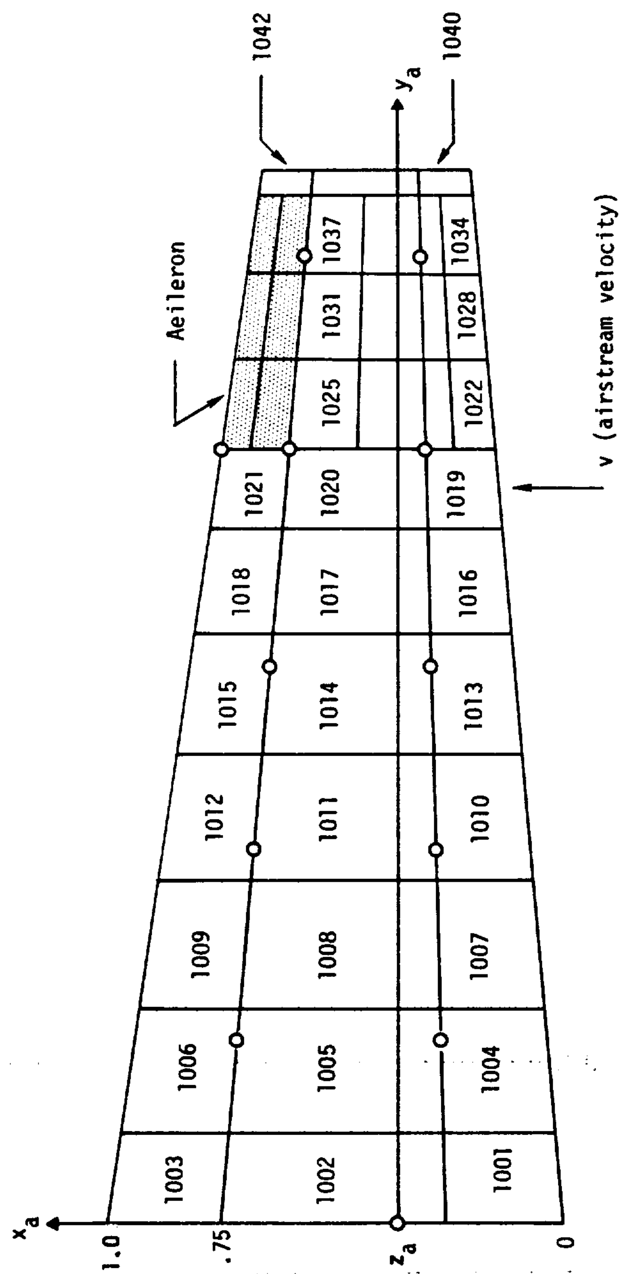

Figure 2. Aerodynamic element model.

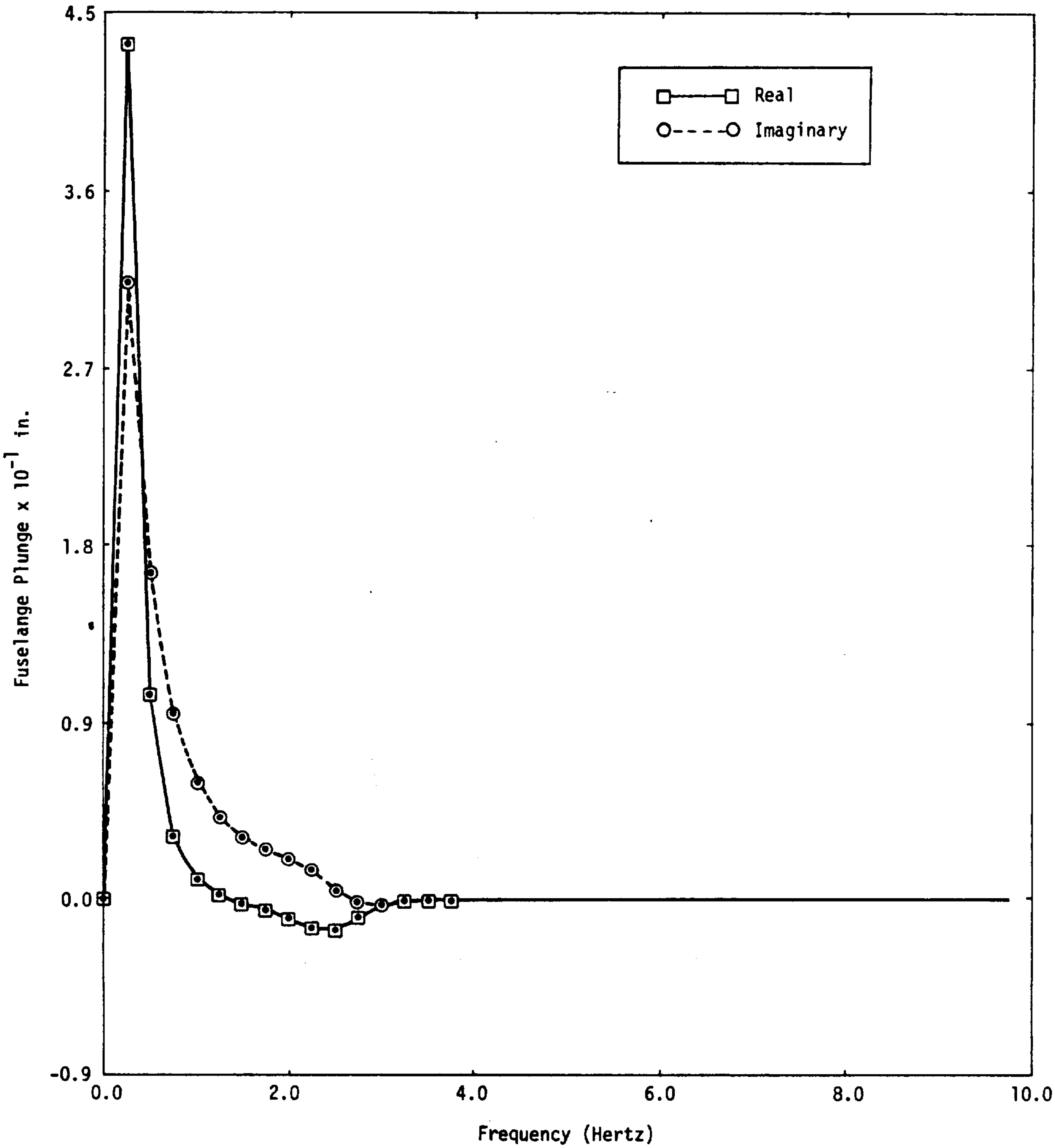

Figure 3. Fuselage plunge versus frequency at grid point 11.

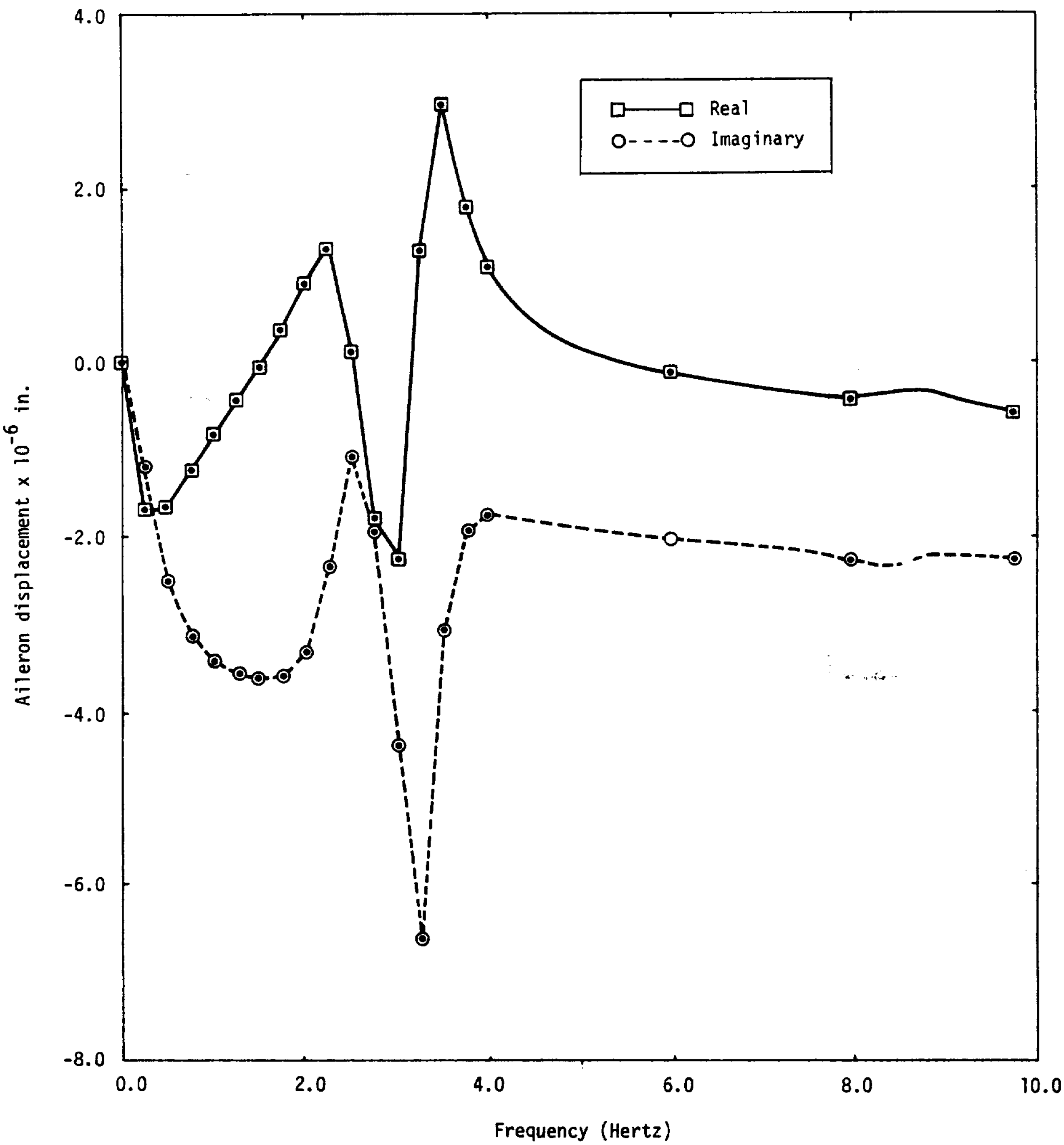

Figure 4. Aileron displacement versus frequency at grid point 12.

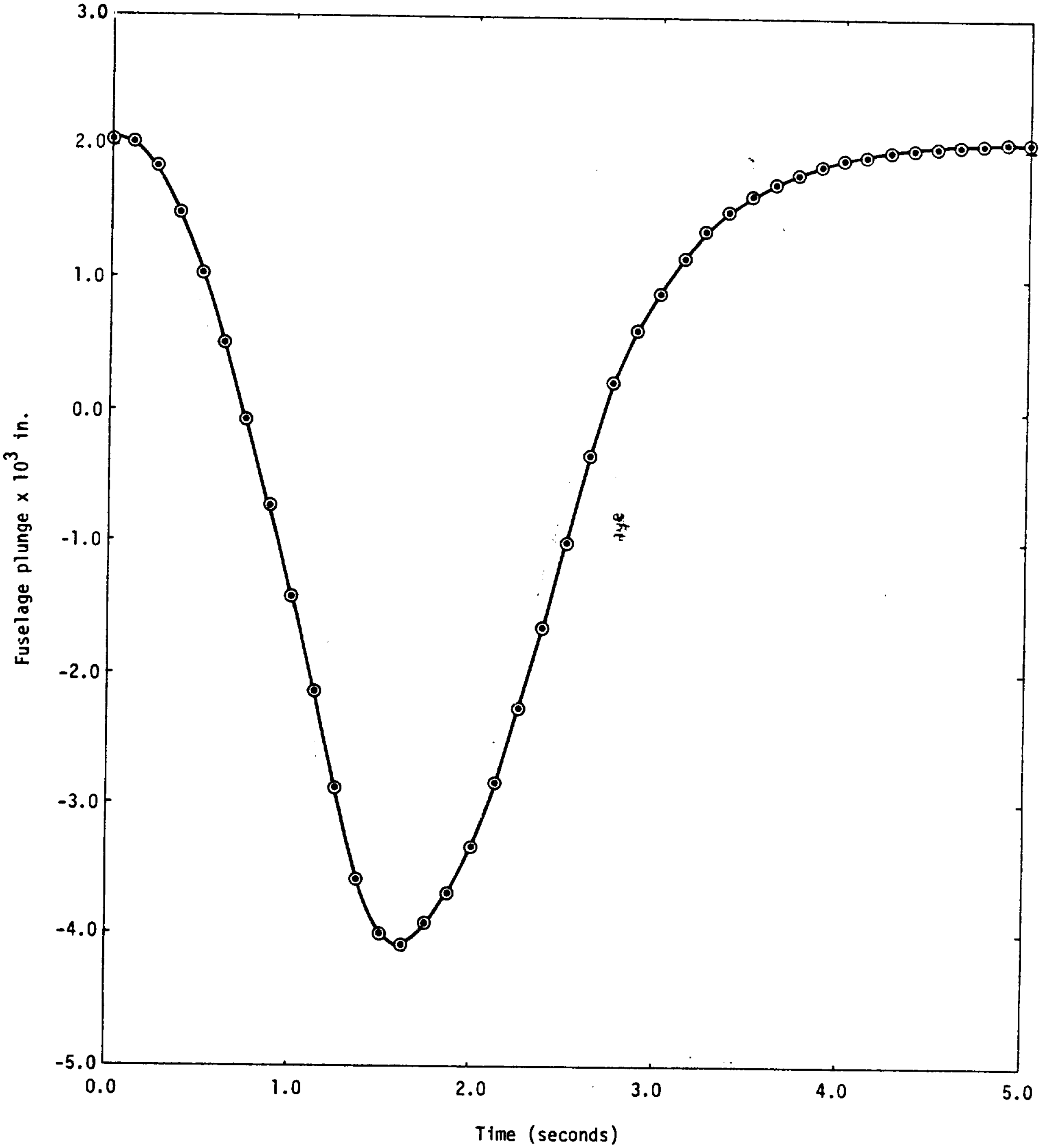

Figure 5. Fuselage plunge versus time at grid point 11.

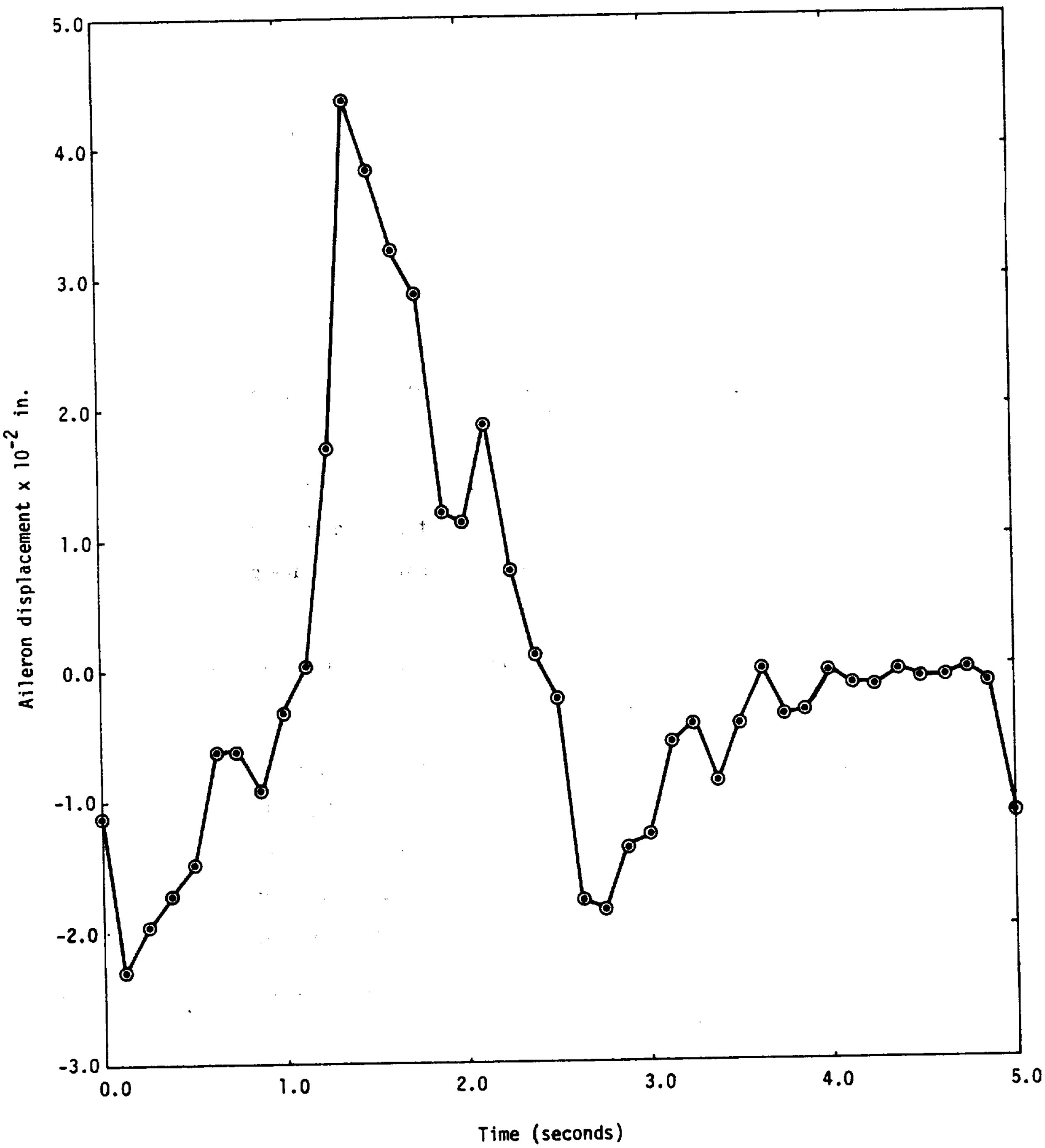

Figure 6. Aileron displacement versus time at grid point 12.

RIGID FORMAT No. 12, Transient Analysis - Modal Formulation

Transient Analysis of a Free One Hundred Cell Beam (12-1-1)

A. Description

The problem demonstrates the transient analysis of a free-body using the integration algorithm for uncoupled modal formulations. The model is a hundred-cell beam with a very large mass attached to one end as shown in Figure 1. Modal damping is included as a function of natural frequency. It does not affect the free-body (zero frequency) modes. The omitted coordinate feature was used to reduce the analysis set of displacements to correspond to eleven grid points.

Both structure plots and curve plots are requested. The types are as follows:

1. Stereoscopic structure plots of the deformed structure are drawn for a specified time step.
2. Orthographic projections of the deformed structure are plotted. However, two variations are plotted on each frame. The bottom region of the frame shows the deformed shape and the top region shows vectors at every tenth grid point which are proportional to the z-displacement at each specified time step.
3. Curve plots and printout of displacement versus time and of acceleration versus time are requested.

When a structure is used without additional transfer functions or direct matrix inputs, the transient analysis solves exact equations for the uncoupled modes. The only errors will be in the discarded modes and the straight line approximation of the loads between time steps. The speed of this solution is offset by the fact that the eigenvalue calculation is relatively costly and the transformation of the vectors to and from modal coordinates could be time consuming.

The mass and inertia on point (1) were selected to be much larger than values of the beam. The answers will therefore approximate a beam with a fixed end.

B. Input

1. Parameters

Beam:

ℓ = 20 (Length)

I = .083 (Bending inertia)

A = 1.0 (Cross sectional area)

E = 10.4×10^6 (Modulus of elasticity)

ρ = $.2523 \times 10^{-3}$ (Mass density)

Lumped Mass:

$$m_1 = 10.0, \quad I_{22,1} = 1666.66$$

2. Damping:

The damping coefficient for each mode is a function of the natural frequency. The function is:

$$g = 10^{-3} f$$

3. Load:

$$P_{z,101} = 100 \sin(2\pi \cdot 60t)$$

4. Real Eigenvalue Data

Method: Inverse Power

Region of Interest: $0 < f < 1000$

Normalization: Mass

D. Results

The NASTRAN results are compared in Figure 2 to the analytic results which use one mode. The modal mass may be calculated using the formula for the mode shape given in Reference 8. The modal displacement is a single degree of freedom response with a closed form solution.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0    NASTRAN FILES=(UMF,PLT2)
  1    ID        DEM12011,NASTRAN
  2    UMF       1977      120110
  3    APP       DISPLACEMENT
  4    SØL       12,3
  5    TIME      10
  6    CEND

  7    TITLE = TRANSIENT ANALYSIS ØF A FREE ØNE HUNDRED CELL BEAM
  8    SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 12-1-1
  9            DLØAD = 516
 10            SDAMP = 15
 11            TSTEP = 516
 12            METHØD = 2
 13    ØUTPUT
 14            SET 1 = 1, 26, 51, 75, 100
 15            SET 2 = 1, 26, 76
 16            DISPLACEMENT = 2
 17       STRESS = 1
 18    PLØTID = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 12-1-1
 19    ØUTPUT(PLØT)
 20    PLØTTER SC
 21            CAMERA 3
 22            SET 1 INCLUDE BAR,
 23              EXCLUDE GRID POINTS 1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,18,
 24              19,20,22,23,24,25,26,27,28,29,30,32,33,34,35,36,37,38,39,40,
 25              42,43,44,45,46,47,48,49,50,52,53,54,55,56,57,58,59,60,62,63,
 26              64,65,66,67,68,69,70,72,73,74,75,76,77,78,79,80,82,83,84,85,
 27              86,87,88,89,90,92,93,94,95,96,97,98,99,100
 28            MAXIMUM DEFØRMATIØN  2.0
 29    STEREØ PRØJECTIØN
 30            FIND SCALE, ØRIGIN 100, VANTAGE PØINT, SET 1
 31    PTITLE = PAPER CØPY ØF STEREØSCØPIC PRØJECTIØN ØF DEFØRMATIØNS
 32            PLØT TRANSIENT DEFØRMATIØN 1, TIME 0.012, 0.013,
 33            MAXIMUM DEFØRMATIØN 0.76, SET 1, ØRIGIN 100, SHAPE
 34    ØRTHØGRAPHIC PRØJECTIØN
 35            FIND SCALE, ØRIGIN 1, REGIØN 0.0,0.0,1.0,0.5
 36            FIND SCALE, ØRIGIN 2, REGIØN 0.0,0.5,1.0,1.0
 37    PTITLE = DEFLECTIØNS ØF BARS WITH VECTØRS
 38            PLØT TRANSIENT DEFØRMATIØN 1, TIME .012, .016,
 39            MAXIMUM DEFØRMATIØN 1.0
 40            SET 1, ØRIGIN 1, SHAPE ,
 41            SET 1, ØRIGIN 2, VECTØR Z
 42    $
 43    $
 44    ØUTPUT(XYØUT)
 45    PLØTTER = SC
 46            CAMERA = 3
 47            SKIP BETWEEN FRAMES = 1
 48        YGRID LINES = YES
 49        XGRID LINES = YES
 50        YDIVISIØNS = 10
 51        XDIVISIONS = 10
 52        XVALUE PRINT SKIP = 1
 53        YVALUE PRINT SKIP = 1
 54        XTITLE =                         TIME (SECØNDS)
 55        YTITLE =     D I S P  * INCH *
 56        TCURVE = * * * * * * *  G R I D   5 1    * * * * * * * * * * * * * *
```

Card
No.

```
57   XYPLØT,XYPRINT,DISP RESP /  51(T3)
58       TCURVE = * * * * * * *  G R I D   1 0 1  * * * * * * * * * * * * *
59   XYPLOT,XYPRINT,DISP RESP / 101(T3)
60       YTITLE = ACCELERATION
61       TCURVE = * * * * * * *  G R I D   5 1  * * * * * * * * * * * * * *
62   XYPLØT,XYPRINT,ACCE RESP /  51(T3)
63       TCURVE = * * * * * * *  G R I D   1 0 1  * * * * * * * * * * * * *
64   XYPLØT,XYPRINT,ACCE RESP / 101(T3)
65   BEGIN BULK
66   ENDDATA
```

1	2	3	4	5	6	7	8	9	10
BARØR					10.0	.0	100.0	1	
CBAR	1	17	1	2					+M1
CØNM2	20	1		10.0					
+M1			1666.66						
DAREA	1	101	3	100.					
EIGR	2	INV	.0	1500.	5	6			PEG
+EG	MASS								
GRDSET							1246		
GRID	1		.00	.00	.00				
MAT1	1	10.4+6	4.+6		.2523-3				+MAT1
+MAT1	111.111	11.1111							
ØMIT1	53	2	3	4	5	6	7	8	+100
+100	9	10	12	13	14	15	16	17	+200
PARAM	GRDPNT	0							
PARAM	LMØDES	6							
PBAR	17	1	1.	.083	.083				+PBAR
+PBAR	1.11111	-1.11111							
SUPØRT	1	3	1	5					
TABDMP1	15								+TD11
+TD11	10.	.01	100.	.1	3000.	.1	ENDT		
TLØAD2	516	1			.0	.1	60.		
TSTEP	516	104	.001388	1					

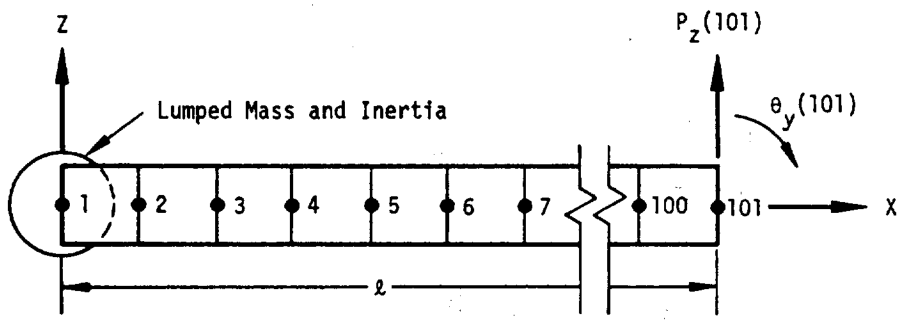

Figure 1. 100 cell free beam.

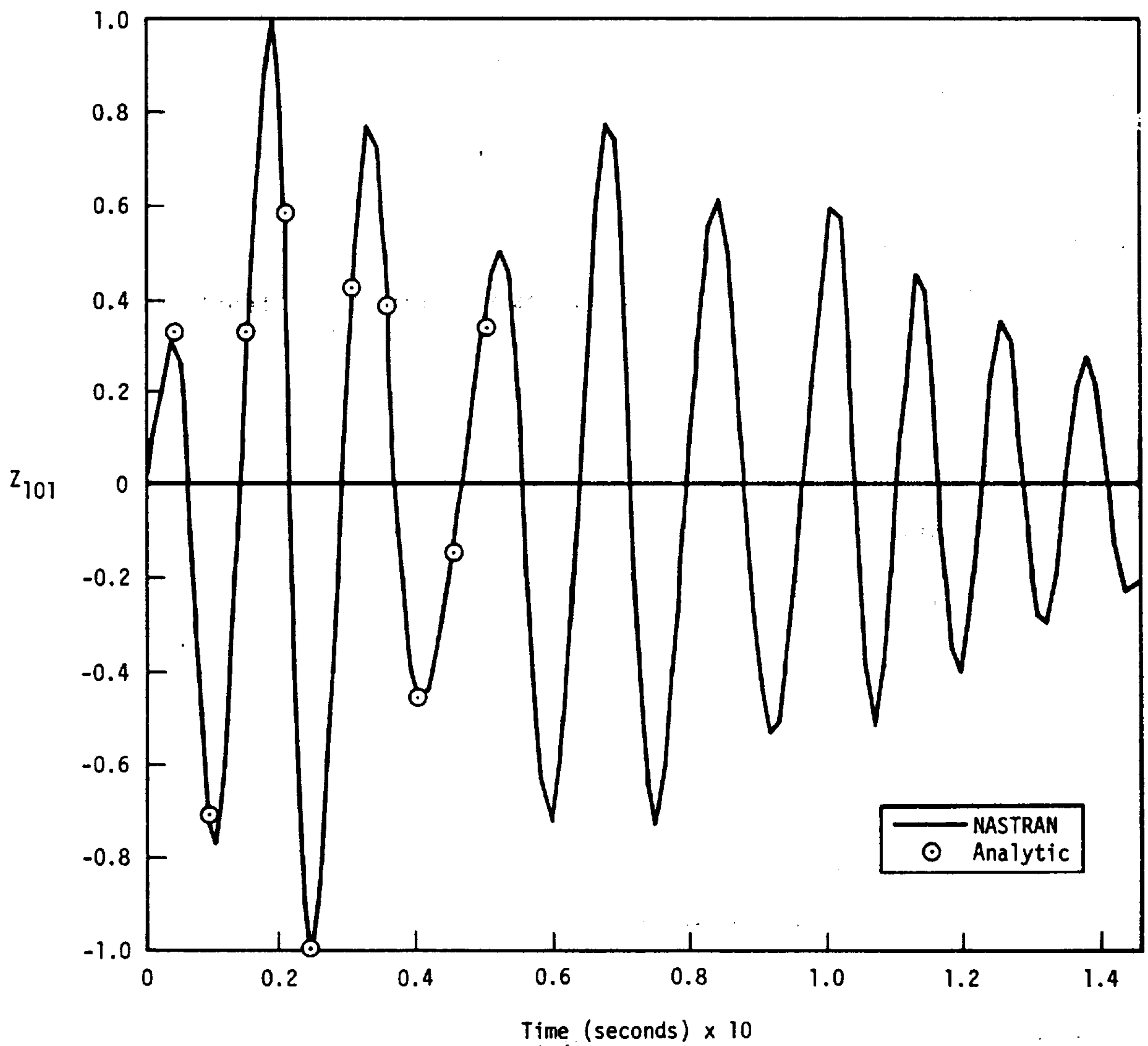

Figure 2. Comparison of NASTRAN and analytic displacements versus time.

RIGID FORMAT No. 13, Normal Modes with Differential Stiffness
Normal Modes of a 100-Cell Beam with Differential Stiffness (13-1-1)

A. Description

This problem illustrates the effects of differential stiffness on the solution for the normal modes of a beam under axial compression.

The natural frequencies of the beam are affected by this load as shown in Reference 23. The loading specified here is one half of the Euler value for compression buckling which decreases the unloaded natural frequency, w, proportional to

$$\left[\frac{\pi^2 EI}{\ell^2} - F\right]^{1/2} ,$$

where F is the applied load.

The structural model illustrated in Figure 1 is a uniform 100 cell beam hinged at both ends.

B. Input

1. Parameters:

$A = 2.0$ (cross sectional area)
$I = 0.667$ (bending inertia)
$E = 10.4 \times 10^6$ (modulus of elasticity)
$\ell = 100.0$ (length)
$\rho = 2.0 \times 10^{-4}$ (mass density)

2. Constraints:

$u_z = \theta_x = \theta_y = 0$ (all points)
$u_y = 0$ (point 101)
$u_x = u_y = 0$ (point 1)

3. Loads:

$F_{101,x} = 3{,}423.17$
$B = 1.0$ (default load factor)

C. Theory

The theoretical natural frequency for the first mode is given by

$$f = \left[\frac{1}{4\rho A\ \ell^2}\ \left(\frac{\pi^2 EI}{\ell^2} - F\right)\right]^{1/2} \text{ Hertz} \tag{1}$$

For this loading of one half the Euler buckling value, the theoretical value is 14.6269 Hertz for the bending mode.

D. Results

The natural frequency computed using NASTRAN is 14.62325 Hertz.

E. Driver Decks and Sample Bulk Data

Card No.

```
NASTRAN FILES=UMF
ID        DEM13011,NASTRAN
UMF       1977      130110
APP       DISPLACEMENT
SØL       13,0
TIME      6
CEND

TITLE = NØRMAL MØDES ANALYSIS WITH DIFFERENTIAL STIFFNESS
SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 13-1-1
          SPC = 2
    SET 1 = 11,21,31,41,51,61,71,81,91
          DISPLACEMENT = 1
          ELFØRCE = 1
SUBCASE 20
    LABEL = STATICS SØLUTIØN.
    LØAD = 100
    ØLØAD = ALL
SUBCASE 40
    LABEL = SECØND ØRDER STATICS SØLUTIØN.
    DSCØEFFICIENT = DEFAULT
SUBCASE 80
    LABEL = NØRMAL MØDES WITH DIFFERENTIAL STIFFNESS EFFECTS
    METHØD = 101
BEGIN BULK
ENDDATA
```

1	2	3	4	5	6	7	8	9	10
BARØR					.0	1.	.0	1	
CBAR	1	1	1	2					
EIGR	101	INV	.0	200.0	3	3	3	1.-4	+EIG1
+EIG1	MAX								
FØRCE1	100	101	3423.17	101	1				
GRDSET							345		
GRID	1		.0						
MAT1	22	10.4E6		.3	2.0E-4				
PBAR	1	22	2.0	.666667	.666667				
SPC	2	1	12	.0	101	2	.0		

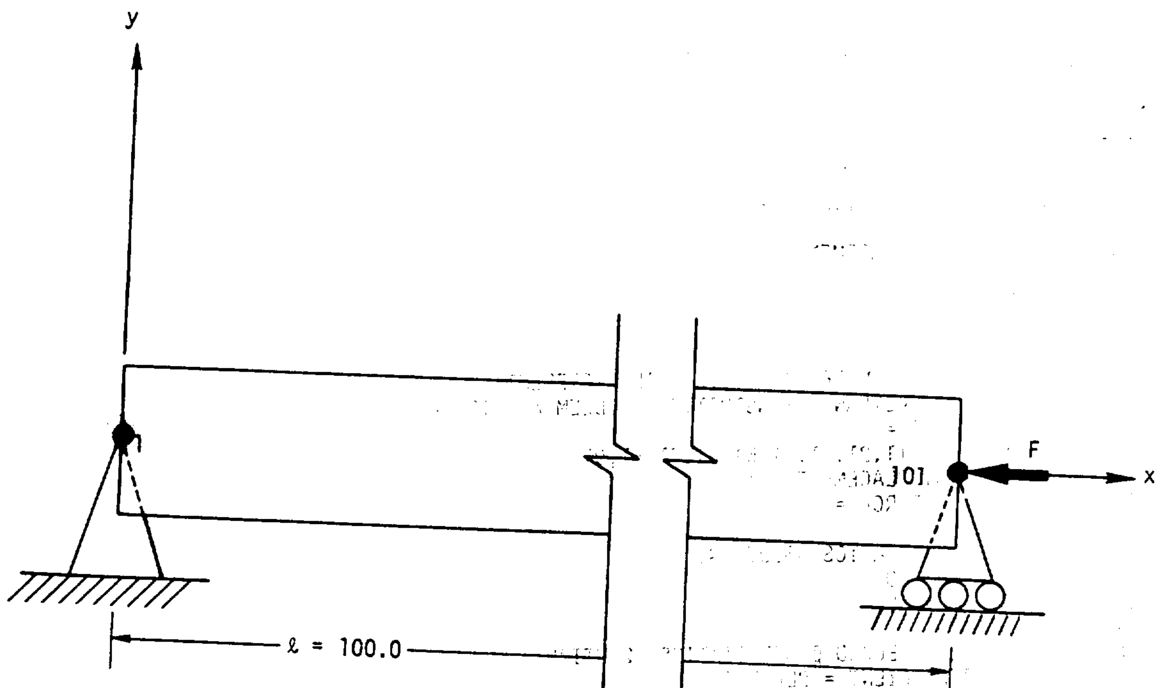

Figure 1. One hundred cell beam.

RIGID FORMAT No. 14, Static Analysis with Cyclic Symmetry
Circular Plate Using Cyclic Symmetry (14-1-1)

A. Description

A constant thickness circular plate with six radial stiffeners and a central hole, shown in Figure 1, is analyzed using dihedral symmetry. The plate is subjected to a uniform pressure load applied over a 60° segment of the plate.

The finite element model is shown in Figure 2. The stringers are 60° apart but only 30° of the structure needs to be modeled when using the dihedral symmetry option. There are 12 subcases since these are 2 half segments in a 60° segment and only one loading condition. The CYJØIN bulk data card defines those points in the middle of the segment (SIDE 2) and those points on the boundary between segments (SIDE 1).

B. Input

1. Parameters:

   R_o = 1.0 (outside radius)

   R_i = .14 (inside radius)

   t = .01 (plate thickness)

   a = .06 (height and width of stiffeners)

   E = 10.6×10^6 (modulus of elasticity)

   ν = .325 (Poisson's ratio)

2. Boundary Conditions:

   $U_r = U_\theta = \theta_z = 0$ (all points)

   $U_z = \theta_r = 0$ (along r = 1.0)

3. Applied loads:

   Pressure = 200.0 between θ = 60° and 120°

4. Cyclic symmetry parameters:

   CTYPE = DRL

   KMAX = 2

   NSEGS = 6

   NLØAD = 1

C. Results

The structure can be analyzed using rotational symmetry or dihedral symmetry described here and the results will be identical.

The results for the normal displacements are given in Table 1 for r = 0.46.

D. Driver Decks and Sample Bulk Data

```
Card
 No.

  0     NASTRAN FILES=UMF
  1     ID         DEM14011,NASTRAN
  2     UMF        1977      140110
  3     APP        DISPLACEMENT
  4     SØL        14,0
  5     TIME       5
  6     CEND

  7     TITLE = STATIC ANALYSIS ØF A CIRCULAR PLATE
  8     SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 14-1-1
  9     LABEL = DIHEDRAL CYCLIC SYMMETRY
 10     SPC = 101
 11     ØUTPUT
 12     ØLØAD = ALL
 13     DISP = ALL
 14     SPCF = ALL
 15     SUBCASE 1
 16     LABEL     = SEGMENT  1 RIGHT
 17     SUBCASE 2
 18     LABEL     = SEGMENT  1 LEFT
 19     SUBCASE 3
 20     LABEL     = SEGMENT  2 RIGHT
 21         LØAD = 102
 22     SUBCASE 4
 23     LABEL     = SEGMENT  2 LEFT
 24         LØAD = 102
 25     SUBCASE 5
 26     LABEL     = SEGMENT  3 RIGHT
 27     SUBCASE 6
 28     LABEL     = SEGMENT  3 LEFT
 29     SUBCASE 7
 30     LABEL     = SEGMENT  4 RIGHT
 31     SUBCASE 8
 32     LABEL     = SEGMENT  4 LEFT
 33     SUBCASE 9
 34     LABEL     = SEGMENT  5 RIGHT
 35     SUBCASE 10
 36     LABEL     = SEGMENT  5 LEFT
 37     SUBCASE 11
 38     LABEL     = SEGMENT  6 RIGHT
 39     SUBCASE 12
 40     LABEL     = SEGMENT  6 LEFT
 41     BEGIN BULK
 42     ENDDATA
```

1	2	3	4	5	6	7	8	9	10
CBAR	1	1	10	20	.0	.0	1.	1	
CNGRNT	10	11							
CØRD2C	1	0	.0	.0	.0	.0	.0	1.	+C1
+C1	1.	.0	.0						
CQUAD2	10	1	10	11	21	20			
CYJØIN	1	C	10	20	30	40	50	60	CYC SYM
CYJØIN	2	C	12	22	32	42	52	62	CYC SYM
GRDSET		1				1			
GRID	10		1.0	.0	.0				
MAT1	1	10.6 +6		.325	2.59 -4	12.9 -6			
PARAM	CTYPE	DRL							CYC SYM
PARAM	KMAX	2							CYC SYM
PARAM	NLØAD	1							CYC SYM
PARAM	NSEGS	6							CYC SYM
PBAR	1	1	1.8 -3	5.4 -7	5.4 -7	1.0 -6			+PB1
+PB1	.0	.03	.03	.0	.03	.03	.03	-.03	
PLØAD2	102	200.	10	20	30	40	50		
SPC1	110	12346	10	11	12				
SPCADD	101	110	112						

Table 1. Displacements of circular plate under pressure load at r = 0.46

θ	DIHEDRAL METHOD		Value
	Subcase	Grid	
0	1	30	1.365
15	1	31	1.379
30	1 2	32 32	
45	2	31	1.412
60	2 3	30 30	1.430
75	3	31	1.464
90	3 4	32 32	1.484
105	4	31	
120	4 5	30 30	1.430
135	5	31	1.412
150	5 6	32 32	1.396
165	6	31	1.379
180	6 7	30 30	1.365
195	7	31	1.359
210	7 8	32 32	1.354
225	8	31	1.349
240	8 9	30 30	1.345
255	9	31	1.344
270	9 10	32 32	1.345
285	10	31	1.344
300	10 11	30 30	1.345
315	11	31	1.349
330	11 12	32 32	1.354
345	12	31	1.359
360	12	30	1.365

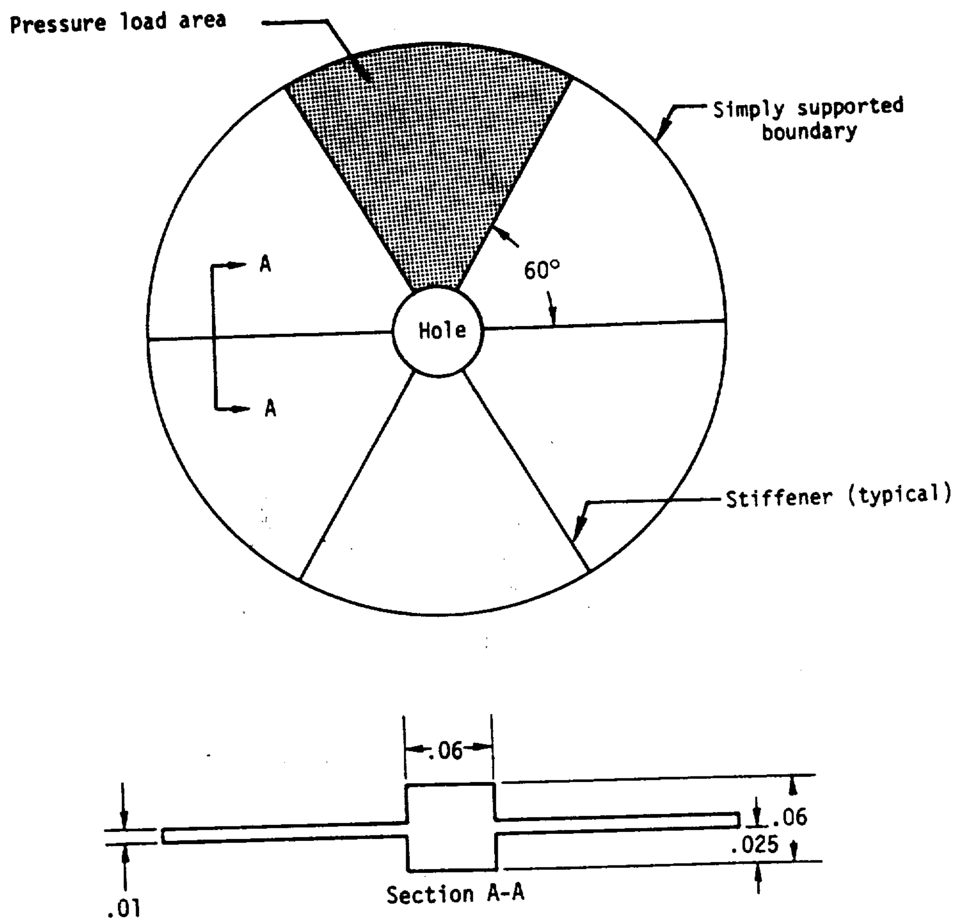

Figure 1. Circular plate with stiffeners.

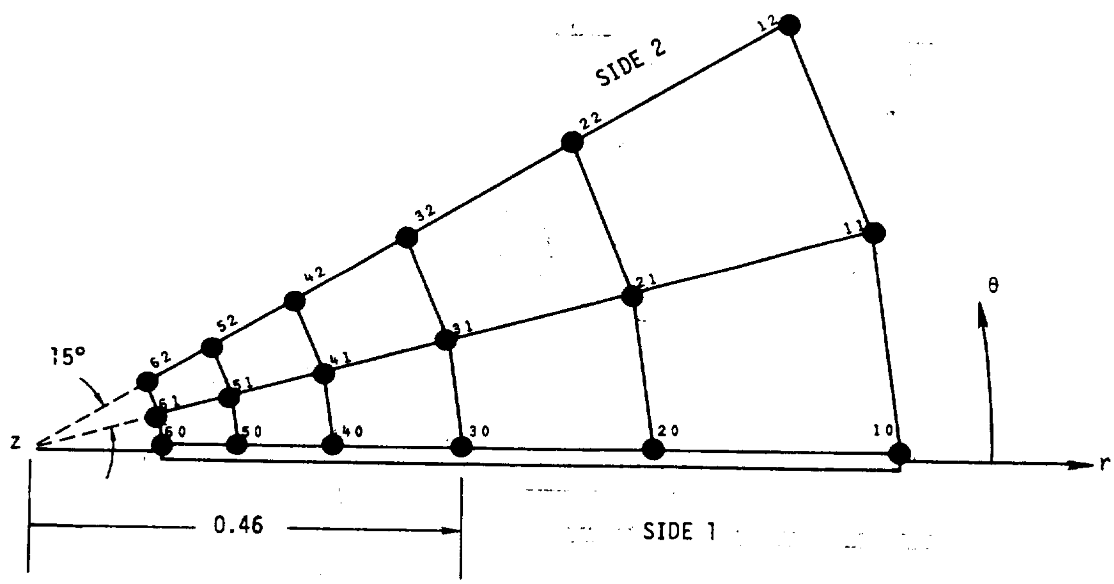

Figure 2. Finite element model.

RIGID FORMAT No. 15, Normal Modes Analysis Using Cyclic Symmetry
Modal Analysis of a Circular Plate Using Cyclic Symmetry (15-1-1)

A. Description

The natural frequencies of a constant thickness circular plate with six radial stiffeners and a central hole are obtained using the rotational symmetry option. The structure, shown in Figure 1, is simply supported at the outer circumference.

The finite element model is shown in Figure 2 representing only sixty degrees of the plate. Note that since the stiffeners are on the symmetry boundary, only 1/2 of the actual properties are used. The bulk data cards demonstrated are the CYJØIN and PARAM.

B. Input

1. Parameters:

   R_o = 1.0 (outside radius)
   R_i = .14 (inside radius)
   t = .01 (plate thickness)
   a = .06 (height and width of stiffeners)
   $E = 10.6 \times 10^6$ (modulus of elasticity)
   ν = .325 (Poisson's ratio)
   $\rho = 2.59 \times 10^{-4}$ (mass density of plate and stiffeners)

2. Boundary conditions:

   $u_r = u_\theta = \theta_z = 0$ (all points)
   $u_z = \theta_r = 0$ (along $r = 1.0$)

3. Eigenvalue extraction data:

   Method: Inverse power
   Region of interest: $0.0 \leq f \leq 8000$
   Number of desired roots: 3
   Normalization: maximum

4. Cyclic symmetry parameters:

   CTYPE RØT
   KINDEX 2
   NSEGS 6

C. <u>Results</u>

Solutions can be obtained using the dihedral symmetry or rotational symmetry described here. Results are accurate to approximately six significant figures.

Table 1. Natural Frequencies

Mode	Frequency (H_z)
1	4288.2
2	4288.2
3	6844.3
4	6844.3
5	11524.3
6	11524.3

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D. Driver Decks and Sample Bulk Data

Card No.		
1	ID	DEM15011,NASTRAN
2	UMF	1977 150110
3	APP	DISPLACEMENT
4	SØL	15,3
5	TIME	3
6	CEND	
7	TITLE = NØRMAL MØDES ANALYSIS ØF A CIRCULAR PLATE	
8	SUBTITLE = NASTRAN DEMØNSTRATIØN PRØBLEM NØ. 15-1-1	
9	LABEL = RØTATIØNAL CYCLIC SYMMETRY	
10	SPC = 101	
11	METHØD = 1	
12	ØUTPUT	
13	VECTØR = ALL	
14	BEGIN BULK	
15	ENDDATA	

1	2	3	4	5	6	7	8	9	10
CBAR	1	1	10	20	.0	.0	1.	1	
CNGRNT	1	11							
CØRD2C	1	0	.0	.0	.0	.0	.0	1.	+C1
+C1	1.	.0	.0						
CQUAD2	10	1	10	11	21	20			
CYJØIN	1	C	10	20	30	40	50	60	CYC SYM
CYJØIN	2	C	14	24	34	44	54	64	CYC SYM
EIGR	1	INV	.0	12000.0	3	3			+EIG1
+EIG1	MAX								
GRDSET		1				1			
GRID	10		1.0	.0	.0				
MAT1	1	10.6 +6		.325	2.59 -4	12.9 -6			
PARAM	CTYPE	RØT							CYC SYM
PARAM	KINDEX	2							CYC SYM
PARAM	NSEGS	6							CYC SYM
PBAR	1	1	1.8 -3	5.4 -7	5.4 -7	1.0 -6			+PB1
+PB1	.0	.03	.03	.0	.03	.03	.03	.03	
PQUAD2	1	1	.01						
SPC1	110	12346	10	THRU	14				
SPCADD	101	110	112						

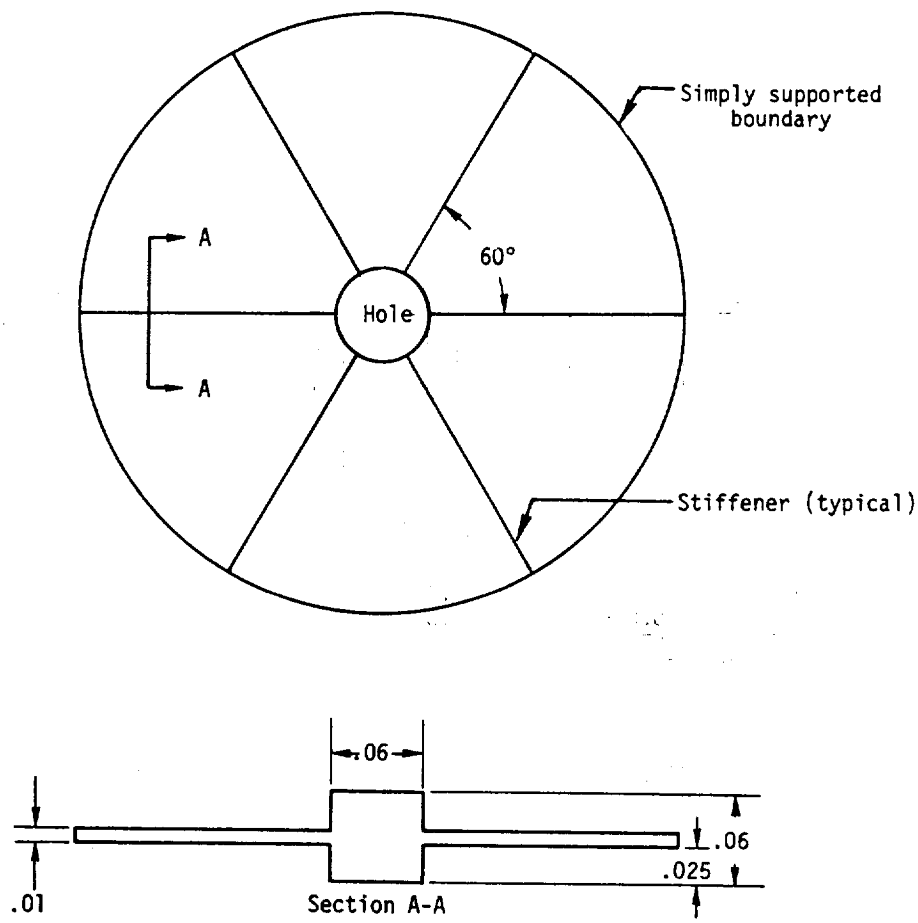

Figure 1. Circular plate with stiffeners.

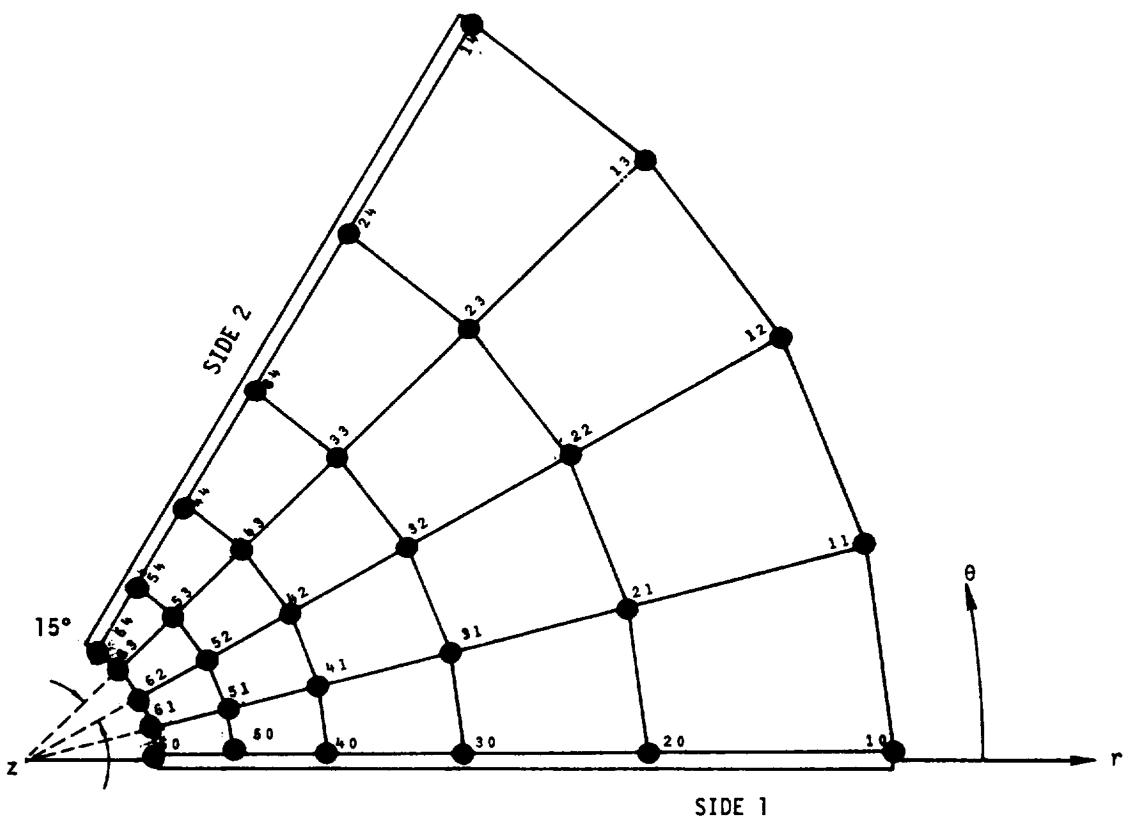

Figure 2. Finite element model.